https://doi.org/10.18778/1689-4286.34.10

HYBRIS nr 34 (2016)
ISSN: 1689-4286

BARTOSZ JANIK
UNIWERSYTET JAGIELLOŃSKI

KRIPKENSTEIN FROM THE MATHEMATICAL POINT OF VIEW: A PRELIMINARY SURVEY

1 After the beginning

The aim of this paper is very modest. In the first few sections I would like to review and comment on the debate about non-factualism in the philosophy of mathematics. First, I will explain whether this kind of debate – closely tied to the debate about meaning – is applicable to such an exceptional field of inquiry. I would argue that, albeit allowing that some clarifications and alternative definitions should be settled, the debate about non-factualism is crucial in this field. Second, I would like to outline the boundaries of the general inquiry and, in order to do so, I will list some of the main problems connected to the aforementioned debate and try to put some of them aside with the proper commentary, the rationale being the introductory character of this paper.

Although a certain interpretation of the skeptical solution to Kripke's puzzle will be provided in this paper, a more precise formulation and critique of the dispositionalism in the philosophy of mathematics will not appear. The aim of this study is to deal with the (non-)factualist stance and the debate concerning dispositions in this area is far too wide ranging for this elaboration. The paper will also touch briefly on the problem of meaning in the philosophy of mathematics. I will provide the necessary definitions, especially when dealing with Dummett's version of intuitionism, but in my opinion the problem is much deeper and probably unsolvable in the first-order language of mathematics. For a fuller analysis of this problem, one could see [Shapiro 1991] The last topic that I would like to indicate here is the problem of the relation between global and local viewpoints in the philosophy of mathematics. I will address this

issue during my elaboration of (non-)factualism and it should be regarded as purely an introductory survey.

2 Kripke's skeptical paradox?

In his famous and broadly commented upon essay about Wittgenstein and rule–following, Saul Kripke struck at the very foundations of meaning determinism in the theory of meaning [Kripke 1982] [Kusch 2006]. In the first part of this essay I would like to outline and explicitly state what was the target of Kripke's interpretation of Wittgenstein's rule-following considerations and to which concept much of the damage was done.

The firsts step would be a brief presentation of meaning determinism based on M. Kusch's elaboration of the starting point of Kripke's critique [Kusch 2006]. Meaning determinism could be briefly stated as the thesis that:

Definition. 1 *[Kusch 2006, 4]*[1] *Meaning determinism holds that sentences of the form 'person x means Y by sign 'z" are true if, and only if, x has certain mental state that constitutes x's meaning Y by 'z'.*

Having stated meaning determinism, we can see how it was attacked by Kripke: suppose that we define 'plus' as an addition function in usual way and we further define 'quus' function: $x \oplus y = x+y$ if $x,y<57$ and $x \oplus y = 5$ otherwise

The skeptical question is: in the past, do we mean plus or quus by the symbol '+'? It appears that in my past, there was no fact justifying the meaning of the '+' sign. To face this paradox, one could formulate a 'straight' or 'skeptical' solution to it. The straight solution to the paradox tries to dissolve the paradox by pointing out the flaw of the original argument[2]. The skeptical solution accepts the paradoxical conclusion that there are no genuine facts about meaning and thus different criteria must be built for meaning-discourse. Skipping the prima-facie answers to the challenge, Kripke, after an elaboration of the various types of high-

1 This formulation is called by Kusch low–brow meaning determinism, while views that attempt to give it theoretical precision are called high–brow meaning determinism. The usability of this distinction will not be pursued in this essay but I will be adopting this distinction

2 "A straight, or normal, solution, he says, would be the discovery of some mistake in the argument—for example, the calling of attention to some further kind of fact, not previously noticed, that would satisfy the adequacy conditions." [Horwich 1998, 213]

brow meaning determinism, concludes that there is no fact that could support the answer that anyone could mean something by uttering a sentence [Kripke 1982, 9].[3] The view that we could not answer the skeptical question fuels Kripke's non-factualism and, from mere epistemological non-factualism, Kripke is deriving the thesis that non-factualism is constitutional (metaphysical) in essence. I will not discuss at this point the obvious connection of Kripke's survey on the high-brow meaning-determinisms to the philosophy of mathematics i.e. Kripke's objection to the meaning-platonism, this issue will be elaborated in the next part. On this basis Kripke is trying to provide a skeptical solution to his puzzle:[4]

> The skeptical solution has two parts that are usefully distinguished. The first consists in the suggestion that we replace the notion of truth conditions, in our intuitive picture of sentence meaning, by that of assertability conditions. The second consists in a description of the assertability conditions for meaning-attributing sentences, in the course of which it is argued that it is essential to such sentences that their assertability conditions advert to the actions or dispositions of a community. [Boghossian 1989, 518]

First of all, it appears that if we accept Kripke's thesis that there can be no mental states that are meaning, we have to also accept the thesis that no classical-realist fact whatsoever can play this role [Kusch 2006, 25]. Kripke's idea is to apply the assertability conditions to interpret discourse and to turn to the communal usage of the language. The second main thesis advocated by Kripke is that the answer to the question about meaning in skeptical problems forces an answer to the metaphysical question about facts. This thesis will be subsequently withdrawn and replaced with more appropriate one. In addition, the minimal factualist account of the Kripkean solution will be advocated. Agreeing with the thesis that meaning plays a central role in Kripke's inquiry, we are now ready to draw some distinctions concerning the problem of meaning in mathematics.

---

3 [Kusch 2006] priovides the necessary introduction and [Ryle 1951] could serve as basic reading for dispositionalist account.

4 One could discuss the meaning of the term truth-conditions. Truth- conditions are part of the intuitive picture of meaning determinism, and as such, should be addressed by a skeptical solution.

3 Meaning and objectivity in mathematics

The first point of the analysis of Kripke's move in mathematics will be a short elaboration of the problems with meaning and objectivity in mathematics. According to Dummett's interpretation, we could distinct different types of interpretation of the concept of meaning in mathematics. First of all, we could adopt the skeptical solution sketched above to its full extent and advocate that the meaning of mathematical statement determines and is exhaustively determined by its use [Dummett 1973, 216]. I will delay dwelling on the criticism of this interpretation until the next section. The second type of the interpretation of meaning is interpretation via the learning of the mathematics. Dummett argues that this interpretation is closely connected to the computational aspect of mathematics [Dummett 1973, 216]. For this account we could provide two crucial problems. First of all, if we are denying the classical notion of truth in mathematics, to what extent is this interpretation more suitable than the first one? Secondly, if we hold with the notion of truth then one may ask if the truth-value of the simple arithmetical problem is settled with proper computation, how could we be sure that the equation was true before the computation or how could it be true? [Dummett 1973, 233]. The third type of interpretation involves a Platonic account that will tie the notion of the grasping of meaning to the notion of truth. Potential problems with this interpretation are similar to the problems with meaning-determinism according to Kripke's critique. Prima-facie, Platonism cannot be saved from the skeptical argument but, as it will be argued later, a certain version could be salvaged from it [Dummett 1973, 233]. The fourth type of interpretation is that the meaning of a mathematical statement consists of the capacity to recognize the proof of it. I will withhold the precise elaboration of the problems tied to the 'use is proof' thesis for this essay — the most important issue is to distinguish the differing conception of meaning in mathematics between those interpretations.

Meaning in mathematics is inseparable from the notion of truth and is closely tied to the notion of objectivity. Before the final part of this essay, three concepts of objectivity should be elaborated and their different mathematical stances distinguished. According to [Wright 1993] we could speak of three different notions of objectivity. First, we could talk

about the objectivity of truth, defined by saying that a class of statements may be fully intelligible to the speaker although resolving truth-values may be beyond the scope of an agent's cognitive abilities. Second, we could recognize the objectivity of meaning by saying that: "the notion of the meaning of a statement is a real constraint, to which we are bound […] and to which verdicts about its truth-value may objectively conform, or fail to conform, quite independently of our considered opinion on the matter." [Wright 1993, 5] And third, we could speak of an objectivity of judgement, the feature that statements possess when they record the features of the real world (statements with "genuine factual" subject matter) [Wright 1993, 6]. In this setting, the objectivity of truth implies the objectivity of meaning and judgement.[5]

Returning to the main problem, we should ask about the possible connection between meaning and objectivity in mathematics. The current inquiry places us in a position to only deal with the objectivity of meaning in mathematics. Why is this so? First of all, the issue of the objectivity of judgement will need the close elaboration of the notion of 'mathematical' reality and so will require us to deal with the problem of the existence of mathematical objects which we put aside. Second, the problem of the objectivity of truth, implying both the objectivity of meaning and judgement, will require an analogous effort from us. The objectivity of meaning, treated independently, is vulnerable via the notion of truth-conditions to Kripke's skeptical argument. What is more, the transition from the local to a global viewpoint and from the thesis about meaning to the thesis about metaphysics is the subject of extensive critique [Kusch 2006] [Posłajko 2012] and one should not adopt it in mathematical discourse without further inquiry.

5 One important distinction should be added and a whole class of possible problems excluded from the present inquiry. In this paper, the thesis that the objectivity of truth in mathematics and the status of mathematical objects are separate is adopted. The question concerning the reality of mathematical objects and its relation to the notion of the truth in mathematics is far more complex than it is possible to deal with in this essay and we could argue that the matter of the objectivity of truth is more important in the subsequent discussion [Wright 1993, 9] [Dummett 1973, 228].

4 The consequences of Kripke's move

Having presented the 'classical' interpretation of Kripke's problem and the solution to it, we have adopted metaphysical non-factualism as advocated by Kripke [6]. Before we move to a possible interpretation of this thesis in the philosophy of mathematics, its irrefutability should be settled yet this is impossible. Boghossian, in his critique of Kripke's solution, presented proof that local non-factualism could not be the case and from this he derived the thesis that only robust realism could serve as the right conception of meaning [Boghossian 1989, 507 and 549].[7] From this conclusion we could derive the view that the Kripke-like analysis of mathematical discourse will result in the adoption of the Platonistic stance yet this is not the case as well. M. Kusch's precis of a C. Wright argument [Kusch 2006, 151] against Boghossian's interpretation opened up the issue of the factualist interpretation of Kripke's problem and its solution.[8] The core of the simplest of the two arguments is the claim that the non-factualist thesis about meaning must apply to itself and, as such, it cannot acclaim to have made a discovery about language [Kusch 2006, 155]. Boghossian's refutation of non-factualism fails and his argument in favor of robust realism is therefore not valid. [Kusch 2006] pointed out that the alleged inconsistency of the interpretation of the skeptical solution could be avoided. Realism could still be challenged by the classical Kripke argument, but the skeptical solution would be stated in a non-factual way.[9]

---

6 Part of the strategy to refute local non-factualism is to maintain the thesis that local non-factualism leads straight to global non factualism according to Kripke [Boghossian 1989]. Boghossian, however, fails to elaborate this 'straight' way as pointed out by [Kusch 2006, 151–154] and [Wilson 1994].

7 An important methodological remark should be made. Boghossian and Kripke use negative-enumeration arguments to attack opponents. The positive thesis of the opponent is formulated and its possible extensions are surveyed. From this, conclusions are drawn and possible viewpoints supporting this thesis are formulated and the inconsistency of them is proven. On that basis, the alternative thesis to the positive thesis stated in the beginning is adopted as the only solution to the problem. Both of them are not generating general arguments against the main theses of the opponent. This was pointed out in [Posłajko 2012].

8 See also [Posłajko 2012].

9 One comment should be made to make things clear. We are talking about the refutation of Boghossian's argument against the non-factualism and yet we are providing another argument supporting the same conclusion, why is this so? The

Many of the commenting authors advocate the use of the minimal factualist thesis as the proper interpretation of Kripke's account [Kusch 2006, 175–176] [Posłajko 2012, 156]. The thesis could be stated that meaning scepticism advocates a minimal, deflationary[10] form of factualism:

> (i) A normal declarative sentence ("normal" judged by communal criteria) is meaningful for an interpreter if, and only if, he is able to make out, in his own terms, what the sentence says (here the interpreter will be guided by assertability conditions). If it is correct to say: "what the sentence says is true", then it is also correct to say "what the sentence says is a fact". A sentence is true if what it says is true.
>
> (ii) A speaker understands a sentence s if he knows what that sentence says or states (in the present context). And hence he knows that s is true if he knows that what the sentence says or states is true. [Kusch 2006, 176][11]

Unfortunately this thesis – due to the limited scope of this essay – should be established without proof[12]. It is not difficult to see that this thesis blocks from one perspective the Platonic interpretation of meaning and, at the same time, also saves meaning from non-factualism and this turns out to be inconsistent. The implications for mathematics will be provided in the next section.

---

first point is that Boghossian's formulation of the problem supports the robust realism conclusion and robust realism could be simply attacked by Kripke's sceptic. Secondly, a better argument against non-factualism was formulated by [Wright 1984] and approached by [Kusch 2006, 155–156] and this one was used to support a thesis distinct from the robust realism.

10 If we adopt a "deflationary" view of truth we deny that "true" stands for a language-independent property and insist that its meaning is exhausted by the certain syntactic–semantic operations it allows for [Kusch 2006, 151].

11 Deflationary and inflationary theses about meaning were elaborated in [Boghossian 1990]. The variety of deflationistic theories of truth could be developed within a mathematical framework, within this essay a particular version of deflationism will be used.

12 For a detailed view on that matter see: [Kusch 2006, 148–176] [Posłajko 2012, 156–181]

5 What does it mean for mathematics?

In this section I would like to deal with the possible interpretations of solutions for meaning scepticism. I will try to briefly introduce Platonism in two variations and intuitionism[13].

Full-blooded Platonism could be stated as a simple thesis that:

Definition 2 the *subject matter of mathematics consists of non-reducible, objective abstract entities*

The first thing to notice is that the standard formulation of this thesis is rather about the objectivity of truth issue than objectivity of meaning. One should keep in mind that we will be dealing with the meaning problem and this thesis should be interpreted as a thesis about meaning. I will not be specifying the general problems connected with this view but would like to settle for its Kripkean interpretation. In the simplest possible view, by accepting full-blooded Platonism we are condemned to (robust) factualism and to the meaning as grasping thesis which was attacked by Kripke's sceptic. The first part could be overcome due to Kusch's interpretation of Kripke's problem, but the second part is immediately attacked and crushed by the sceptic:

> For Wittgenstein, Platonism is largely an unhelpful evasion of the problem of how our finite minds can give rules that are supposed to apply to an infinity of cases. Platonic objects may be self-interpreting, or rather, they may need no interpretation; but ultimately there must be some mental entity involved that raises the skeptical problem. [Kripke 1982, 54]

An important thing to mention is that if Boghossian's account were refined and defended, we would have no problems with robust realism and Platonism. However, being unable to refine Boghossian's position, we could try to refine Platonism. The second version of Platonism will be far removed from Boghossian's view and this is achieved by adopting deflating Platonism [Tait 2005]. This view could be stated as:

Definition 3 [...]*proposition A is true hen there is an object of type A, and that a proof of it is the construction of such an object* [Tait 2005, 79]

13 I will not be discussing Logicism and the different versions of constructivism for two reasons. Firstly, Logicism seems to be a purely extrinsic view on the relation of mathematics and logic, dealing mostly with some formal aspects and not a metaphysical stance. Secondly, the problem of Kripke's interpretation could be accurately interpreted within an intuitionistic framework which could be perceived as canonical for constructivism. The problem of the interpretation of other constructive accounts is beyond the scope of this inquiry.

We have a clear connection between the notion of truth and the notion of proof which resembles Dummett's account but Tait also claims that the existence of the object states a fact but from within the mathematical framework. Tait is closely related to the internal/external questions of the framework, a distinction made by Carnap [Horsten 2012]. Deflating Platonism fits perfectly into the framework determined by Kusch's minimal interpretation of Kripkenstein. Moreover, Tait's view on the external questions of the mathematical framework (mainly philosophical) supports the view that mathematics needs no philosophical inquiry and could settle the problem of the existence of its objects by itself. This is the main advantage and weakness of Tait's proposition. One could ask about the criterion of the existence of mathematical objects in different domains and for the possibility of the unification of mathematics and Tait would say that the distance between them prevents the possibility of unification. The criterion of existence is beyond the current limits of investigation but unification should truly be the case. For the Platonist, this absence is counterintuitive. The major objection to Tait's criterion of existence is that for Platonism the existence of the object always seems to be external to the proposed semantic framework. For Tait, everything could be settled from within this domain and thus we are presented with the vicious circle problem.[14]

The next possible interpretation of meaning in mathematics is the "meaning as proof" thesis connected with the intuitionist stance in the philosophy of mathematics. The main advocate of this stance would be M. Dummett who advocates adopting assertabilist semantics to all languages in general and thus to mathematical language as a special case [Shapiro 2007, 324]. Dummett has two strong arguments in favor of his account. Firstly, he introduces the language acquisition argument, namely that during language acquisition we only learn assertability conditions and they define our grasp of the meaning, and secondly, a pragmatic argument that only assertability conditions allows us to demonstrate an understanding of language [Shapiro 2007, 324]. Having those arguments in support of some interpretation of Kripke's solution one could advocate that:

14 Tait's deflationism should not be confused with deflationism about mathematics, developed by H. Field [Field 1980]. The latter argues that the whole of mathematics is deflationary, not only the notion of truth within mathematics.

> mathematical statement is intuitionistically true if there exists an (intuitionistic) proof of it, where the existence of a proof does not consist in its platonic existence in a realm outside space and time, but in our actual possession of it [Dummett 1973, 239]

Dummett's view on the nature of Kripke's skeptical problem which was provided earlier strongly suggests this conclusion. S. Shapiro argues that the adoption of the use theory of meaning does not necessarily lead to Dummett's conclusions [Shapiro 1991, 212] but it follows from it that ontology and truth conditions must be compatible with language acquisition i.e. the understanding and learnability of it. The second point is that Shapiro's view is compatible with the very model–theoretical semantics which was attacked by Dummett. This could be refuted by saying that the grasp of the model theory is mediated by its informal usage and thus it is a practice of mathematicians to grasp concepts by usage [Shapiro 1991, 213]. This proves that we have more than the Kusch-related problems to consider with the basics of the intuitionism formulated by Dummett. The main problem with Dummet's stance, and of relevance to Kripke's skeptical problem, could be tied to the notion of fact. Dummett's intuitionism is not fully compatible with the skeptical solution because he is not devoted to the non-factualist thesis. He rather argues that facts about meaning consist of facts about assertability conditions.[15] Dummett is also advocate of the use of the thesis that can be formulated as follows:

Thesis 1 [*Use Thesis] One understands the concepts embodied in a language to the extent that one knows how to use the language correctly* [Shapiro 1991, 211].

The notion of understanding ties the notion of meaning to the problem of learning and grasping the concepts. This leads to the problem with semantics by the need for replacement of truth conditions by proof conditions [Dummett 1973].

Dummett's view is incompatible with the skeptical solution to the paradox but the question of whether it is incompatible with a minimalist interpretation is more complex. One could argue that the factual nature of Dummet's intuitionsm is similar to the minimalist interpretation of factualism but, at the same time, Dummett's program is against any realist interpretation of the discourse and is thus incompatible with this

15 I am grateful to the anonymous reviewer for pointing out this issue.

interpretation. But again, we should point out the issue of the relation between the objectivity of truth and objectivity of meaning. The objectivity of meaning thesis, by limiting the scope of the inquiry, could serve as a common ground for adopting both a minimalist interpretation and Dummett's factual use thesis but the point of divergence will still be present in the usage of truth-conditions in the definition. I will leave this elaboration with the question of whether a change in the definition could be made to enable the possibility of merging those two viewpoints. By using the notion of meaning facts, intuitionism is vulnerable to the skeptical paradox but, by a close elaboration of it and thanks to a minimalist answer, it could remain unaffected by it. Sadly, intuitionism is also incompatible with the proposed solution and the validity of its core theses are a constant object of philosophical debate.

From those brief considerations about the possible interpretation of Kripke's skeptical problem and its solution from the mathematical point of view we could draw the following . If we are interpreting the thesis metaphysically from the realm of meaning then we would obtain strong constraints for any mathematical theory which we would like it to encompass. That is because the concept of meaning in mathematics is vulnerable to Kripke's skeptical problem. I have been arguing that from Kusch's point of view we would rather obtain a peculiar version of Platonism in the philosophy of mathematics but the peculiarity of this solution could be non-intuitive and, as such, could not be satisfactory. The prominent stances in the philosophy of mathematics (full-blooded Platonism and intuitionalism) are vulnerable to Kripke's critique and should be revisited and refined. Of those two, only deflationary Platonism is consistent with Kusch's interpretation of the solution to Kripke's problem. Due to the limited scope of this paper I would like to advocate the thesis that the problem of non-factualism should be addressed and resolved by any philosophical stance in mathematics that would like to be global in character.

6 Conclusions

Thus we can draw some conclusions. Having analyzed Kripke's skeptical problem and its skeptical solution, I then turned to the issue of meaning in mathematics and concluded that it is vulnerable to Kripke's skeptical problem. Through the possible changes in the (non-)factualist

interpretation of Kripke's analysis I was able to formulate a framework in which the main positions in the philosophy of mathematics could be analyzed and I conducted an analysis. Finally, I was able to identify the most accurate interpretation of mathematics that turns out to be globally unintuitive but which fits Kusch's interpretation of Kripke's skeptical problem well. The scope of this survey enforced the conclusion that any global theory of the philosophy of mathematics should resolve Kripke's (non-)factualist account of meaning.

This paper is a part of a research project financed by National Centre of Science (Poland) on the basis of the decision no. UMO-2016/20/T/HS5/00232.

REFERENCES

Boghossian, P., (1989). The Rule-Following Considerations. Mind, (98(392)).

Boghossian, P., (1990). The Status of Content. The Philosophical Review, Vol. XCIX, No. 2.

Dummett, M., (1973). Truth and other enigmas. Harvard Univ Press.

Horsten, L., (2012). Philosophy of mathematics. In Zalta, E. N., editor, The Stanford Encyclopedia of Philosophy. Summer 2012 edition.

Horwich, P., (1998). Meaning. Oxford, Clarendon Press.

Kripke, S., (1982). Wittgenstein on rules and private language. Harvard Univ Press.

Kusch, M., (2006). A Sceptical Guide to Meaning and Rules: Defending Kripke's Wittgenstein. Acumen and McGill-Queen's.

Posłajko, K., (2012). To wszystko nic nie znaczy. Faktualizm i nonfaktualizm w teorii znaczenia. OBI.

Ryle, G., (1951). The Concept of Mind. Hutchinsons University Library.

Shapiro, S., (1991). Foundation without Foundationalism: A Case for Second-order Logic. Clarendon Press.

Shapiro, S. (ed.) (2007). The Oxford Handbook of Philosophy of Mathematics and Logic. Oxford University Press.

Tait, W., (2005). The Provenance of Pure Reason. Oxford University Press.

Wilson, G., (1994). Kripke on Wittgenstein on Normativity. Midwest Studies in Philosophy, 19 (1994).

Wright, C., (1984). Kripke's Account of the Argument Against Private Language. Journal of Philosophy 71.

Wright, C., (1993). Realism, Meaning and Truth. Wiley-Blackwell.

ABSTRACT

KRIPKENSTEIN FROM THE MATHEMATICAL POINT OF VIEW: A PRELIMINARY SURVEY

This paper deals with the problem of the impact of Kripke's skeptical paradox on the philosophy of mathematics. By perceiving mathematics as a huge rule-following discipline, one could argue that the Kripkean non-factualist thesis should be adopted within the philosophy of mathematics en bloc to imply a refutation of objectivity and an enforcement of a particular view on the nature of mathematics. In this paper I will discuss this claim. According to Kripke's skeptical solution we should reject the notion of fact and adopt the use theory of meaning that could be stated as follows: 'One understands the concepts embodied in a language to the extent that one knows how to use the language correctly.' [Shapiro 1991, 211] [Kripke 1982]. Focusing on mathematical discourse, we should ask: what are the implications of the use theory of meaning for the philosophy of mathematics? Furthermore, is the answer to the skeptical paradox consistent with selected views in philosophy of mathematics? The supposed answer to the first question is that it demands the view that mathematics should be perceived as a strictly pragmatic discipline and the rules of mathematical discourse are mere conventions. But this is too simplistic a view and the matter at hand is far more complicated.

KEYWORDS: Kripke, philosophy of mathematics, non-factualism, Platonism