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ATOMISTIC UNIVERSES OF INDIVIDUALS

At the beginning of this paper I would like to refer to certain selected theses of B. Russell's philosophical conception. These theses can be presented as a part of the general „logical atomism programme". Further on, an outline and analysis of N. Goodman's modern nominalistic theory is presented, in the form as it was interpreted by R. Eberle. Within this interpretation the concept of an „atomistic universe of individuals" is defined. I consider this concept to be a certain specific realization of the quouted theses of Russell's „logical atomism programme".

As an important point of the atomism programme one may accept the pluralistic thesis about the multiplicity of separate and autonomous things appearing in the world. Russell writes: „I share the common-sense belief that there are many separate things. I do not regard the apparent multiplicity of the world as consisting merely in phases and unreal divisions of a single undivisible Reality"[1].

The second significant idea of Russell's conception is, as is known, the belief concerning the real („external") existence of relations, standing in opposition to Leibniz's monadology and Bradley's global monism.

As the third important thesis of the programme one may account the opinion about a relative simplicity of ontological objects. These objects can have different properties, and can bear different relations, but the majority of these relations do not take part in establishing of the identity of an object.

Struggling with the so called axiom of internal relations Russell assumes a simplicity of objects which are related to each other: „The view which I reject holds, if I understand it right, that the fact that an object x has certain relation R to an object y implies complexity in x and y, i.e. it implies something

[1] B. Russell, *Logic and Knowledge*, Allen and Unwin, London 1971, p. 178.

in the nature of x and y in virtue of which they are related by the relation R"[2].

I will outline now N. Goodman's „calculus of individuals", as it was presented by R. Eberle in his book *Nominalistic Systems*[3]. Every nominalism, including Goodman's, prohibits admittance of other beings than individuals. This principle needs explication and, above all, answering the question what an individual is. The answer to that question takes on a double form; firstly, the construction of a certain formal system and secondly, non-formal remarks concerning the difference between individuals and classes, and the consequence of refuting all beings except individuals.

Unfortunately, Goodman's explanations concerning the mentioned problems are far from being unequivocal. The nominalistic decision of not accepting other items but individuals does not automatically determine what kind of beings could be admitted as individuals. Goodman has nothing against the decision that the individuals would be abstract as well as concrete items, singular and collective beings, physical and phenomenal objects. In one of his articles he writes: „Whatever can be construed as a class can be indeed construed as an individual"[4]. Besides, Goodman claims that any individual may be presented and construed as a class or a set. One can do that, for instance, by the identification of a physical object with the class of its macroscopic, atomic or sub-atomic parts, or with a certain class of events which set up the history of a given object. One can also construe individuals as classes through the translation of all statements concerning them into the statements about unit sets, containing as the only element the given object („singletons"). Eberle claims that Goodman distinguishes individuals from classes first of all at the level of a theory, which means that the theory of individuals differs from the theory of classes or sets. Individuals are distinguished from classes neither by the fact that they are made from a special kind of material, nor by their spatio-temporal character. There are also no specific epistemological criteria which would let differentiate them. It is not the case that individuals could be perceived while classes could not, nor that classes are only mental constructs while the individuals are „given". The possibility of differentiation should be searched at the theoretical level by the analysis of formal features.

An important principle for individuals is the „principle of sum formation". This kind of objects can be put together, summed up, aggregated making up as

[2] B. Russell, *Some Explanations in Reply to Mr. Bradley*, Mind 1910, p. 373–374.

[3] R. A. Eberle, *Nominalistic Systems*, Reidel, Dordrecht 1970.

[4] N. Goodman, *A World of Individuals*, [in:] *The Problems of Universals. A Symposium*, Notre Dame Univ. Press, Notre Dame, Ind., 1956; reprinted in: *Philosophy of Mathematics*, ed. P. Benacerraf, H. Putnam, Printice Hall, 1964.

a result the other individuals which are certain wholes. The operation of summing up individuals has no physical character, and the result of it does not need to be a whole preserving spatio-temporal continuity. It can be an object with parts dispersed in space or existing in different periods of time. Because of this liberal and almost abstract character of the principle of individuals summation it becomes quite alike the set-theoretic operation of union formation: „it seems the principle of sum formation is quite analogous to the set-theoretic principle governing unions of singletons. For example, the set of all red objects (the union of all singletons of red objects) has for its counterpart a certain individual, namely the sum of all red objects"[5]. Therefore, the question arises what other basis can there be for the differentiation between classes and individuals?

As the main criterion distinguishing individuals from classes Goodman suggests the „principle of individuation". As is known, the principle of individuation for sets is the extensionality principle:

$$A = B \longleftrightarrow \hat{x}\,(x \in A \longleftrightarrow x \in B).$$

That principle in the above form does not apply to these objects from the domain of set theory which do not have members (with the exception of the empty set). To ensure the universal validity of the extensionality postulate in the domain of set-theoretical objects one can either assume that all objects have elements, or introduce one-place predicate „is a set" and relativise with the help of it the principle of extensionality.

Consequently, we identify classes and sets through pointing out the correlates of the relation $\in$ with respect to the given set (class); when two sets (classes) have the same elements, then they are identical. Should the similar rule be applied in the calculus of individuals, given that one introduces the relation „being a part" in place of the relation „being a member", and postulates that two individuals are identical just in case when they have the same parts? But should one take here into consideration all actual and possible parts of a given individual object? It seems that in the calculus of individuals such a criterion would be too strong, and that it is not necessary to point out all parts to identify two individuals with each other. Goodman and Eberle claim that for individual wholes it is sufficient to set forth a condition requiring that objects which have the same „ultimate constituents" are identical.

Another important difference between set theory and the calculus of individuals is as follows: if we have a given object A, then the transitions $A \rightarrow \{A\} \rightarrow \{\{A\}\} \rightarrow \ldots$ in accordance with the set-theoretical principle of sets formation, i.e. from elements to sets, sets of sets etc., lead to objects

[5] Eberle, *Nominalistic System...*, p. 18.

non-identical with A: A $\neq$ {A} $\neq$ {{A}} etc. In this manner, starting from one object we can obtain whole infinite wealth of objects. This is applied in the reconstruction of natural numbers as sets hierarchically founded on the empty set. In the calculus of individuals such a situation is excluded. For example, from a pair of objects A and B one can build up only one new object , i.e. the whole containing as its immediate proper parts only these two individuals A and B.

In connection with this matter Eberle writes: „the principle of extensionality differentiates classes which have different «immediate constituents» relative to membership-chains; that is to say, classes which have different members. Individuals which have the same content are to count as identical [...] and having the same content is here taken to mean «having the same ultimate constituents»"[6]. When we look at the question of „constituents" of sets from a more general point of view – not only refering to the relation $\in$ itself, but also to the ancestral relation from the membership relation – then it turns out that the extensionality principle differentiates sets which have different „immediate constituents" with respect to that ancestral relation. On the other hand, when individuals are concerned, one may acknowledge as identical those of them which have the same „content". Habing the same content means here having the same „ultimate constituents". Which kind of constituents would be recognized as ultimate depends on what relations they bear to each other.

The essential task which is assigned to the calculus of individuals is, therefore, the explication of the concept of a fundamental relation between individuals. For instance, the mereology of S. Leśniewski is interpreted as a theory of the relation „being a part". Goodman introduces a more general concept of a „generating relation" which is to include both the set-theoretic ancestral relation from the relation $\in$ and the relation „being a proper part". Thus, at that stage of the development of his theory he wants to cover mereological as well as set-theoretic concepts[7]. In the later formulation Goodman's calculus of individuals is based on the primitive term *to overlap*, i.e. on the concept of a partial covering of one individual by another. The principal postulate of this version of the calculus looks as follows:

$$x \text{ ov } y \leftrightarrow \underset{z}{\vee} \underset{w}{\wedge} (w \text{ ov } z \rightarrow w \text{ ov } x \wedge w \text{ ov } y)$$

where ov is a symbol for the relation of a partial covering of individuals or, in other words, of having a common part[8]. The reason, why Goodman chooses as a primitive the symmetric predicate to *overlap*, and not the better known

[6] *Ibid.*, p. 26.

[7] Goodman, *A World of Individuals...*,

[8] N. Goodman, *The Structure of Appearance*, 3rd ed., Reidel, Dordrecht 1977, p. 34.

predicate „is a part", is the greater formal simplicity of the former. The relation „being a part" in this version of the calculus is defined by means of the relation of overlapping:

$$x \text{ is a part of } y \overset{\text{df}}{\longleftrightarrow} z \ (\ z \text{ ov } x \rightarrow z \text{ ov } y \ ).$$

In words: the individual x is a part of the individual y, if and only if, when every individual having a common part with x has also a common part with y.

According to Eberle, there are three elements most important for the concept of individual introduced by Goodman: the concept of the „generating relation", the principle of summation, and the principle of individuation. From a formal point of view the part-whole relation should fulfil conditions put forward by the following definition:

Def. R is a part-whole relation, if and only if the following conditions are satisfied[9]:

1. R is a partial ordering.
2. Ø (the empty set) is not a member of the field of R.
3. There exists a set A meeting the following requirements:

a) for every non-empty subset S of A, and for all x in A, if x bears R to $\sup_R S$, then x is in S;

b) the field of R is equal to the set of all items x such that for some non-empty subset S of A, $x = \sup_R S$;

c) A is infinite.

Let us note that in the quouted work Eberle uses the set-theoretic apparatus on the meta-language level describing the nominalistic calculus of individuals. Thus, he describes nominalistic systems in non-nominalistic language. The condition 2 is, according to him, equivalent to the claim that for every non-empty subset S of the set A the $\sup_R S$ exists, and all descriptions formulated by means of the part-whole relation theory terms have definite character. The point 3, specifies requirements which should be fulfilled by the distinguished subset A of the domain of the relation R. This subset is interpreted as a set of R-atoms (atoms with respect to the part-whole relation), i.e. a set of minimal elements with respect to R.

Now, let me underline that in the definition of the part-whole relation one employs a specific idea of summation of elements. This idea, which is a realization of the principle of summation for individuals, uses the concept of a supremum of a set with respect to the relation R (R is a partial ordering). The concept of a supremum can be applied both to the finite and infinite subsets of the field of the part-whole relation, e.g. to the whole distinguished set of atoms A. The introduction of a generalized operation of summation for

[9] Eberle, *Nominalistic Systems...*, p. 33.

individuals ($sup_R S$) has a definite goal in Goodman-Eberle's theory. Namely, they want to stay neutral with respect to the problem of finiteness or infiniteness of the relation part-whole domain. That problem shouldn't be decided at the level of introducing one or another summation operation. The problem of the possibility of reconstructing the whole universe of individuals as the sum of all its constituents also comes into play here. In this connection, Eberle writes: „Can we be assured that there are indeed «ultimate constituents» in the whole field of physical objects relative to this relation (between a physical part and a physical whole)? Suppose that physical objects turn out to be infinitely divisible; should we then be prepared to admit that physical objects are not individuals? Goodman does not preclude, on principle, that there may be an infinite number of least physical constituents"[10]. The nominalistic standpoint should be formulated in such a general way that a disagreement between it and the content of a physical theory would not be possible.

The above consideration is to justify the condition 3c which states that the distinguished set A, representing the class of atoms, is infinite. In addition to this Eberle justifies the assumption about the infiniteness of the set of atoms in the following way: „the atoms in question are concerned as possible, rather than as actual objects. And it does not seem counter-intuitive to require that there shall be infinitely many possible entities. On the other hand, since universes of individuals are conceived as comprising actual individuals, we shall refrain from imposing a condition on such universes which would imply that every universe of actual things is infinite"[11]. If I understand this intention correctly, one should define the part-whole relation in the most general way and eventually limit this generality later while applying that relation to specify the concept of an actual universe of individuals.

The point 3 also claims that the set A consists of discrete atoms, i.e. R-minimal elements having no common parts. An atom can be neither a part of another atom, nor a part of a sum of two or more atoms. Beside atoms, individuals are wholes generated from atoms with the help of the summation operation.

Let us proceed now to the principle of individuation which is another factor constituting, according to Goodman and Eberle, the concept of an individual. The principle of individuation together with the part-whole relation and the summation principle characterize objects which we want to reckon among individuals. They describe what certain collectives of individuals are, rather than what a single individual is. According to Eberle, between such collectives a special attention deserve so-called atomistic universes of in-

[10] *Ibid.*, p. 30.

[11] *Ibid.*, p. 39.

dividuals. He thinks that a definition of such universes constitute a principal explication of the concept of an individual. In general, an universe of individuals is a subset of the field of the part-whole relation in which the appropriate conditions concerning summation of objects and their individuation are satisfied. An individual is characterized in a round-about way as the element of a certain universe of individuals.

Eberle tries different alternative versions of the principle of individuation, and finally assumes that the task of distinguishing in the field of relation R the universes of individuals is best fulfilled by the following principle:

For every x and y belonging to U, $x = y$ iff for every z, if z is R-least in U, then z R x iff z R y;

U symbolize here a certain 'universe of individuals[12].

In other words, individuals belonging to U are identical just in case, when they have the same atoms in relation to U as their parts. Such a formulation of the principle of individuation imposes certain restrictions on the universes of individuals which are not imposed by the other, more liberal formulations. For instance, more general principle of individuation, which identifies individuals when they have the same atoms with respect to the whole field of the relation R, does not impose restrictions on the universe U; every subset of the field of the relation R could then be accepted as an universe of individuals U. What reasons are there for choosing such a principle of individuation? This problem boils down to the question of the role which is played by the universes of individuals within the field of the part-whole relation. Let us remind that Eberle interprets the field of the part-whole relation as the set of all objects which could be parts or wholes. He writes: „By contrast the elements of a particular universe of individuals are regarded as those individuals which happen to be actualized in that universe. To provide a suggestive example: suppose that we conceive of an infinite class of items all of which satisfy a physicist's description of an atom. Let a part-whole relation be conceived between these atoms and all possible compositics of the atoms. Any selection of these possible atoms or composities might be actualized in some universe which is a «universe of individuals» if for every composite object which is actualized in it a sufficient variety of parts are also actualized, so that different actual composities have in the universe different actual parts. It is logically possible that the simplest physical objects which happen to be actualized in such a universe are molecules, while all proper parts of molecules remain unactualized possibles"[13].

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[12] *Ibid.*, p. 38.
[13] *Ibid.*, p. 37–39.

The selected principle of individuation suggests that Eberle wants to restrict the variety of possible items. Individuals are to be actualized objects which consist of actualized parts. According to the principle of individuation, different actual individuals are composed in the last resort of different actual atoms. Being actual is conceived here in a specific manner as an attachment to certain distinguished universe of individuals which is a subset of the part-whole relation field. At this point one can raise the question, whether arbitrary groups of individuals from the universe can be put together by the summation operation, giving as the result in each case new individual wholes?

Goodman — as is known — has answered this question quite positively: „Although not every individual has a negate and not every two individuals have a product, every two individuals *do* have a sum. Bearing in mind that only individuals are values of our variables, we can affirm the unconditional statement:

$$\hat{x}\,\hat{y}\,\check{z}\,(z = x + y)$$

as a postulate or theorem of our calculus"[14]. The negation and product Goodman writes about, as well as the symbol + denoting the summation operation, are terms defined in the version of the calculus of individuals presented in the quouted work.

Goodman was often criticized for adopting the above principle of summation, and in this case Eberle joins his critics: „we would depart from Goodman's conception by admitting other relations which qualify intuitively as part-whole relations but fail to generate actual sums of arbitrary individuals"[15]. Eberle imposes the following weaker condition on the operation of summing the elements of the universe of individuals:

For every x which belongs to U, there exists a set S consisting of elements R-minimal in U, such that $x = \sup_R S$.

In other words, every individual from the universe of individuals U is a sum of elements which are atoms with respect to the relation R in U. If any object is an individual belonging to the universe U, then it must have a decomposition into atomic parts within U. However, such a condition does not assume that every sum of atoms or any other individuals belonging to U is again an element of U, i.e. an individual in this universe. That formulation stresses analytic, rather than synthetic function of the individual summation operation.

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[14] Goodman, *The Structure of Appearance...*, p. 36.

[15] Eberle, *Nominalistic Systems...*, p. 41.

Let us comment here on one more matter. In B. Russell's logical atomism programme an essential role plays the quouted conviction that „the world does not consist merely in phases and unreal divisions of a single undivisible Reality". Thus, Eberle is in a better agreement with Russell's programme than Goodman, since he does not assume that every possible sum of individuals is an actual individual. In this way he restrains himself from the assumption that the whole world is one maximal, global individual, and that — possibly — all properties and external relations of objects in the world are reducible to the properties and internal relations of the world itself.

After selecting the appriopriate principles of individuation and summation Eberle defines the central concept of his reconstruction of Goodman's calculus – an „atomistic universe of individuals". The both above-mentioned principles assume that in every universe of individuals exist atoms; hence, the expression „universe of individuals" is supplemented with the adjective „atomistic". Since, according to Eberle, the chosen principle of summation implies the principle of individuation, in the definition of an „atomistic universe of individuals" one can take into account only the former.

Def. U is an atomistic universe of individuals for R iff

1) R is a part-whole relation,

2) U is included in the field of R,

3) for every x in U, there exists a set S such that all members of S are R-least in U, and $x = \sup_R S$[16].

After presenting the above outline of Goodman-Eberle's theory some comments suggest themselves. As I have written at the beginning of this article, that conception seems to be a certain realization of selected theses of Russell's logical atomism programme. It assumes that there exist many separate and independent individuals, atoms and wholes, while restraining itself from concluding the matter of existence of a maximal global individual, identical perhaps with the whole reality. Thus, it represent a standpoint of pluralism. At the same time a fundamental role plays here the „generating" part-whole relation which is assumed independently from individual objects. One does not attempt to reduce that relation to internal properties of individuals but the other way round, the introduction of it is constitutive of the concept of an individual. This approach is in accordance with Russell's standpoint rejecting internal relations axiom and postulating external relations independently from objects' properties. Finally, there is certain kind of simplicity in domains which qualify as atomistic universes of individuals, viz. their members can be uniquely presented as relatively simple wholes composed of elementary constituents, and such a composition must allow for their complete identification. This aspect of simplicity could be expressed by the statement that

[16] *Ibid.*, p. 42.

atomistic universes of individuals have the structure pertaining to the relation „simpler than", which can be interesting in the meaning analysis of the concept of simplicity[17].

The fundamental nominalistic claim postulates refutation of abstract entities, in particular existence of classes. Goodman writes: „Whatever we are willing to recognize as an entity at all may be construed as an individual [...] we can construe anything as an individual"[18]. It is rather semantic than ontological approach: the nominalistic thesis could be formulated in Eberle's conceptual framework as the statement that every domain of objects can be interpreted as a certain atomistic universe of individuals. Let us consider the soundness of that statement.

Things and material objects do not seem to fulfil the nominalistic principle of individuation. From the same things-parts we can construct different wholes in different moments of time; a little child does that while playing with building blocks. Material objects are not individuals in Goddman-Eberle's sense, in order to attain this status the time dimension should be taken into account. Thus, for instance, a table is not an individual but the table-hour, table-minute, and table-second are. Sixty table-minutes summed up together give as a result an individual which is one table hour. The question arise, what would in this case atoms be. The same common-sense table, taken into consideration for the period of one minute yesterday and today, consists of two completely different nominalistic, individuals – two separate table-minutes. They are only connected by the other intermediate table-minutes which adjoin to each other or succeed one after another. The identity of two table-minutes separated in time does not come into play, although they can be parts of the same table-week or table-month, since they consist of completely different – let's say – particle-seconds (the minimal distinguished space-time regions). The only kind of identity which can occur between two individuals separated in space or time is the genidentity, which has not much to do with the identity in a nominalistic sense. It is also not difficult to see that language expressions do not fulfil the nominalistic principle of individuation either; from the same signs we usually may built up different expressions.

One can obviously construct domains which would be atomistic universes of individuals; one could also do that with the help of set-theoretic concepts. For example, the power set of some non-empty set Z (the empty set excluded) with the operation of union and the inclusion relation, i.e. the relation structure $<2^Z, \cup, \subset>$, is an atomistic universe of individuals. The atoms here are the unit sets formed from the elements of the set Z. If the empty set was

[17] Strawiński, *A Formal Definition of the Concept of Simplicity*, [in:] *Polish Essays in the Philosophy of the Natural Science*, ed. W. Krajewski, Reidel, Dordrecht 1982, p. 195–197.

[18] Goodman, *A World of Individuals...*; *Philosophy of Mathematics...*, p. 199.

included, then it would have been the only atom in this universe which, however, would not have been able to generate other elements. Nevertheless, the construction of such domains seems to be a rather weak justification of Goodman's conviction that „we can construe anything as an individual".

It appears that we do not meet atomistic universes of individuals too often. Things and material objects do not seem to be individuals in this sense. It is rather the entities of eventistic ontology, consisted of spatio-temporal events, which satisfy the conditions required from individuals by Goodman and Eberle.

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ATOMISTYCZNE UNIWERSA INDYWIDUÓW

Autor przedstawia krytyczną analizę pewnych idei nominalistycznej teorii N. Goodmana w interpretacji R. Eberlego. Teoria ta, czyli „rachunek indywiduów", oparta jest na trzech podstawowych pojęciach: zasadzie sumowania, relacji „część-całość" i zasadzie indywidualizacji. Owe pojęcia wzięte razem charakteryzują przedmioty, które chcemy zaliczyć do indywiduów, przy czym charakterystyka ta określa raczej czym jest pewien zespół indywiduów, niż to czym jest pojedyncze indywiduum.

Według Eberlego na specjalną uwagę zasługują tzw. atomistyczne uniwersa indywiduów. Definicja takich uniwersów ma stanowić właśnie określenie tego, czym są indywidua. W ogólności „atomistyczne uniwersum indywiduów" to podzbiór pola relacji „część-całość", w którym są spełnione odpowiednie warunki dotyczące sumowania i indywidualizacji przedmiotów. Pojęcie to autor wiąże z atomizmem logicznym B. Russella oraz rozważa jego możliwe zastosowania.