PROBABILISTIC APPROACH TO EPISTEMIC MODALS IN THE FRAMEWORK OF DYNAMIC SEMANTICS

Montague Semantics — General Introduction

According to the classical Montagovian theory, meaning of a sentence is given by its truth conditions, i.e. to know the meaning of a sentence amounts to knowing what the world should be like for a given sentence to be true. Montague semantics is an approach to the natural language that attempts to provide a very precise, explicit and formal account of the notion of meaning (understood in the previously described way). Montague believed that a formal theory of natural language can be developed, despite the fact that natural languages (as it had earlier been stressed by Tarski) have no specified structure, are semantically closed\(^1\), contain indexicals, etc.

In the paragraphs that follow, I will quickly overview Montague’s approach to the semantics of ordinary language to make easier the understanding of the difficulties it encounters and the later developments and extensions of the theory that are supposed to deal with such difficulties.

Montagovian theory attempts to provide an account of the relation between the world and natural language in a very precise and formal

\(^1\) Languages without specified structure do not contain an unambiguous characterization of those expressions that are to be considered meaningful; a semantically closed language is the one which contains not only propositions, but the names of the propositions and expressions that can be used to refer to the propositions in the language; indexicals are those expressions that are highly dependant on the context, e.g. I, here, now.
manner. In doing so Montague is utilizing the means of model theory. In other words, he defines a set of objects (“the universe”), interprets linguistic expressions as elements of this set and describes relations among them employing set theory, first order logic, modal logic and intensional logic [Dowty et al. 1981, 10].

I will explain in more details how Montague’s approach is supposed to work by presenting his account of the semantic value of names first, since it seems that in the case of names the aforementioned relation between the world and our language seems to be quite straightforward. In Montague’s semantics a name, for instance “Plato” stands for an object, an individual — Plato. Similarly, the semantic value of a predicate e.g. “talks” is given by a set of individuals who talk, or, more precisely, by a characteristic function\(^2\) that takes an individual (Plato)\(^3\) and yields the result true if the individual actually talks (belongs to the set of objects that talk) and returns the value false if that is not the case. Finally, semantic value of the sentence “Plato talks” is either truth or falsity (Dowty et al 1981, 5).

This is a view that was first put forward by Frege. Another principle derived from Frege’s philosophy of language and closely related to the semantic value of complex expressions is compositionality. According to the compositionality principle, semantic value of complex expressions can be derived based on the formal and syntactic combinations of the basic linguistic units. Compositionality is one of the basic presuppositions of Montague’s theory. It is supposed to explain the productivity and creativity of our language competence — the fact that humans are even though finite beings capable of producing and understanding potentially infinite number of language expressions.

Besides compositionality, Frege’s theory provides motivation for including another framework into Montague’s theory — the framework of

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2 Characteristic function takes individuals as arguments and yields either one or zero (true or false) as their value. Taking characteristic functions rather than sets of individuals that the predicate applies to as the meaning of the predicate makes the computation somewhat easier.

3 This is a simplified view, though. In actual Montague’s semantics a noun phrase is thought to denote a set of properties; consult footnote 6.
possible worlds. Frege noticed that there is a difference in cognitive value between the statements of identity such as “Socrates is Socrates” and “Socrates is the teacher of Plato”. This fact is hard to explain in the referential framework because both sentences are supposed to state self-identiy of the same object. Frege’s two-dimensional semantics — the statement that meaning of words is consisted of their sense and reference and that the aforementioned expressions share their reference, but differ in sense — was supposed to explain away this problem. Nevertheless, Frege provided only a metaphorical definition of sense; he claimed that sense is just a “mode of representation of a referent” [Frege 2001, 7–10]. Montague tried to provide a formal account of this notion by employing the framework of possible worlds. Intension of a proper name in his framework then is a constant function which picks out its bearer in every possible world. Intension of a definite description is a function from sets of properties within possible worlds and moments of time to truth values. Such approach makes us able to at the same time stay within the truth-conditional/referential framework, but also to capture our intuition that the aforementioned propositions actually differ in cognitive value.

Problems for Montague semantics — (cross-sentential anaphora and epistemic modals)

In Montague’s theory semantic value of a sentence is given by the set of assignments, i.e. it is a function from possible worlds and moments of time to truth values. Because of this property, Montague’s framework is only capable of treating sentences in isolation. Consequently, it faces difficulties in treating certain phenomena that require taking entities that are larger than sentences as minimal semantic units.

The first example of a phenomenon that cannot be adequately accommodated within Montague’s framework is the cross-sentential anaphora. Cross-sentential anaphoric pronouns cannot be understood either as referring expressions that inherit their referents from other referring expressions, or as variables bound by the quantified antecedents in Montague’s semantics.
Example of a cross-sentential anaphora: A man comes in. He sees a dog.

The obvious translation of this sentence in the Montagovian framework would be the following one:

$$\exists x \ (\text{man}(x) \land \text{comes-in}(x)) \land \exists y \ (\text{dog}(y) \land \text{sees}(x, y))$$

But, this translation is not correct since the second variable $x$ is not bound by the first existential quantifier. Since quantifiers function only as intra-sentential operators within this framework the fact that the pronoun “he” refers to the same man mentioned in the first sentence cannot be captured.

More informally, this is due to the fact that propositions are functions that map possible worlds to truth values in Montague’s semantics. Classical framework does not provide us with any means to connect two of such functions in the way that is needed to account for the relation between anaphoric pronoun and its antecedent.

The correct translation of the sentence “A man comes in. He sees a dog” would be something like:

$$\exists x \ (\text{man}(x) \land \text{comes-in}(x)) \land \exists y \ (\text{dog}(y) \land \text{sees}(x, y)))$$.

The problem with this translation is in the fact that it cannot be produced in a compositional manner from the formalization of “A man comes in”, that is — $\exists x \ (\text{man}(x) \land \text{comes-in}(x))$.

Example of a donkey sentence:

Another example of a failure to capture the intended meaning of a sentence within the classical framework is by Peter Geach [Geach 1962]. That is the example of the so-called donkey-sentences that have the following form:

$$\exists x \ (\text{man}(x) \land \text{comes-in}(x)) \land \exists y \ (\text{dog}(y) \land \text{sees}(x, y)))$$.

Typed lambda calculus that is being used in Montague’s semantics will be substituted by the first order logic for the sake of clarity.
If a farmer owns a donkey, he beats it.

Just like in the previous case, an intuitive translation of this sentence would be:

$$\exists x \ (\text{farmer}(x) \land \exists y \ (\text{donkey}(y) \land \text{owns}(x, y))) \rightarrow \text{beats}(x, y)$$

But, again, just like in the previous case — the two variables in the consequent are free.

The correct translation would be something like:

$$\forall x \ (\text{farmer}(x) \rightarrow \forall y \ ((\text{donkey}(y) \land \text{owns}(x, y)) \rightarrow \text{beats}(x, y)))$$

Again, just like in the case of cross-sentential anaphora this translation cannot be obtained compositionally from Geach’s donkey sentence. The problem here is that the pronoun “it” cannot be explained as a referring expression since no referring object, no particular donkey is mentioned in the sentence. Besides that, similarly to the previous example it cannot be understood as a variable bound by the antecedent existential quantifier since it is not under its scope — it is free. In addition, even if the quantifier “donkey”\(^5\) could do this, assuming that it is an existential quantifier, we still would not get the intuitive truth conditions which require that a farmer beats every donkey he owns.

In conclusion, we cannot derive a translation of the donkey sentence into the language of predicate logic in a systematic way; in order to capture the intuitive meaning we have to assume that the indefinite expression buried in a subordinate position ends up having wide scope and universal force. A theory based on such assumptions might capture the facts, but it is clearly ad hoc, and can generate a host of false predictions, as well.

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\(^5\) In Montague’s framework, noun phrases are treated as quantifiers; this approach allows for their uniform treatment; their denotation should be understood as a set of properties [Janssen 2012].
Epistemic modals example:

Another problem for Montague's semantics was presented by Seth Yalcin in 2007. Yalcin claims that epistemic modals (such as might), can sometimes give rise to something like Moore's paradox. He presents two sentences:

1) It might not be raining and it is raining.
2) It is raining and it might not be raining.

The first one sounds odd and almost contradictory, whereas the second one does not seem equally odd.

The aforementioned sentences can be formalized in the following way:

1) $\Diamond \neg P \land P$
2) $P \land \Diamond \neg P$

Bearing in mind the fact that in classical framework conjunction is commutative then these two sentences should have the same truth conditions. Truth-conditional semantics cannot account for this difference in the oddity of the two sentences then — it treats the two sentences in just the same way. They are both equally consistent in Montague's grammar and classical modal logic and (given that conjunction is a commutative connective) there are no means to explain the fact that the order of conjuncts turns out to be relevant for our understanding of the sentences, ascription of truth value, consistency, etc.

It has been widely agreed that this problem can be addressed by employing a different framework, i.e. dynamic semantics. It is a theory in which meanings are treated not as truth-conditions, but as context change potentials. What follows is a general overview of this approach.

Dynamic Semantics — General Introduction

The aforementioned problems led to the development of dynamic semantics in which meaning of a sentence is not given by its truth conditions, but rather, sentences are understood as instructions for changing any already existing context (understood as a set of possible worlds) [Groenendijk, Stokhof 1996, 106]. Meaning is given not as in the
case of the standard semantics of predicate logic as the interpretation of a formula (i.e. as a set of assignments, those assignments that verify the formula), but as a set of ordered pairs of assignments. Due to such understanding of semantic value existential quantifiers in dynamic framework have extendible scope and the compositional translation of the problematic sentences can be obtained.

**Existential quantifier**

Intuitively, an existential quantifier in dynamic semantics introduces a new variable and then eliminates all the possible worlds without that variable. More precisely, the dynamic interpretation of $∃xφ$ will consist of those pairs of assignments $(g, h)$ such that there is some assignment $k$ which differs from $g$ at most in $x$ and which together with $h$ forms a possible input-output pair for $φ$. The interpretation clause for existentially quantified formulas then reads as follows: $[[∃xφ]] = \{(g, h) | ∃k: k[x]g & (k, h) ∈ [[φ]]\}$ [Groenendijk 1991, 47].

**Conjunction**

Conjunction in the dynamic predicate logic passes on values of variables from the first conjunct to the second one. Its formal definition is as follows:

$[[φ ∧ ψ]] = \{(g, h) | ∃k: (g, k) ∈ [[φ]] & (k, h) ∈ [[ψ]]\}$

According to this definition, the interpretation of $φ ∧ ψ$ with input $g$ may result in output $h$ iff there is some $k$ such that interpreting $φ$ in $g$ may lead to $k$, and interpreting $ψ$ in $k$ enables us to reach $h$ [Groendijk 1991, 45-48]. It clearly follows from such definition that the conjunction connective is not commutative.

**Modal operator**

Its role is to test possible worlds and if the set of possible worlds expressed by the proposition is consistent with the information state — its
output is the very same information state; if it is not consistent — it returns an empty set as an output [Veltman 1996, 231]. Formally: 
\[ c[\diamond \phi] = \{ w \in c | c[\phi] \neq \emptyset \} \]

**How dynamic semantics accounts for problematic anaphora**

When the first sentence, “A man came in”, is accepted by the hearer, what happens is that a new variable is added to the domain of the assignments in the hearer’s information state, a certain object is assigned to that variable and then the information state is reduced to those assignments in which that object is in the extension of “came in”. The second sentence reduces the information state to those assignments in which the object assigned to the new variable is in the extension of “he sees a dog”.

Resolution of donkey sentences is quite similar; implication is also understood as a dynamic connective — the one that passes on values from the antecedent to the consequent. Universal force of the existential quantifier is achieved by requiring that for every pair of assignments \(<h, g>\) in the interpretation of the antecedent there is some assignment \(k\) such that \(<h, k>\) is in the interpretation of the consequent [Groendinijk 1991, 45–48].

**How dynamic semantics accounts for epistemic modals**

Once again, the problem is that sentences “It is raining and it might not rain” are not inconsistent in classical logic and semantics since rain in the actual world is not precluded by there being no rain in some merely possible world. But, the problem is that they intuitively seem inconsistent. In the context of dynamic semantics, we can account for this intuition as follows: Agent’s knowledge is a set of finitely many sentences \((A)\), and information state is a subset of such set \((\delta)\); a priori possibilities are represented as a

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\(^6\) C denotes a context (information state) and \(w\) a possible world.
power set of the set A. Updating such information state means eliminating certain possible worlds/possibilities [Veltman 1996, 229].

Furthermore, the corresponding update action of the modal operator on the current context is to check whether an update with $\varphi$ in the context yields a non-empty set of possibilities. In the affirmative case, the update with $\Diamond \varphi$ returns the whole context otherwise it returns the empty set. The idea behind the analysis of might is the following one — one has to agree with “might P” if P is consistent with one’s knowledge.

How does this account of modal operators and consistency help us to capture and explain the fact that “It is not raining and it might not be raining” seems inconsistent? The idea is that “It might not be raining” will be consistent with the information state only if there are some non-raining worlds in the information state. But it is not consistent since in the previous step — while stating “It is raining” — I have already eliminated all of the non-rain worlds. Simply, when I assert “It is raining”, the information state is updated with the proposition that it is raining. That means that all worlds without rain are removed from the information state — that is — after eliminating the $\neg p$-worlds (by asserting “p”) the $\Diamond \neg p$ test will fail. This pair of sentences is inconsistent: it turns any state into $\emptyset$; it crashes the context. Naturally, once the context comes to accept p — it will fail the test corresponding to $\Diamond \neg p$.

Further problem — Gauker’s criticism

To sum up — there is a kind of inconsistency of the sentences of the form: $(p, \Diamond \neg p)$; this inconsistency depends on the order of the appearance of the sentences in a discourse (since the sentence of the form: $(\Diamond \neg p, p)$ is supposed to be consistent). Such phenomenon cannot be captured by the classical logic and formal semantics, but it can, according to Veltman’s opinion, be captured within the framework of dynamic semantics.

On the other hand, according to Gauker’s criticism there might be something inconsistent about the second sequence as well $(\Diamond \neg p, P)$. Gauker illustrates this difficulty in the following way: "As I walk out of my house, I say “It’s going to rain”. But then as I step outside I look up at the
sky and say, “It might not rain”. It does seem as though, in saying “It might not rain”, I have taken back what I first said, namely, “It will rain”. So there is a kind of inconsistency in “It will rain” followed by “It might not rain”. But likewise: Suppose as I leave the house, I grab an umbrella, saying to my wife, “It might rain”. But then, as I leave the house, I look up at the sky and say, “Nah, it won’t rain” and toss the umbrella back inside. Again, my second sentence amounts to taking back what I first said” [Gauker 2007, 10].

Why is that a problem for the dynamic semantics?

As I have previously said — modal operators in the dynamic semantics function as tests that are supposed to check if the proposition at question is consistent with the given information state and yield either the same information state if the proposition is consistent with it or an empty set if it is not. Hence, ◊¬P simply checks if there are any non-raining possible worlds consistent with the information state and it yields the very same information state, but it does not eliminate all raining worlds from the information state, i.e. there are some possible worlds left that are not ruled out by the first conjunct and, consequently, the second sentence is predicted to be consistent in the dynamic semantics framework.

In other words, even if it is compatible with the information state that there might be some (at least one) non-raining worlds, it does not mean that the raining worlds have been ruled out [Willer 2013, 10–11].

There is another independent result that can motivate searching for a more adequate account for the semantic of epistemic modals except accounting for the intuition that the conjunction “It might not rain and it is raining” seems odd similarly to its reverse form (nevertheless to a smaller degree). That is the result presented by Egan who claims that usually transition from ◊˥p to p requires non-monotonic information growth [Egan 2007]. This result is not incorporated in the dynamic semantic framework — no such information growth is required. Modal operator just checks if the given proposition is consistent with the information state and if it is —

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7 Paradoxical character of the second sentence has been noticed by other authors as well [Willer 2013, 20; Sorensen 2009; Dorr and Hawthorne 2012].
it just returns the very same information state. Then the second sentence updates the starting information state. Hence, the sentence that involves “might” does not contribute anything to our knowledge. An example presented by Goodman and Lassiter might make this point clearer. Imagine the following discussion:

A: What will happen in the game tonight between team X and team Y?
B: Team X is likely to win.

As Lassiter notices it is quite easy to see that “likely” in this case does convey some information about the world, but it is quite hard to create a model that will devise this seemingly banal fact [Goodman, Lassiter, manuscript].

Hence, the case of non-monotonic information growth (in the case of ◊¬P and P sentence) might just reflect this general worry about the lack of informational content carried by epistemic modals as defined in the dynamic semantics framework.

Possible solution — Probabilistic approach to epistemic modals in dynamic semantics?

In the final part I will try to show that endorsing probabilistic approach to dynamic semantics can help us to rule out the aforementioned problems and attain a more comprehensive and accurate theory about the semantics of epistemic modals.

In a probabilistic semantics — truth conditions are exchanged for probability conditions; more precisely — information states are interpreted as probability distributions over propositions [Van Eijck, Lappin 2014].

Formally — it can be a function from possible worlds (sets of assignments) to the information states to set of real numbers: \( I \rightarrow K \rightarrow [0, 1] \), where \( I \) is the set of intensions, \( K \) is the set of knowledge representations, and \([0, 1]\) is the set of reals \( p \) with \( 0 \leq p \leq 1 \).

In that case then a probabilistic model \( M \) is a tuple \((D, W, P)\) with \( D \) a domain, \( W \) a set of worlds for that domain (predicate
interpretations/assignments in that domain), and \( P \) a probability function \( P \) over \( W \), i.e., for all \( w \in W \), \( p(w) \in [0, 1] \).

In a dynamic framework that employs a model that involves this probability distribution — a meaning of a sentence could be represented as a change it introduces in the probability distribution of an information state. In other words — meanings would be defined as relations between probability distributions of the information states.

**Definition of a modal operator in a probabilistic dynamic semantics framework:**

As I have said in dynamic semantics — modal operator “might” would take a sentence — check if there are possible worlds consistent with the given information state and yield either the very same information state in the case of affirmative answer or an empty set.

If we include probability distribution in our model, we are equipped to claim that epistemic modals operate on the sets of probabilities; it is not the case that they either return the whole information state or an empty state — they can also affect the probability distribution in the model.

This leads to a new viewpoint on the way contexts are updated in general: it is not the case that the contexts are updated only by eliminating possibilities, they are also updated by shifting the admissible probabilities over the possibilities. As Seth Yalcin puts it — the conception of information this picture recommends is not as radical as it may appear. He claims that in the standard information theory the amount of information a signal carries is not just a function of the possibilities it eliminates; rather — it is also a function of how it shifts the probabilities over the open possibilities. Hence, it can be claimed that information is a fundamentally probabilistic notion \[Yalcin 2012, 20\].

Analogously to updating information states by eliminating possible worlds — employing might operator decreases probability of accepting the opposite proposition. Then — when I say it might rain — it does not just detect the existence of such possibilities and then returns the context intact, but also it reduces the probability of accepting the sentences that involve reference to the non-raining worlds.
Update Mechanism — Bayes Rule

The way contexts are updated in the probabilistic dynamic semantics can be described using means of the mathematical probability theory, i.e. transition from prior to posterior probabilities can be regulated using means of classical probability calculus (applying the Bayesian rule)\(^8\). Using this framework, we can capture the fact that both sentences sound odd (the fact that could not be explained in non-probabilistic dynamic semantics). Moreover, the difference in the degree of their oddity can be captured by the degree of the change that is produced in the probability distribution. Finally, the fact that ◊˥p and p requires non-monotonic information growth can be captured, again, by the change in the probability distribution that is produced.

Conclusion

Dynamic framework without the involvement of probabilities is advantageous for analyzing epistemic modals because it can explain the fact that the order of conjuncts in the sentences

“It might be raining and it is not raining” versus “It is raining and it might not be raining” matters. Nevertheless, it turned out to be unsuccessful in terms of accommodating our intuition that both sentences sound odd — even though they differ in the degree of their oddity as well as the fact that transition from ◊˥p to p intuitively requires non-monotonic information growth. Dynamic semantics cannot accommodate this finding since epistemic modals function as operators on information states that take an information state and either returns all of it or none, depending whether the condition is satisfied. If epistemic modals are understood as operators on the probability distributions over information states, then the

\(^8\) Even though some problems with regard to the use of the Bayesian rule as a general mechanism for the belief updating have been noticed [Baltag, Smets 2008, 182], it seems that in this very limited framework that focuses only on the epistemic modals Bayesian rule might be a sufficient tool.
aforementioned gradience in oddity and information growth (in the case ♦\(\neg\)p and p) can be captured.

Besides providing a possibility to model a solution to the problem in the treatment of the epistemic modals in the dynamic semantics framework denoted by Gauker, probabilistic approach has been proven to be successful in ruling out certain invalid patterns of reasoning that could be validated in an earlier framework that was supposed to account for the semantics of epistemic modals\(^9\) [Holliday, Icard 2013] as well as in modeling higher order beliefs in the epistemic and dynamic epistemic logic [Smets, Baltag 2008].

It seems that even though it might seem unusual as a proposal for a basis for formal semantics — we might refer to the famous Kratzer’s claim: “Our semantic knowledge alone does not give us the precise quantitative notions of probability and desirability that mathematicians and scientists work with” [Kratzer 2012, 25], mathematical theory of probability can be employed to resolve certain issues within semantics and philosophy of language. It seems that this can also be a sufficient motivation for putting more effort in merging traditional ideas of belief revisions and more quantitative/probabilistic ones. Even though it is unclear at the moment how this general probabilistic semantics should work, there have been some tentative proposals — mainly by Goodman, Lassiter and Lappin. Finally, an advantage of such approach, besides capturing the pervasiveness of the uncertainty in natural language is that it is a promising theory that can bridge a gap between the formal theory of natural language and natural language processing and it can help in explaining the process of semantic learning.

\(^9\) The earlier framework I have in mind is Kratzer’s semantics for might [Yalcin 2010]; also it is noteworthy that the authors who propose the solution based on probabilities have in mind qualitative and not quantitative probabilities.
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ABSTRACT

PROBABILISTIC APPROACH TO EPISTEMIC MODALS IN THE FRAMEWORK OF DYNAMIC SEMANTICS

In dynamic semantics meaning of a statement is not equated with its truth conditions but with its context change potential. It has also been claimed that dynamic framework can automatically account for certain paradoxes that involve epistemic modals, such as the following one: it seems odd and incoherent to claim: (1) “It is raining and it might not rain”, whereas claiming (2) “It might not rain and it is raining” does not seem equally odd (Yalcin, 2007). Nevertheless, it seems that it cannot capture the fact that statement (2) seems odd as well, even though not as odd as the statement (1) (Gauker, 2007). I will argue that certain probabilistic extensions to the dynamic model can account for this subtlety of our linguistic intuitions and represent if not an improved than at least an alternative framework for capturing the way contexts are updated and beliefs revised with uncertain information.

KEYWORDS: dynamic semantics, probabilistic semantics, epistemic modals