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POWERFUL PARACONSISTENT LOGIC

Coming to Łódź we have had a friendly discussion in the train. Imagine that there was a little green logician in the our compartment too, who wrote down anything we said. So he protocoled e.g. my assertion „To come to Łódź, the train will go through Kunowice'' as well as the opposite opinion, namely „You must be stupid! By no means it will take that route". As the little green man was a logician, he feels uncomfortable: on his sheet he wrote down a sentence H together with its negation *non* H. In two-valued extensional logic one of them must be false. Whence, according to the classical law *ex falso quodlibet* anything follows from H or from *non* H. Therefore the little green logician decided to stop protocolling the further discussion because of its absurdity.

Was he right to do so? Perhaps not. The quarrel about the route produced an inconsistent situation in which opposite claims were uttered, though it did not result in a so called overfilled situation (that means in a situation in which any uttered sentence would be accepted as true). The behavior of all participants in the discussion was notwithstanding (more or less) rational: we would be very surprised indeed if after that any of us would claim e.g. that both uttered sentences are true or that Kunowice is the very same town as Łódź.

In my opinion, logic should – at least to some extent – handle with complex argumentations of rational speakers. Logic should formalize appropriate fragments of natural language (e.g. the language of empirical theories) and investigate the formal counterparts thus obtained rather than elaborate sophisticated regulations about how to use our language „correctly".

Naturally, it is not easy to specify, what is a rational speaker – or more general: what kinds of natural language texts are suitable for logical investigation. But never mind, for the moment we just suppose that some of them are in fact inconsistent.

There are several attempts to formalize inconsistent situations, that means to construct logical calculi which support inferences from inconsistent sets of premises.

One of those calculi, the system D_2 of so called „discussive logic", was introduced by Stanisław Jaśkowski[1]. He noted a distinction between two properties of a logical calculus, which are usually not discerned within classical logic:

Def. 1: A calculus (**FOR**, Cn) is called **inconsistent**, iff for some $H \in$ **FOR**: $H \in Cn(\emptyset)$ & $\neg H \in Cn(\emptyset)$.

Def. 2: A calculus (**FOR**, Cn) is called **overfilled** (or **trivial**) iff $Cn(\emptyset) =$ **FOR**.

Jaśkowski's aim was the construction of a sentential calculus which meets three conditions: 1) when applied to inconsistent systems it would not always entail their triviality, 2) it would be rich enough to enable practical inference, 3) it would have an intuitive justification.

Theories which are inconsistent but not trivial are called „paraconsistent" (the name was proposed by Miró Quesada, it means „beyond the consistent").

One can say that Jaśkowski not only constructed the first formal paraconsistent system in history but that he made available the metatheoretical background to handle the phenomenon „paraconsistency" formally.

The point of his construction is this: to accept a sentence means to claim its validity, but with hidden restrictions: „someone of the participants (in the discussion) claims that H is true" or „H is true, provided that the terms are used according to some of the admissible meanings" or something like that. Instead of the usual „H is true" we have henceforth „it is possibly true that H" with reference to some concept of possibility.

Jaśkowski decided (not very fortunately, perhaps) to take the Lewis system *S5* as modal basis of his construction.

Very roughly, his original definition can be restated as follows: let $\mathbf{FOR}_d$ be the set of all formulas built up from a denumerable set of propositional variables by means of some boolean complete set of functors and two additional two-argument „discussive" connectives: discussive conjuction $\&_d$ and discussive implication $\gg_d$. Next, let t be a translation from $\mathbf{FOR}_d$ into the modal language. t leaves propositional variables unchanged as well as all

[1] S. Jaśkowski, *Rachunek zdań dla systemów dedukcyjnych sprzecznych*, „Studia Societatis Scientiarum Torunensis" 1948, sec. A, nr 5, p. 57–77.

boolean connectives. For **M** beeing the *S5*-possibility we set: $t(\text{H} \,\&_{\text{d}}\, \text{G}) \equiv_{\text{df}} t(\text{H}) \,\&\, \mathbf{M}t(\text{G})$ and $t(\text{H} \gg_{\text{d}} \text{G}) \equiv_{\text{df}} \mathbf{M}t(\text{H}) \gg t(\text{G})$. (Don't worry about the motivation of that somewhat strange „inclined" functors, other definitions are possible.) Now we are able to define:

$$D_2 =_{\text{df}} \{\text{H} \in \mathbf{FOR}_{\text{d}}\colon \mathbf{M}t(\text{H}) \in S5\}.$$

Jaśkowski's idea can be generalized into several directions: it is possible to use a large class of modal systems, among them even non-normal calculi, to obtain interesting discussive systems. Further, it seems more natural to take in discussive logic as a logical calculus (i.e. a consequence operation in a formal language) rather than as a set of formulas. For each modal logic *S* containing *S3* in *Parakonsistenz in schwachen Modalkalkülen* we explained a consequence operation Cns in the discussive language $\mathbf{FOR}_{\text{d}}$ and gave a direct semantical characterization for the systems D_s thus obtained[2].

Fact 1: $\text{Cn}_{S5}(\emptyset) = D_2$
Fact 2: $\forall S \,\forall \text{X} \subseteq \mathbf{FOR}_{\text{d}} \,\forall \text{H}, \text{F} \in \mathbf{FOR}_{\text{d}}\colon \text{H} \gg_{\text{d}} \text{F} \in \text{Cn}_S(\text{X}) \Longleftrightarrow \text{F} \in \text{Cn}_S(\text{X} \cup \{\text{H}\})$

Usually we interpret the fact, that the deduction theorem holds in a system as a property of the regarded implication. But now we go the other way round: we already know, that the discussive implication posesses a lot of properties expected of a discussive inference. By deduction theorem they are induced to the consequence relation. Moreover, the original system D_2 of Jaśkowski belongs to the class of systems D_s obtained in the above construction.

Therefore we call D_s the class of **discussive Jaśkowski systems**. Each of them generates a class of higher-degree discussive systems. Surprising enough: for normal *S* the whole manifold collapses into D_2. Some further properties of the non-normal based systems are presented in *Parakonsistenz in schwachen Modalkalkülen*[3].

A constitutive property of all Jaśkowski systems is the rejection of Adjunction: none of the systems accept the rule $\text{H}, \text{G} \models_{\text{d}} \text{H} \,\&\, \text{G}$. This is essential for those systems. Very weak assumptions about the underlying modal system allow to prove both the law of excluded contradiction $-(\text{H} \,\&\, -\text{H})$ and Conjunctive Spread $\text{H} \,\&\, -\text{H} \gg_{\text{d}} \text{G}$. Therefore Adjunction would lead from inconsistency to triviality and should be ruled out consequently.

[2] M. Urchs, *Parakonsistenz in schwachen Modalkalkülen*, Konstanzer Berichte Logik and Wissenschaftstheorie Nr. 11, 1990.
[3] *Ibid.*

But worse luck, the superintendent of paraconsistent logic disagrees: „Given a choice of rejecting one or other of Adjunction and Conjunctive Spread to avoid paradoxes and catastrophic spread from an inconsistency, the rejection of Adjunction is the wrong choice"[4]. Priest offers several serious objections to Jaśkowski's construction.

In his opinion the discussive consequence operation is too strong, it is only „half-heartedly" paraconsistent (because of accepting the *ex contradictione quodlibet* principle) and therefore „totally unsuitable as the underlying logic of naive set theory [...] (and) of naive semantics"[5].

That seems not very damaging to discussive logic, because the Jaśkowski systems share this peculiarity with a lot of honourable logical calculi.

However, the second argument looks really dangerous for the non-adjunctive approach to paraconsistency: „The other side of this objection to discursive logical consequence is that it is too weak. To be exact, let Σ be a non-null set of zero degree formulas and let A be a first degree formula. Then if $\Sigma \models_d$ A there is some $B \in \Sigma$ such that $\{B\} \models_d$ A. To see this, suppose for *reductio* that there is no $B \in \Sigma$ such that $\{B\} \models_d$ A. Then for every B we can find a model M_B such that, for some world w in M_B, B is true in w, whilst for no world w, A is true in w. Let M be the collection of all the worlds in every M_B. Then M is counter-model to $\Sigma \models_d$ A"[6].

Whenever it is possible to deduce A from a set Σ of premises, A can be obtained from one single element of Σ. In such a system, Priest concludes, nothing new will be gained by combination of informations or knowledge bases of two or more participants in a discussion. „This shows, that as a logic for drawing inferences in real life situations, discursive logic is useless". And: „the non-adjunctive approach to paraconsistency should be dismissed"[7].

Priest second argument looks quite obvious – but unfortunately, it is not true: the structure M presented above may be inconsistent, i.e. in general it is not a model. Perhaps, the following example will suffies.

Example: For any H, F $\in FOR_d$:

H, F $\models_d \neg(H \gg_d \neg F)$ but neither H $\models_d \neg(H \gg_d \neg F)$ nor F $\models_d \neg(H \gg_d \neg F)$.

To come back to the first argument, for other reason it is not really convincing too. First of all, the discussive systems respect the discussive Adjunction:

H, G $\models$ H $\&_d$ G.

[4] G. Priest, P. Routley, *First Historical Introduction: A Preliminary History of Paraconsistent and Dialethic Approaches*, [in:] *Paraconsistent Logic. Essays on the Inconsistent*, München 1989, p. 48.

[5] G. Priest, R. Routley, *Systems of Paraconsistent Logic*, [in:] *Paraconsistent Logic...*, p. 160.

[6] *Ibid.*, p. 161.

[7] *Ibid.*, p. 162.

This is not dangerous at all because of H $\&_d \neg$H $\not\models$ G. And second, it seems to me that truth-functional Adjunction is not at all so natural as it seems at the first insigth. Imagine once more the quarrel in the train. It is quite normal in any discussion and rather harmless from logical point of view, if there are claimed opposite sentences. But if somebody would claim the conjunction of two opposite sentences simultaneously, he would allow a deep insigth in his intellectual capacities. Perhaps any such speaker should be excluded from rational discussion. Ruling out the truthfunctional Adjunction seems perfectly in keeping with the intuitions underlying Jaśkowski's construction. In other words: there are situations in which rational speakers maintain opposite empirical facts. Such situations should be considered by formal logic. It is possible to modelize them within paraconsistent logic. Hence it is highly entiteled indeed to call in question the universal validity of the classical principle *ex falso quodlibet*.

I don't know whether Duns Scotus really wished to express the property I have in mind. But anyhow, it seems not to be worthless to distinguish *implicational overfillness*, i.e.

H » (¬H » F)

from its *conjunctive version* H & ¬H » F. The last one coincides undoubtedly with the *ex contradictione quodlibet* principle. Hence it makes sense to indentify the *law of implicational overfillness* with the *ex falso quodlibet* principle.

In spite of the part of the *ex falso quodlibet* it is quite another thing with the second principle *ex contradictione quodlibet*. I perfectly agree with Łukasiewicz's conviction (shared by Jaśkowski too): the *law of excluded contradiction* seems to be the keystone of any rational argumentation, it is by all means the criterion for rationality. In my opinion it is exactly there – in the realm of paraconsistent logic – where passes the borderline between the serious investigations of non-classical logics and mysticism. Logical systems, which violate the first principle may be interesting. Systems violating the second one deserve all our suspicion.

Having in mind the enormous methodological power of the *ex contradictione quodlibet* principle, as it was demonstrated in Łukasiewicz's essay[8], we define:

Def. 3: A system (**FOR**, Cn) is called **(methodological) powerful** iff Cn($\perp$) = **FOR**. (As usual, $\perp$ denotes the falsum.)

Paraconsistensists introduced the impressive and crafty notion of an **explosive** system: logical systems are either paraconsistent or explosive. If you

[8] J. Łukasiewicz, *O zasadzie sprzeczności u Arystotelesa*, PWN, Warszawa 1987; comp. Jaśkowski, *Rachunek zdań...*

think of an deductive system as of a vehicle (e.g. a motor bike or a space shuttle) which brings you from premises to conclusions, then it would be calming to know that it is not explosive. On the other hand, to be a good space shuttle, it is not enough to be non-explosive: it should be powerful as well. Now, we are able to present Jaśkowski's discussive calculus as the first non-explosive, but powerful system of paraconsistent logic.

Igor Urbas observed, that some systems (e.g. Johanssons minimal calculus or some of Arruda's set theoretical systems) fulfil the formal criterion of paraconsistency, though they are very close to beeing explosive. Therefore he makes use of a modified concept of „strict paraconsistency"[9] to find out the systems paraconsistent in spirit, not merely formal. He argues[10], that (except in the case of conclusions which are theorems) strictly paraconsistent inferences from contradictory premises satisfy the relevant requirement of shared variables. However, it is not hard to prove:

Fact 3: No D_S fulfils that requirement.

Whence the Jaśkowski discussive systems would be ruled out from paraconsistent logic in the strict sense. Jaśkowski's D_2 is surely not perfect. Nevertheless, if some criterion excludes not only D_2, but the whole family of Jaśkowski's discussive systems simply because they violate truth-functional Adjunction, than the criterion should be tried very carefully. So we have to conclude, that either Urbas conjuncture is wrong, or the concept of strict paraconsistency is misleading. But perhaps, truly paraconsistensists can defend both.

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O „MOCNYCH" LOGIKACH PARAKONSYSTENTNYCH

W pracy prezentowane są niektóre wyniki dotyczące wprowadzonej, w jednej z poprzednich prać autora, klasy tzw. dyskusyjnych systemów Jaśkowskiego. W szczególności zwraca się uwagę na parakonsystentność owych systemów, brak reguły dołączania prawdziwościowej koniunkcji oraz respektowanie reguły dołączania tzw. koniunkcji dyskusyjnej w każdym z tych systemów. Jako specjalne systemy parakonsystentne stanowią one punkt wyjścia do dyskusji nad pewnymi własnościami logik parakonsystentnych. Autor poddaje krytyce stanowisko Priesta na temat roli dołączania koniunkcji dla logik parakonsystentnych oraz sensowność pojęć ścisłej parakonsystencji.

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[9] D. B a t e n s, *Paraconsistent Extensional Propositional Logic*, „Logique et Analyse" 1980, No. 90–91, p. 195–234.

[10] I. U r b a s, *Paraconsistency*, „Studies in Soviet Though" 1990, No. 39, p. 343–354.