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AN ALGEBRAIC APPROACH TO A CONCEPT OF PROPOSITION

INTRODUCTION

The analysis of natural language, resulting in the so called illocutionary logic[1] needs an adequate concept of proposition. Such a concept in the simplest form was already presented in D. Vanderveken, *What is a Proposition*[2] using model-theoretical methods. Moreover, a large philosophical background related to that concept is contained there.

In this paper, the same concept of proposition is analysed from a different point of view, using some algebraic methods.

Generally speaking, a proposition is an ordered pair, whose first element, called its „content", is a set of so called atomic propositions, and the second one, called „truth conditions" is a set of some subsets of the set of atomic propositions. The propositions form an algebra similar to some formal language, that language, for which the set of propositions is the set of senses. The analysis of that algebra results on the one side, in enlarging the notion of „strong implication"[3] to the notion of special consequence relation, on the other side, in some representation of that algebra and conceiving a proposition in a new way.

[1] Cf. D. Vanderveken, *Meaning and speech acts*, Vol. 1–2, Cambridge University Press, 1990.

[2] Cf. D. Vanderveken, *What Is a Proposition*, „Cahiers d'épistémologie" 1991, No. 9103, Université du Québec à Montréal.

[3] *Ibid.*

I. THE ALGEBRA OF PROPOSITIONS

Let U be any non-empty set of objects and I be any non-empty set of indices or points, which represent possible worlds or contexts of utterances. Then following Carnap, U^I (the set of all functions C: $I \to U$) is the set of individual concepts, and for any n = 1, 2, ..., $(\wp(U^n))^I$ (the set of all functions R_n: $I \to \wp(U^n)$) is the set of n-ary relations in intension or simply attributes.

First we define the set U_a of the so called atomic propositions. An *atomic proposition* $u \in U_a$ is an ordered pair, whose first element is the union of two sets: one-element set containing any single attribute, and finite set of individual concepts; the second element of an atomic proposition is some subset of the set I, as follows:

$$U_a = \{\langle\{R_n, C_1, ..., C_n\}, \{i \in I: \langle C_1(i), ..., C_n(i)\rangle \in R_n(i)\}\rangle : R_n \in (\wp(U^n))^I,\ C_1, ..., C_n \in U^I,\ n = 1, 2, ... \}.$$

Now we can define inductively the set of *propositions* as the smallest subset U_p of the set $\wp(U_a) \times \wp(\wp(U_a))$ satisfying the following conditions:

(i) $\{\langle\{u\}, [\{u\}]\rangle : u \in U_a\} \subseteq U_p$, where for any $W \subseteq U_a$, $[W] = \{W' \in \wp(U_a): W \subseteq W'\}$;

(ii) for any $P \in U_p$, $\langle id_1(P), \wp(U_a) - id_2(P)\rangle \in U_p$;

(iii) for any P, $Q \in U_p$, $\langle id_1(P) \cup id_1(Q), id_2(P) \cap id_2(Q)\rangle \in U_p$, where for any $\langle A, B\rangle \in \wp(U_a) \times \wp(\wp(U_a))$, $id_1(\langle A, B\rangle) = A$, $id_2(\langle A, B\rangle) = B$.

Now, the algebra $\mathbf{U}_p = (U_p, \neg, \wedge, \vee, \to)$ generated by the set $\{(u): u \in U_a\}$, where for any P, $Q \in U_p$:

$\neg P = \langle id_1(P), \wp(U_a) - id_2(P)\rangle$,

$P \wedge Q = \langle id_1(P) \cup id_1(Q), id_2(P) \cap id_2(Q)\rangle$,

$P \vee Q = \neg(\neg P \wedge \neg Q) = \langle id_1(P) \cup id_1(Q), id_2(P) \cup id_2(Q)\rangle$,

$P \to Q = \neg P \vee Q = \langle id_1(P) \cup id_1(Q), (\wp(U_a) - id_2(P)) \cup id_2(Q)\rangle$,

and for any $u \in U_a$, $(u) = \langle\{u\}, [\{u\}]\rangle$, will be said to be *the algebra of propositions*.

It is seen that any element P of the algebra of propositions i.e. a proposition P is an ordered pair, whose first element is the finite non-empty subset of the set of atomic propositions U_a; it will be called in the sequel as *the content of the proposition* P, the second element of that pair is a subset of $\wp(U_a)$ which will be called as *the truth conditions of the proposition* P.

2. LANGUAGE AND ITS INTERPRETATIONS

It is well known[4] that a proposition should be considered simultaneously as a constituent of a conceptual thought (independently on the language) and as a sense of a sentence – that sentence, which expresses that thought. In § 1 we have just tried to give a formal concept of a proposition independently on the language, now we can try to describe a proposition as a sense of a sentence. To that aim we choose a special formal language such that the set U_p of propositions would be the set of senses of the formulas of that language. The language is a part of the usual first-order language – without quantifiers, individual variables and functional symbols.

Let Const and Pred be the set of individual constants and predicate symbols respectively. By *the language* we will understand the algebra $\underline{L} = (L, \neg, \wedge, \vee, \rightarrow)$ freely generated by the set At of free generators of the form: $r_n(c_1, ..., c_n)$, where r_n is n-ary predicate symbol and $c_1, ..., c_n$ are individual constants, $n = 1, 2, ...$

By *the interpreting function of the language* $\underline{L}$ we understand an assignment

$$s: \text{Const} \cup \text{Pred} \cup \text{At} \rightarrow U^I \cup \bigcup \{(\Phi(U^n))^I: n = 1, 2, ...\} \cup U_a$$

such that for any $c, c_1, ..., c_n \in$ Const, $r_n \in$ Pred:

$s(c) \in U^I$, $s(r_n) \in (\Phi(U^n))^I$, $s(r_n(c_1, ..., c_n)) = \langle\{s(r_n), s(c_1), ..., s(c_n)\}, \{i \in I: \langle s(c_1)(i), ..., s(c_n)(i)\rangle \in s(r_n)(i)\}\rangle$.

Taking into account the homomorphism $h_s: \underline{L} \rightarrow \underline{U}_p$ defined as follows: for any $A \in At$, $h_s(A) = (s(A))$, we can say that for any $\alpha \in L$, the proposition $h_s(\alpha)$ is *the sense of sentence* α *with respect to s.*

3. A CHARACTERIZATION OF THE CONTENT AND OF THE TRUTH CONDITIONS OF A PROPOSITION

In order to characterize the content of a proposition let us introduce the obvious definition of the occurrence of an atomic proposition in a proposition, as follows: for any atomic proposition v:

(1) v occurs in (u) iff v = u,

(2) v occurs in $\neg$P iff v occurs in P,

(3) v occurs in $P \wedge Q$ iff v occurs in P or v occurs in Q.

Then: for any $P \in U_p$, $id_1(P)$ is the set of all atomic propositions occurring in P.

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[4] Cf. for instance: *ibid.*

Notice that the content of any proposition is always a finite non-empty subset of U_a.

To the aim of characterizing the truth conditions we will use the concept of proposition as a sence of sentence.

For any interpreting function s consider the function $g_s: \mathcal{P}(U_a) \to \{0, 1\}^L$, where $\{0,1\}$ is the set of truth-values, as follows: for any $W \in \mathcal{P}(U_a)$, $g_s(W): L \to \{0,1\}$ is the classically admissible valuation on L such that for any $A \in At$,

$g_s(W)(A) = 1$ iff $s(A) \in W$.

Lemma 3.1. For any interpreting function s, for any $\alpha \in L$ and $W \subseteq U_a$: $W \in id_2(h_s(\alpha))$ iff $g_s(W)(\alpha) = 1$.

Proof (induction on the length of α). Let s be any fixed interpreting function of L and $W \subseteq U_a$.

1. Let $\alpha \in At$. Then $h_s(\alpha) = (s(\alpha))$ and consequently $W \in id_2(h_s(\alpha))$ iff $W \in [\{s(\alpha)\})$ iff $s(\alpha) \in W$ iff $g_s(W)(\alpha) = 1$.

2. Let α be of the form: $\neg\beta$, where $\beta \in L$ is such that

(*) $W \in id_2(h_s(\beta))$ iff $g_s(W)(\beta) = 1$.

Then $W \in id_2(h_s(\neg\beta))$ iff $W \in id_2(\neg h_s(\beta))$ iff $W \notin id_2(h_s(\beta))$ iff $g_s(W)(\beta) = 0$ iff $g_s(W)(\neg\beta) = 1$ by (*) and the fact that $g_s(W)$ is classically admissible.

3. Let α be of the form: $\beta \wedge \gamma$, where (*) for β and for γ is assumed. Then $W \in id_2(h_s(\beta \wedge \gamma))$ iff $W \in id_2(h_s(\beta) \wedge h_s(\gamma))$ iff $W \in id_2(h_s(\beta)) \cap id_2(h_s(\gamma))$ iff $g_s(W)(\beta) = g_s(W)(\gamma) = 1$ iff $g_s(W)(\beta \wedge \gamma) = 1$. ■

Lemma 3.1 enables to give a simple characterization of the set $id_2(P)$ for any proposition P. Indeed, for given P one can choose the formula α and the interpreting function s such that $P = h_s(\alpha)$. So if for instance we consider the proposition P of the form: $(\neg(u_1) \wedge (u_2)) \to (u_1)$, then we should take into account the formula $(\neg A_1 \wedge A_2) \to A_1$, $A_1, A_2 \in At$, and the interpreting function s such that $s(A_i) = u_i$, $i = 1, 2$. Then $id_2(P)$ is the family of all $W \subseteq U_a$ such that the functions $g_s(W)$ associated with W form the set of all classically admissible valuations on L which take the value 1 on the formula $(\neg A_1 \wedge A_2) \to A_1$.

4. SOME PROPERTIES OF A PROPOSITION

First of all we should define when a proposition P is true or false. If we consider a proposition of the simplest form:

$(<\{R_n, C_1, ..., C_n\}, \{i \in I: <C_1(i), ..., C_n(i)> \in R_n(i)\}>)$, we can obviously say that it is true in a point $i \in I$ iff $<C_1(i), ..., C_n(i)> \in R_n(i)$.

Taking into account the classical way of defining the truth for the propositions of the form: $\neg P$ and $P \wedge Q$ we obtain the following definition:

(i) for any $u \in U_a$, (u) is true in i iff $i \in id_2(u)$,

(ii) for any $P \in U_p$, $\neg P$ is true in i iff P is false in i,

(iii) for any $P, Q \in U_p$, $P \wedge Q$ is true in i iff P and Q are true in i.

However we should connect the fact that a proposition is true or false with its truth conditions. The following Lemma establishes such connection:

Lemma 4.1. Let for any $i \in I$, $U_a^i = \{u \in U_a: i \in id_2(u)\}$. Then for any proposition P, P is true in $i \in I$ iff $U_a^i \in id_2(P)$.

P r o o f. Straightforward by induction concerning on the form of a proposition P. ∎

We can introduce another important properties of a proposition as follows:

– a proposition P is said to be a *tautology* iff $id_2(P) = \mathcal{P}(U_a)$;

– P is a *contradictory proposition* iff $id_2(P) = \emptyset$;

– P is a *necessary proposition* iff for each $i \in I$, P is true in i;

– P is an *impossible proposition* iff for each $i \in I$, P is false in i.

According to Lemma 4.1 it is easily seen that any tautology is a necessary proposition, but not conversely, and similarly any contradictory proposition is always impossible, although not conversely.

5. THE CONSEQUENCE RELATIONS ON THE SET OF PROPOSITIONS

Now we intend to define two concepts of consequence relations on the set of propositions: one of them, called „strict" or simply „usual" consequence relation (it is related to the connective of strict implication, so we use the term „strict") although defined, let say, in the natural way, is not realized from the point of view of human being carrying out the practical reasonings; the second consequence relation, called „strong", posesses such properties that it can be taken as a formal ground of the practical reasonings.

Let $\Gamma \subseteq U_p$ and $P \in U_p$. We will say that Γ *strictly entails* P ($\Gamma \dashv\!\!\in P$ in symbols) iff for any $i \in I$, P is true in i whenever each $Q \in \Gamma$ is true in i.

In that way we have for instance: $\{P\} \dashv\!\!\in P \vee Q$, which is not good from the point of view of practical reasoning.

The strong consequence relation is closely related to the algebraic structure of the set of propositions. So first we will start from some properties of the algebra $\underline{U}_p$.

Lemma 5.1. For any equality σ in the signature $(\neg, \wedge, \vee, \rightarrow)$, σ is an equality in the algebra $\underline{U}_p$ iff σ is a Boolean equality and the set of variables occurring in the left term of σ is identical with the set of variables occurring in the right term.

P r o o f. Assume that we have the following variables: $x_0, x_1, \ldots$, and let σ be of the form: $f(x_{i_1}, \ldots, x_{i_n}) = g(x_{j_1}, \ldots, x_{j_m})$, where $x_{i_1}, \ldots, x_{i_n}$ ($x_{j_1}, \ldots, x_{j_m}$) are all the different variables occurring in the term $f(x_{i_1}, \ldots, x_{i_n})$ ($g(x_{j_1}, \ldots, x_{j_m})$).

($\Rightarrow$): Assume that σ holds in the algebra $\underline{U}_p$. First suppose that $\{x_{i_1}, \ldots, x_{i_n}\} \neq \{x_{j_1}, \ldots, x_{j_m}\}$. Let $x_{i_k} \notin \{x_{j_1}, \ldots, x_{j_m}\}$ for some $k \in \{1, \ldots, n\}$. Notice that according to the assumption, for any propositions $P_1, \ldots, P_n, Q_1, \ldots, Q_m$, $id_1(f(P_1, \ldots, P_n)) = id_1(g(Q_1, \ldots, Q_m))$, which implies that $id_1(P_1) \cup \ldots \cup id_1(P_n) = id_1(Q_1) \cup \ldots \cup id_1(Q_m)$. Thus, substituting: $x_{j_\ell} \mapsto P$ for any $\ell = 1, \ldots, m$, $x_{i_\ell} \mapsto P$ for any $\ell = 1, \ldots, n$, $\ell \neq k$, where P is any proposition, and $x_{i_k} \mapsto Q$, where Q is such that $id_1(Q) \nsubseteq id_1(P)$, we obtain that $id_1(P) \cup id_1(Q) = id_1(P)$, which is impossible. Analogously if $\{x_{j_1}, \ldots, x_{j_m}\} \nsubseteq \{x_{i_1}, \ldots, x_{i_n}\}$.

In order to show that σ must be Boolean equality, notice that for any propositions $P_1, \ldots, P_n$: $id_2(f(P_1, \ldots, P_n)) = f'(id_2(P_1), \ldots, id_2(P_n))$ for any function f of n variables in the signature $(\neg, \wedge, \vee, \rightarrow)$, where f' is like f but set-theoretical operation. So the equality: $f(x_1, \ldots, x_n) = g(x_1, \ldots, x_n)$ holds in $\underline{U}_p$ iff it is Boolean.

($\Leftarrow$): by the last argument of the proof ($\Rightarrow$). ■

Following Lemma 5.1, the equalities:

$x \wedge x = x$,

$x \wedge y = y \wedge x$,

$x \wedge (y \wedge z) = (x \wedge y) \wedge z$,

are satisfied in $\underline{U}_p$, so we can consider the reduct $(U_p, \wedge)$ of $\underline{U}_p$ as a meet-semilattice.

We shall say that for any $\emptyset \neq \Gamma \subseteq U_p$, $P \in U_p$, Γ *strongly entails P* ($\Gamma \mapsto\!\!\!\!\in P$ in symbols) iff $P \in [\Gamma)$, where $[\Gamma)$ is the filter generated in the semilattice $(U_p, \wedge)$ by the set Γ. We also put $\{P \in U_p: \emptyset \mapsto\!\!\!\!\in P\} = \emptyset$.

The following obvious lemma explains strong consequence relation in terms of the content and of the truth conditions:

Lemma 5.2. For any $\emptyset \neq \Gamma \subseteq U_p$, $P \in U_p$: $\Gamma \mapsto\!\!\!\!\in P$ iff there exists $\{P_1, \ldots, P_n\} \subseteq \Gamma$ such that

$id_1(P) \subseteq id_1(P_1) \cup \ldots \cup id_1(P_n)$ and

$id_2(P_1) \cap \ldots \cap id_2(P_n) \subseteq id_2(P)$.

P r o o f. Notice that for any $\emptyset \neq \Gamma \subseteq U_p$, $P \in U_p$, $P \in [\Gamma)$ iff $P_1 \wedge \ldots, \wedge P_n \leqslant P$ for some $P_1, \ldots, P_n \in \Gamma$, where $\leqslant$ is the partial ordering of the semilattice $(U_p, \wedge)$, i.e. it is defined as follows: for any $P, Q \in U_p$: $P \leqslant Q$ iff $P \wedge Q = P$ iff $id_1(Q) \subseteq id_1(P)$ & $id_2(P) \subseteq id_2(Q)$. ■

One can show using Lemmas 4.1 and 5.2 that for any $\emptyset \neq \Gamma \subseteq U_p$, $P \in U_p$, $\Gamma \mapsto\!\!\!\!\in P$ implies that $\Gamma \dashv\!\!\!\!\in P$, but not conversely; for instance in general $\{P\} \mapsto\!\!\!\!\in P \vee Q$ does not hold.

6. A REPRESENTATION OF PROPOSITIONS

Now we are going to give another but equivalential to just presented, an algebraic approach to the concept of proposition. A proposition will be conceived less intuitively but its structure will turn out more simple – we would be able to identify a proposition with an ordered pair consisted of two finite sets.

Let us introduce for any non-empty and finite set $W \subseteq U_a$ the following equivalence relation on the set $\mathcal{P}(U_a)$: for any $V, V' \in \mathcal{P}(U_a)$, $V \equiv V'(W)$ iff $W \cap V = W \cap V'$.

We will need the following lemma concerning with the propositions:

Lemma 6.1. For any proposition P and any $W \in id_2(P)$: $[W]_{id_1(P)} \subseteq id_2(P)$, where for any finite $\emptyset \neq V \subseteq U_a$ and any $W \subseteq U_a$, $[W]_V = \{W' \subseteq U_a: W \equiv W'(V)\}$.

P r o o f. Straightforward by induction on the length of a proposition P. ∎

We will use Lemma 6.1 in the proof of the following:

Lemma 6.2. For any finite $\emptyset \neq W \subseteq U_a$ and any $\mathcal{W} \subseteq \mathcal{P}(W)$: $< W, \bigcup \{[W']_W: W' \in \mathcal{W}\} > \in U_p$.

P r o o f. Let $W = \{u_1, ..., u_n\} \subseteq U_a$ and $\mathcal{W} = \{W_1, ..., W_k\}$, $0 \leqslant k \leqslant 2^n$, be any family of subsets of the set W.

1. Let $k > 0$, that is $\mathcal{W} \neq \emptyset$. For any $j = 1, ..., k$ let $W_j = \{u^j_1, ..., u^j_{f(j)}\}$, $W - W_j = \{u^j_{f(j)+1}, ..., u^j_n\}$, where $f(j) \in \{0, 1, ..., n\}$ (in case when $f(j) = 0$, $W_j = \emptyset$ and similarly when $f(j) = n$, $W - W_j = \emptyset$).

We show that: $<\{u_1, ..., u_n\}, [W_1]_W \cup ... \cup [W_k]_W> = ((u^1_1) \wedge ... \wedge (u^1_{f(1)}) \wedge \neg (u^1_{f(1)+1}) \wedge ... \wedge \neg (u^1_n)) \vee ... \vee ((u^k_1) \wedge ... \wedge (u^k_{f(k)}) \wedge \neg (u^k_{f(k)+1}) \wedge ... \wedge \neg (u^k_n))$.

Denote the last proposition as P_0. It is obvious that $id_1((u^j_1) \wedge ... \wedge (u^j_{f(j)}) \wedge \neg (u^j_{f(j)+1}) \wedge ... \wedge \neg (u^j_n)) = \{u_1, ..., u_n\}$, any $j = 1, ..., k$, which means that $id_1(P_0) = \{u_1, ..., u_n\}$. Next notice that $W_j \in id_2((u^j_1) \wedge ... \wedge (u^j_{f(j)}) \wedge \neg (u^j_{f(j)+1}) \wedge ... \wedge \neg (u^j_n))$, which implies that for any $j = 1, ..., k$, $W_j \in id_2(P_0)$. So let $V \in [W_1]_W \cup ... \cup [W_k]_W$, then $W_j \equiv V(W)$ for some j, hence due to Lemma 6.1: $V \in id_2(P_0)$. And conversely, if $V \in id_2(P_0)$, then $V \in id_2((u^j_1) \wedge ... \wedge (u^j_{f(j)}) \wedge \neg (u^j_{f(j)+1}) \wedge ... \wedge \neg (u^j_n))$ for some j, which implies that $W_j \cap V = W_j$ and $(W - W_j) \cap V = \emptyset$, so $W \cap V = (W_j \cup (W - W_j)) \cap V = (W_j \cap V) \cup ((W - W_j) \cap V) = W_j = W \cap W_j$, that is $V \in [W_j]_W \subseteq [W_1]_W \cup ... \cup [W_k]_W$. Finally: $[W_1]_W \cup ... \cup [W_k]_W = id_2(P_0)$.

2. Let $k = 0$, that is $\mathcal{W} = \emptyset$. It is obvious that in case:

$<\{u_1, ..., u_n\}, \emptyset> = ((u_1) \wedge \neg (u_1)) \wedge (u_2) \wedge (u_3) \wedge ... \wedge (u_n)$. ∎

Now consider the following algebra $\underline{V}_p$ similar to $\underline{U}_p$:

$\underline{V}_p = (V_p, \neg, \wedge, \vee, \rightarrow)$

where $V_p = \{<W,\mathcal{W}>: W \in \mathcal{P}_{fin}(U_a), \mathcal{W} \subseteq \mathcal{P}(W)\}$, $\mathcal{P}_{fin}(U_a)$ is the family of all non-empty and finite subsets of U_a, and for any $<W_1,\mathcal{W}_1>$, $<W_2,\mathcal{W}_2> \in V_p$:

$\neg <W_1,\mathcal{W}_1> = <W_1, \mathcal{P}(W_1) - \mathcal{W}_1>$,

$<W_1,\mathcal{W}_1> \wedge <W_2,\mathcal{W}_2> = <W_1 \cup W_2, \{V_1 \cup V_2 : V_1 \in \mathcal{W}_1, V_2 \in \mathcal{W}_2, V_1 \cap W_2 = V_2 \cap W_1\}>$,

$<W_1,\mathcal{W}_1> \vee <W_2,\mathcal{W}_2> = \neg(\neg <W_1,\mathcal{W}_1> \wedge \neg <W_2,\mathcal{W}_2>)$,

$<W_1,\mathcal{W}_1> \rightarrow <W_2,\mathcal{W}_2> = \neg <W_1,\mathcal{W}_1> \vee <W_2,\mathcal{W}_2>$.

Theorem 6.3. The algebras $\underline{U}_p$, $\underline{V}_p$ are isomorphic.

P r o o f. We show that the function g: $\underline{U}_p \rightarrow \underline{V}_p$ defined as follows: for any $P \in U_p$, $g(P) = <id_1(P), \{W \cap id_1(P) : W \in id_2(P)\}>$, is the required isomorphism.

Following Lemma 6.2 we can consider the function f: $V_p \rightarrow U_p$ defined as follows: for any $<W,\mathcal{W}> \in V_p$,

$f(<W,\mathcal{W}>) = <W, \bigcup\{[W']_W: W' \in \mathcal{W}\}> = <W, \{V \subseteq U_a: V \cap W \in \mathcal{W}\}>$.

Then for any $<W,\mathcal{W}> \in V_p$, $g(f(<W,\mathcal{W}>)) = <W, \{V \cap W : V \in \{V' \subseteq U_a: V' \cap W \in \mathcal{W}\}\}> = <W,\mathcal{W}>$.

Moreover, for any $P \in U_p$, $f(g(P)) = <id_1(P), \bigcup\{[W']_{id_1(P)}: W' \in \{W \cap id_1(P) : W \in id_2(P)\}\}> = <id_1(P), \bigcup\{[W]_{id_1(P)}: W \in id_2(P)\}>$. According to Lemma 6.1, $\bigcup\{[W]_{id_1(P)}: W \in id_2(P)\} \subseteq id_2(P)$, the converse inclusion is obvious, so $f(g(P)) = P$.

Thus the function g is 1-1 and onto. In order to show that g preserves the operation $\neg$ notice that:

(1) $\mathcal{P}(id_1(P)) = \{W \cap id_1(P): W \in id_2(P)\} \cup \{W \cap id_1(P): W \notin id_2(P)\}$,

and

(2) $\{W \cap id_1(P): W \in id_2(P)\} \cap \{W \cap id_1(P): W \notin id_2(P)\} = \emptyset$, any $P \in U_p$ (in order to prove (2) suppose that it does not hold; then there exist $W_1 \in id_2(P)$, $W_2 \notin id_2(P)$ such that $W_1 \equiv W_2(id_1(P))$, so by Lemma 6.1 we obtain a contradiction).

In that way we have for any $P \in U_p$:

$g(\neg P) = <id_1(\neg P), \{W \cap id_1(\neg P): W \in id_2(\neg P)\}> = <id_1(P), \{W \cap id_1(P): W \notin id_2(P)\}> = <id_1(P), \mathcal{P}(id_1(P)) - \{W \cap id_1(P): W \in id_2(P)\}> = \neg g(P)$, due to (1) and (2).

And further, for any $P,Q \in U_p$ we have:

$g(P \wedge Q) = <id_1(P \wedge Q), \{W \cap id_1(P \wedge Q): W \in id_2(P \wedge Q)\}> =$
$= <id_1(P) \cup id_1(Q), \{(W \cap id_1(P)) \cup (W \cap id_1(Q)) : W \in id_2(P) \& W \in id_2(Q)\}>$. But obviously the following inclusion holds:

$\{(W \cap id_1(P)) \cup (W \cap id_1(Q)) : W \in id_2(P) \& W \in id_2(Q)\} \subseteq$
$\subseteq \{V_1 \cup V_2 : V_1 \in \{W \cap id_1(P): W \in id_2(P)\} \&$
$V_2 \in \{W \cap id_1(Q): W \in id_2(Q)\} \& V_1 \cap id_1(Q) = V_2 \cap id_1(P)\}$.

And in order to show the converse inclusion notice that for any $W_1 \in id_2(P)$, $W_2 \in id_2(Q)$:

(3) $W_1 \cap id_1(P) = ((W_1 \cap id_1(P)) \cup (W_2 \cap id_1(Q))) \cap id_1(P)$, and

(4) $W_2 \cap id_1(Q) = ((W_1 \cap id_1(P)) \cup (W_2 \cap id_1(Q))) \cap id_1(Q)$, whenever $(W_1 \cap id_1(P)) \cap id_1(Q) = (W_2 \cap id_1(Q)) \cap id_1(P)$. Further put $W = (W_1 \cap id_1(P)) \cup (W_2 \cap id_1(Q))$. According to Lemma 6.1, from (3) and (4) we obtain that $W \in id_2(P)$ and $W \in id_2(Q)$. Thus $g(P \wedge Q) = \langle id_1(P) \cup id_1(Q), \{V_1 \cup V_2 : V_1 \in \{W \cap id_1(P): W \in id_2(P)\} \;\&\; V_2 \in \{W \cap id_1(Q): W \in id_2(Q)\} \;\&\; V_1 \cap id_1(Q) = V_2 \cap id_1(P)\}\rangle = \langle id_1(P), \{W \cap id_1(P): W \in id_2(P)\}\rangle \wedge \langle id_1(Q), \{W \cap id_1(Q): W \in id_2(Q)\}\rangle = g(P) \wedge g(Q)$. ■

Obviously we can treat a proposition as an ordered pair of the form $\langle W, \mathcal{W}\rangle$. One can express all the properties of the propositions and of the consequence relations in the new way, for instance, $\langle W, \mathcal{W}\rangle$ is a tautological (contradictory) proposition iff $\mathcal{W} = \mathcal{P}(W)$ ($\mathcal{W} = \emptyset$); for any $\langle W_1, \mathcal{W}_1\rangle$, $\langle W_2, \mathcal{W}_2\rangle \in V_p$, $\{\langle W_1, \mathcal{W}_1\rangle\} \vdash\!\!\!\!\prec \langle W_2, \mathcal{W}_2\rangle$ iff $W_2 \subseteq W_1$ & $\{V \cap W_2 : V \in \mathcal{W}_1\} \subseteq \mathcal{W}_2$ etc.

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ALGEBRAICZNE UJĘCIE POJĘCIA „PROPOSITION"

W artykule analizuje się pojęcie „proposition" (sądu w sensie logicznym) wprowadzone w pracy D. Vandervekena *What Is a Proposition*, stosując metody algebraiczne. Analiza ta umożliwia głębsze zrozumienie tego pojęcia, prowadzi m. in. do uogólnienia pojęcia „mocnej implikacji" (§5), jej głównym rezultatem jest pewna reprezentacja pojęcia „proposition" (§ 6).