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Selected GARCH-type Models in the Metals Market – Backtesting of Value-at-Risk

Abstract: Risk analysis in the financial market requires the correct evaluation of volatility in terms of both prices and asset returns. Disturbances in quality of information, the economic and political situation and investment speculations cause incredible difficulties in accurate forecasting. From the investor's point of view, the key issue is to minimise the risk of huge losses. This article presents the results of using some selected GARCH-type models, ARMA-GARCH and ARMA-APARCH, in evaluating volatility of asset returns in the metals market. To assess the level of risk, the Value-at-Risk measure is used. The comparison between real and estimated losses (in terms of VaR) is made using the backtesting procedure.

Keywords: volatility, GARCH-type models, risk, Value-at-Risk, metals market

JEL: G01, G11, G31

1. Introduction

The assessment of investment risk is mostly based on the analysis of changes observed in prices or returns on financial assets. In the classical approach, the volatility of those changes is defined in terms of deviations from expected levels. Nevertheless, the characteristics of time series observed in real financial markets clearly reject the use of symmetric measures. There are many factors determining volatility. According to the basic classification, there are two groups of factors: systematic and specific. The first group is represented by changes observed in the majority of macroeconomic indicators (i.e. GDP, inflation, political policy, etc.). It is not possible to reduce such a kind of volatility by efforts of individuals. The other group is determined by factors directly related to the undertaken investment. These factors are called typical or specific. Each investor, if a proper investment strategy is used, is able to reduce their influence on the final result of investment.

Prices and returns volatility is not detached from behavioural attitudes of investors. Volatility represents a general mood observed in the market which affects the level of prices and returns. Each investor reacts subjectively and it is usually difficult to predict his or her behaviour. All these factors together cause the final result of the investment to be uncertain and possibly different from the expected one. In other words, the investment becomes risky. Of course, the difference between the real and expected future value of investment might be understood ambiguously. The most popular approach to risk defining is to look at this problem in its neutral or negative meaning. The neutral approach indicates that the final value of investment is different from the investor's expectations, whereas the negative aspect always assumes the loss of undertaken investment.

To describe properly the volatility of the studied phenomena, it is necessary to choose a suitable statistical model. The applied model depends on the type of volatility. If financial markets are of interest, two types of volatility can be distinguished: historical and implied volatility. The first type is associated with the identification of volatility observed in prices or returns on the basis of historical data. In turn, the implied volatility is associated with the activity of the investor in the area of derivatives, especially options (the implied volatility is calculated on the basis of prices for options issued for a specified underlying asset) (Parasuraman, Ramudu, 2011: 112–120).

As it can be seen, models describing volatility depend mainly on how volatility is defined. The analysis based only on variance (as a volatility measure) is insufficient. Taking into account the specific characteristics of time series in financial markets, the most popular models describing volatility belong to the class of GARCH models.

2. Models for conditional volatility: GARCH and APARCH

The autoregressive heteroscedasticity model of order q ($ARCH(q)$) was proposed as a theoretical tool for volatility modelling by Engle in 1982 (Engle, 1982: 987–1007). Due to the need of determining high orders of lags in the ARCH model (and hence a large number of unknown parameters to be estimated), in 1986 Bollerslev (1986: 307–327) proposed its generalisation called the GARCH model. Mathematically, any $GARCH(p, q)$ model can be described using the following equations[1]:

$$r_t - \mu = a_t = \sigma_t \varepsilon_t, \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{2}$$

where $\alpha_0 \geq 0$, $\alpha_i \geq 0$ for $i > 0$, $\beta_j \geq 0$, . The error term satisfies the assumptions of $\varepsilon_t N(0, 1)$ and of iid.

The class of GARCH models is comprehensively described in the literature. They have many interesting properties, i.e. the ability of modelling the heavy-tailed distribution. The main disadvantage is that they do not describe asymmetry in the data (the impact of positive and negative information), and do not describe the leverage effect or the long-memory effect.

To solve such problems, Ding et al. (1993: 83–106) proposed a new class of models describing the above-mentioned stylised facts of financial time series. This class of models is called APARCH (Asymmetric Power ARCH). It is a wide group of theoretical tools allowing conditional volatility modelling. The mathematical formula is an extension of (2) and takes the following form (Karanasos, Kim, 2006: 116):

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta, \tag{3}$$

where $\alpha_0 \geq 0$, $\alpha_i \geq 0$ for $i > 0$, $\beta_j \geq 0$, .

The error term has to satisfy the assumptions of $\varepsilon_t N(0, 1)$ and of iid. Moreover, in the specification (3) additional parameters δ and γ_i appear. The parameter δ plays the role of a Box-Cox transformation of the conditional standard deviation σ_t, while the parameter γ_i reflects the leverage effect. A positive (negative) value of the parameter γ_i means that past negative (positive) shocks have a deeper impact on the current conditional volatility.

[1] The equation system usually contains a conditional mean model.

Taking into account the class of *APARCH*(p, q) models, it is worth mentioning some detailed types, dependent on parameters (Karanasos, Kim, 2006: 118):

1) for $\delta = 2$, $\gamma_i = 0$, $\beta_i = 0$ – *ARCH*(q),
2) for $\delta = 2$, $\gamma_i = 0$ – *GARCH*(p, q),
3) for $\delta = 1$, $\gamma_i = 0$ – *TS – GARCH*(p, q),
4) for $\delta = 2$ – *GJR – GARCH*(p, q),
5) for $\delta = 1$ – *T – GARCH*(p, q),
6) for $\gamma_i = 0$, $\beta_i = 0$ – *N – ARCH*(q),
7) for $\delta \to \infty$ – log – *ARCH*(q).

As we can see, ARCH and GARCH models are special cases of APARCH. The estimation of unknown parameters is usually conducted using the maximum likelihood method.

Of course, we can point out many other statistical tools used effectively to describe the volatility observed in financial time series (Stochastic Volatility Models, Local/Stochastic Volatility Models, etc.). As this paper presents the results of our initial research in applying volatility models to the data from the precious metals market, we decided to use only the ARCH-based approach.

3. Conditional error distributions and model fit

While parameters of APARCH model are estimated, an important issue is to correctly determine the conditional distribution of error term ε_t. In terms of classical models, this class assumes that the distribution of ε_t is Gaussian (standard). In practice, however (due to the characteristics of processes which describe models of conditional variance), other types of distributions are used, especially those allowing for asymmetry, leptokurtosis or outliers. These conditional distributions of residuals can be described by the following density functions:

$$f_{Norm}(\varepsilon_t, \sigma_t^2; \theta) = \frac{1}{\sigma_t\sqrt{2\pi}} exp\left\{\frac{-\varepsilon_t^2}{2\sigma_t^2}\right\}, \tag{4}$$

$$f_{t-Stud}(\varepsilon_t, \sigma_t^2; \theta) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma_t \Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\left(1 + \frac{\varepsilon_t^2}{(v-2)\sigma_t^2}\right)^{\frac{v+1}{2}}, \tag{5}$$

$$f_{GED}(\varepsilon_t, \sigma_t^2; \theta) = 2^{\frac{-v+1}{v}} \frac{v}{\sigma_t\sqrt{\frac{\Gamma(v^{-1})}{\Gamma(3v^{-1})}2^{\frac{-2}{v}}}\Gamma(v^{-1})} exp\left\{\frac{-1}{2}\left|\frac{\varepsilon_t}{\sigma_t\sqrt{\frac{\Gamma(v^{-1})}{\Gamma(3v^{-1})}2^{\frac{-2}{v}}}}\right|^{v}\right\}, \tag{6}$$

where $\{\varepsilon_t\}$ is the sequence of iid random variables, is the conditional variance, θ is the vector of estimated parameters, ν is the number of degrees of freedom and is the gamma function with the parameter k. The parameter ν has to be estimated if the t-Student distribution and GED distribution are used.

These types of conditional distributions are commonly used in practice. However, it is also possible to include certain modifications in classical distributions to fit them properly to the data. Looking at the fact that empirical distributions of returns are characterised by a significant level of asymmetry, it is possible to modify the symmetric distribution to take into account the asymmetry observed in the data (Piontek, 2005: 300).

Let $g(x)$ be the probability density function of random variable X and let it be defined by the functions $k_1(\varsigma)$ and $k_2(\varsigma)$. Any skewed distribution $f_\varsigma(x)$ is described by the formula:

$$f_\varsigma(x) = \frac{2\left[g\left(\frac{x}{k_1(\varsigma)}\right)I_{x\geq 0} + g\left(\frac{x}{k_2(\varsigma)}\right)I_{x<0}\right]}{k_1(\varsigma)+k_2(\varsigma)}, \tag{7}$$

Where I_x is the indicator function. If $f_\varsigma(x)$ defines the skewed probability distribution, the functions k_I and k_{II} usually have the form:

$$k_I = \begin{cases} k_1(\varsigma) = \varsigma \\ k_2(\varsigma) = \frac{1}{\varsigma} \end{cases} \tag{8}$$

or

$$k_{II} = \begin{cases} k_1(\varsigma) = 1 - \varsigma \\ k_2(\varsigma) = 1 + \varsigma \end{cases} \tag{9}$$

The function k_I uses the parameter ς for modelling asymmetry (in addition $\varsigma \in (0, +\infty)$). For $\varsigma \in (0, 1)$, the final distribution is skewed to the left, whereas for $\varsigma \in (1, +\infty)$, the distribution is skewed to the right respectively. For the function k_{II}, the parameter $\varsigma \in (-1, 1)$. For $\varsigma \in (-1, 0)$, the final distribution is skewed to the left and for $\varsigma \in (0, 1)$, the distribution is skewed to the right. Any symmetric distribution is obtained if $\varsigma = 1$ (for k_I) and if $\varsigma = 0$ (for k_{II}).

The assessment of fitting GARCH-type model to the data is usually based on the information criteria: Akaike (AIC), Schwarz (BIC), and Hannan-Quinn (HQC). The values of information criteria are calculated using the formulas below:

$$AIC = -2ln[LLF(\check{\theta})] + 2k \tag{10}$$

$$BIC = -2ln[LLF(\check{\theta})] + kln(n) \tag{11}$$

$$HQC = -2ln[LLF(\check{\theta})] + 2kln[ln(n)] \tag{12}$$

where $LLF(\check{\theta})$ is the log-likelihood function of the parameters vector $\check{\theta}$, k is the number of estimated parameters, and n is the number of observations. The lower the values of information criteria, the better the model.

4. Value-at-Risk and backtesting of VaR

Value-at-Risk (VaR) is one of the most popular measures of risk and is defined as a representation of a potential loss of investment which can occur within some time interval with an arbitrarily determined tolerance level. The general formula defining VaR is as follows (Piontek, 2002: 477):

$$P(V_t \leq V_{t-1} - VaR_\alpha), \tag{13}$$

where V_t is the value of investment at the end of the analysed period, V_{t-1} is the current investment value, and α is the tolerance level. If the asset return is of interest, the formula (13) has the following representation:

$$VaR_\alpha(X) = r_t F_t^{-1}(x), \tag{14}$$

Where r_t is the return on asset X at the time-point t, whereas is the inverse cumulative distribution function.

There are many different methods of estimating VaR but all of them are strongly related to investment risk management. The most popular methods for estimating VaR are the variance-covariance method, the historical simulation method, and methods based on the Extreme Value Theory, or the quantile-based method. All of these methods are determined by some specific assumptions, such as the form of probability distribution, relations between assets in the created portfolio, etc. However, if we look deeper into the theoretical background of VaR, we can find that this measure is not perfect. Its value answers the question about the minimum loss from the investment in α possibles cases. As VaR represents a threshold, this measure of risk does not take into account possibilities of occurring losses exceeding its level. Therefore, an alternative risk measure is the Expected Shortfall (ES, called Conditional VaR) and Median Shortfall (MS) defined as:

$$ES_\alpha(X) = E[r_t - VaR_\alpha(X) | r_t > VaR_\alpha(X)] \tag{15}$$

$$MS_{\alpha}(X) = Me[r_t - VaR_{\alpha}(X)|r_t > VaR_{\alpha}(X)] \tag{16}$$

The disadvantage of VaR is that as a risk measure it is not coherent. VaR does not satisfy the property of subadditivity which assumes that if a portfolio investment is of interest, the overall risk of the portfolio is not higher than the sum of individual risks. This property is satisfied by ES and MS.

The accuracy of VaR models is assessed using the backtesting procedure. To evaluate the effectiveness of estimating VaR, one of the most popular approaches assumes that the series of failures is used in the form presented below (Ganczarek, 2007: 315–320):

$$[I_{t+1}(\alpha)]_{t=1}^{t=T} = \begin{cases} 1, r_t \leq -VaR_{\alpha} \\ 0, r_t > -VaR_{\alpha} \end{cases} \tag{17}$$

The popular statistical tool used in practice is the Proportion of Failures Test (POF[2]) proposed by Kupiec (1995: 73–84):

$$LR_{POF} = -2ln\left\{\frac{(1-\alpha)^{T-N}\alpha^{N}}{\left[\left(1-\frac{N}{T}\right)^{T-N}\right]\left(\frac{N}{T}\right)^{N}}\right\}, \tag{18}$$

where N is the number of observations exceeding VaR for the series of length T.

5. Precious metals market

This paper presents an alternative approach to the classical financial investments (mainly related to the stock exchange), namely investments in precious metals. This type of assets is one of the oldest financial instruments, but their ability to multiply invested capital is not fully exploited in practice. From the historical perspective, precious metals represent wealth of individuals. Investors that hold this kind of assets are psychologically perceived as more credible or more stable, especially under the conditions of growing market uncertainty. One of the most interesting properties of precious metals is their low correlation with most asset classes and their resistance to unpredicted events which may generate extreme risk. They can therefore be treated as an effective and reliable tool for risk management, especially in terms of portfolio hedging. By adding precious metals to a diversified portfolio, its efficiency can increase, whereby the portfolio risk is lowered, while the portfolio return remains the same or increases (if compared to a diversified portfolio without a precious metals allocation) (Batten et al., 2010: 65–71).

[2] Under the null hypothesis, the LR_{POF} test has χ^2 distribution with 1 degree of freedom.

In general, metal prices are relevant not only for manufacturers and end-users, but also for the economy as a whole. Therefore, their prices/returns volatility has to be examined and well assessed. Taking into account the economic situation of recent years, the importance of precious metals has increased significantly. They provide effective hedging of undertaken investments, especially in times of destabilisation of the economy or a crisis. Precious metals are resistant to changes observed in the market. Over the past four years, the demand for precious metals has been considerably weakened as a result of the improving capital market situation.

When analysing portfolio investments, precious metals are an important part of well-balanced portfolios. They can effectively protect against a high level of volatility and risk observed in the market. Nowadays, we can point out many alternative investment opportunities in the field of precious metals. We can mention direct investments related to the physical purchase of metals, but also indirect investments, in the form of futures contracts, capital market operations, as well as investment funds, Exchange Traded Funds or structured products. Nevertheless, regardless of the choice of form of investment in the investment process, the level and rate of price volatility (and return as well) should be taken into account. It is also worth mentioning the fundamental factors of the price level of precious metals: demand and supply. The main sources of supply are mines, the recovery of precious metal scrap, commercial banks, and central banks. In addition, the world's economic and geopolitical situation, interest rates, central bank policies and the exchange rates associated with them are very significant for precious metal prices (Charles et al., 2015: 284–291).

Recently, investors have many opportunities to invest their money in precious metals. As mentioned before, one of the most popular investment forms is the indirect one. Its main advantage is higher liquidity and security, and the reduction of high volatility resulting mainly from speculative activities.

6. Empirical analysis

The empirical example presents the practical application of APARCH-type models in volatility modelling. The analysed data are the log-return of four precious metals: GOLD, SILVER, PLATINUM, and PALLADIUM. The returns are calculated using daily spot closing prices of these metals quoted on the LME within the period January 2010 – December 2015. The verified models are nested ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-APARCH(1,1) for different conditional error distributions: normal, student and GED. The quality of models was assessed using information criteria of AIC, BIC, and HQC. The backtesting procedure of VaR was conducted for VaR at the confidence level 0.95 and 0.99. First, the time series of returns and squared returns for selected metals is presented in Figure 1 (examples for SILVER and PLATINUM).

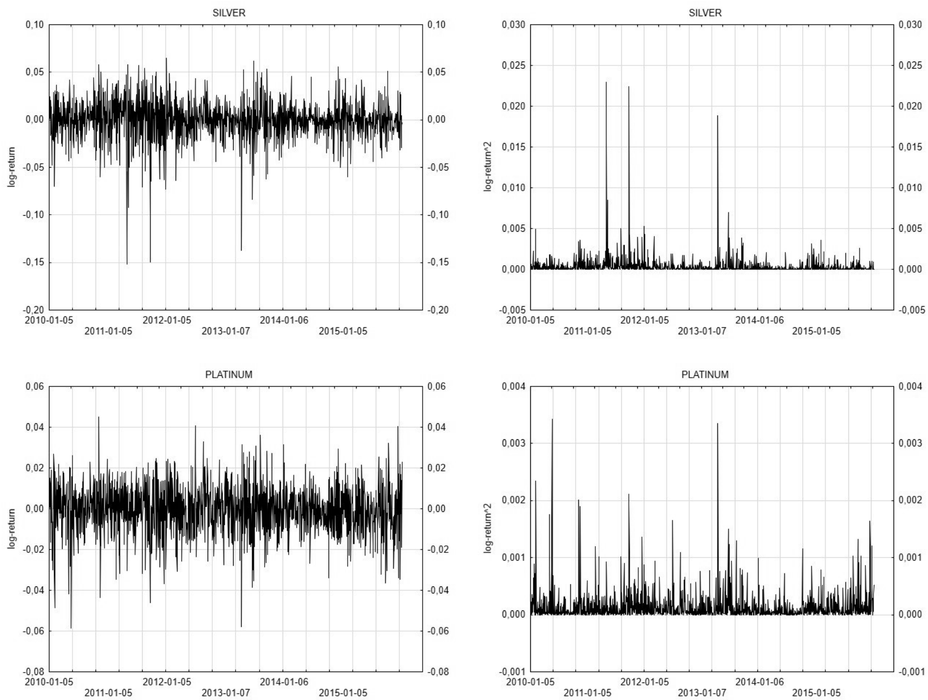

Figure 1. Time series of return (left) and squared return (right) for SILVER (top) and PLATINUM (bottom)

Source: own calculations

As we can see in Figure 1, there are periods characterised by a higher level of volatility. It is easy to show clustering in variance as well. The descriptive statistics for each asset are shown in Table 1.

Table 1. Descriptive statistics

Metal	Mean	Min	Max	Variance	Skewness	Kurtosis
GOLD	–0.000033	–0.092414	0.039691	0.000116	–0.793546	5.692985
SILVER	–0.000152	–0.151437	0.065189	0.000399	–1.213096	7.832212
PLATINUM	–0.000349	–0.058556	0.044835	0.000144	–0.380736	1.392237
PALLADIUM	0.000202	–0.083097	0.060895	0.000326	–0.391324	1.519173

Source: own calculations

It is worth noting that only investments in PALLADIUM generated profits in the average meaning. The highest loss was observed for PLATINUM. All returns are described by empirical distributions which are leptokurtic and skewed to the left. SILVER proved to be the most risky investment. The results show that probably empirical distributions are not normal. Table 2 provides results for testing normality.

Table 2. Normality tests

	Kołmogorow-Smirnow			Shapiro-Wilk		
	Statistics	**df**	**p-value**	**Statistics**	**df**	**p-value**
GOLD	0.075	1512	0.000	0.948	1512	0.000
SILVER	0.090	1512	0.000	0.920	1512	0.000
PLATINUM	0.038	1512	0.000	0.986	1512	0.000
PALLADIUM	0.045	1512	0.000	0.982	1512	0.000

Source: own calculations

None of the empirical distributions belongs to the family of normal distributions, which is confirmed in Figure 2 (examples for GOLD and PALLADIUM).

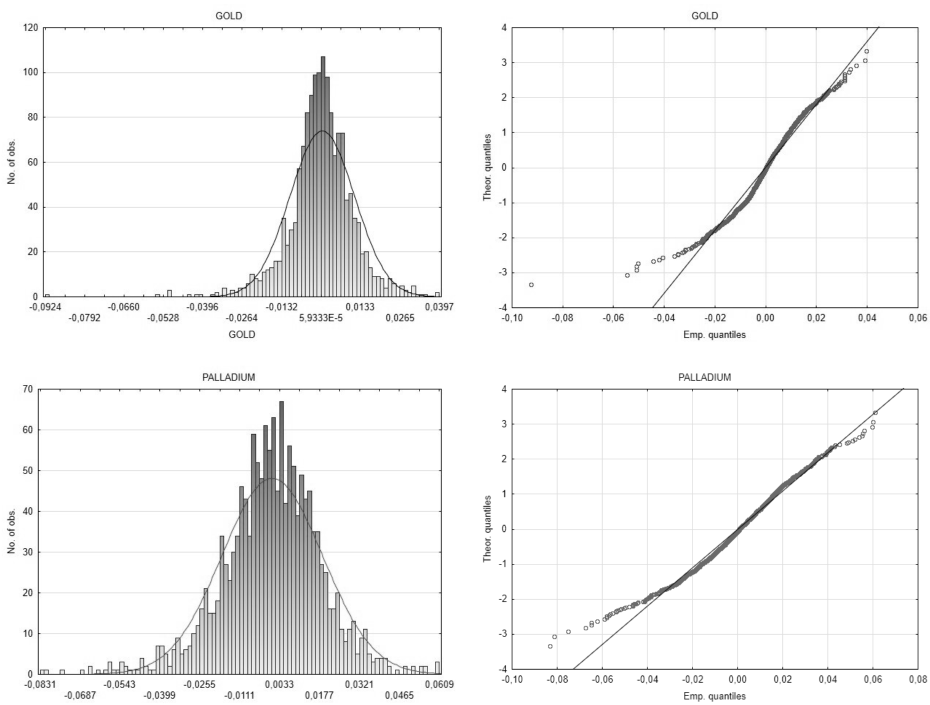

Figure 2. Histogram and QQ-plot for GOLD (top) and PALLADIUM (bottom)

Source: own calculations

The characteristics of financial time series observed in Figures 1–2 suggest a need to use a more sophisticated approach for volatility modelling than the one based on variance. In this paper, a class of mixed models is proposed: ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-APARCH(1,1) for three types of error distributions: normal, student and GED. At this stage, it is worth presenting the charts for ACF and PACF functions (examples for PLATINUM and PALLADIUM).

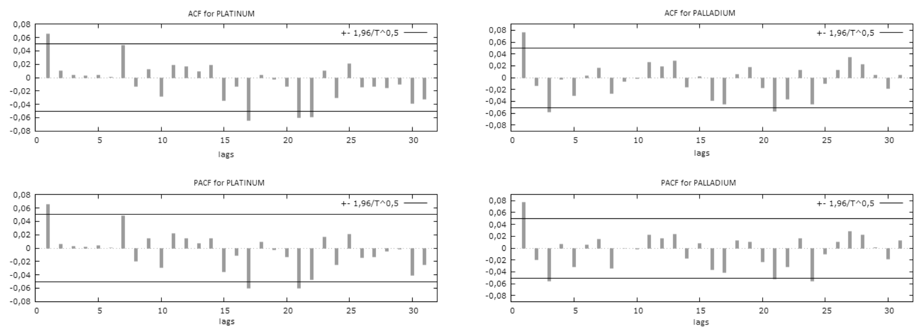

Figure 3. ACF (top) and PACF (bottom) for PLATINUM (left) and PALLADIUM (right)

Source: own calculations

The information presented in the charts of autocorrelation and partial autocorrelation functions indicates the order of lags for the ARMA part of the model. At the next stage, the parameters for both ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-APARCH(1,1) models were estimated, presented respectively in Tables 3 and 4.

Table 3. Estimated parameters for ARMA(1,1)-GARCH(1,1) models using different error distributions

PARAMETERS	μ	φ_1	ϕ_1	α_0	α_1	β_1
GOLD-N	–0.000034	–0.866590*	0.889920*	0.0000034*	0.0521569*	0.9181260*
p-value	0.905	0.000	0.000	0.008	0.000	0.000
GOLD-S	–0.000034	–0.866590*	0.889920*	0.0000016*	0.0393204*	0.9499580*
p-value	0.905	0.000	0.000	0.043	0.000	0.000
GOLD-GED	–0.000034	–0.866590*	0.889920*	0.0000019	0.0410499*	0.9411320*
p-value	0.905	0.000	0.000	0.088	0.007	0.000
SILVER-N	–0.000153	–0.863676*	0.894182*	0.0000127	0.0977150*	0.8740900*
p-value	0.770	0.000	0.000	0.136	0.028	0.000
SILVER-S	–0.000153	–0.863676*	0.894182*	0.0000015	0.0313255*	0.9674500*
p-value	0.770	0.000	0.000	0.306	0.006	0.000
SILVER-GED	–0.000153	–0.863676*	0.894182*	0.0000029	0.0401945	0.9525580*
p-value	0.770	0.000	0.000	0.529	0.281	0.000
PLATINUM-N	–0.000348	0.159606	–0.094631	0.0000091	0.0731035*	0.8640980*

PARAMETERS	μ	φ_1	ϕ_1	α_0	α_1	β_1
p-value	0.295	0.630	0.777	0.284	0.036	0.000
PLATINUM-S	–0.000348	0.159606	–0.094631	0.0000054	0.0609924*	0.9020670*
p-value	0.295	0.630	0.777	0.192	0.009	0.000
PLATINUM-GED	–0.000348	0.159606	–0.094631	0.0000070	0.0654589*	0.8865160*
p-value	0.295	0.630	0.777	0.208	0.014	0.000
PALLADIUM-N	0.000204	–0.023916	0.102124	0.0000047	0.0633423*	0.9231630*
p-value	0.682	0.846	0.408	0.202	0.011	0.000
PALLADIUM-S	0.000204	–0.023916	0.102124	0.0000038	0.0702613*	0.9210760*
p-value	0.682	0.846	0.408	0.239	0.007	0.000
PALLADIUM-GED	0.000204	–0.023916	0.102124	0.0000042	0.0670419*	0.9216080*
p-value	0.682	0.846	0.408	0.206	0.007	0.000

* statistical significance at the level 0.05.
N – normal distribution, S – student distribution, GED – General Error Distribution.

Source: own calculations

As we can infer from the data presented in Tables 3–4, not every model is statistically significant. If the ARMA(1,1)-GARCH(1,1) model is of interest, statistical significance is observed for GOLD and SILVER, regardless of conditional error distribution. Taking into account the APARCH part of the model for describing conditional volatility, the majority of statistically significant parameters observed for SILVER are for the normally distributed error term. It is interesting that if the APARCH model is considered, the parameters describing the leverage effect are statistically significant only for PALLADIUM. The positive value (observed mostly for PLATINUM and PALLADIUM) means that the past negative shocks have a deeper impact on the current volatility. Tables 5–6 show the assessment of quality for both types of models.

Table 4. Estimated parameters for ARMA(1,1)-APARCH(1,1) models using different error distributions

PARAMETERS	μ	φ_1	ϕ_1	α_0	α_1	β_1	γ_1	δ
GOLD-N	–0.000034	–0.866590*	0.889920*	0.000005	0.066052	0.900640*	0.151063	1.626530
p-value	0.905	0.000	0.000	0.569	0.385	0.000	0.720	0.258
GOLD-S	–0.000034	–0.866590*	0.889920*	0.000002	0.046975	0.947334*	–0.044724	1.673590
p-value	0.905	0.000	0.000	0.055	0.054	0.000	0.686	0.068
GOLD-GED	–0.000034	–0.866590*	0.889920*	0.000002	0.044367	0.940010*	0.011149	1.866310*
p-value	0.905	0.000	0.000	0.100	0.086	0.000	0.519	0.036
SILVER-N	–0.000153	–0.863676*	0.894182*	0.0000134	0.1077310*	0.8742940*	–0.033763	1.7082100*
p-value	0.770	0.000	0.000	0.083	0.009	0.000	0.728	0.011
SILVER-S	–0.000153	–0.863676*	0.894182*	0.0000023	0.0467064	0.960134*	–0.113894	1.4226900
p-value	0.770	0.000	0.000	0.514	0.187	0.000	0.387	0.183
SILVER-GED	–0.000153	–0.863676*	0.894182*	0.0000051	0.0651205	0.9357270*	–0.073807	1.4468700*
p-value	0.770	0.000	0.000	0.553	0.293	0.000	0.537	0.039
PLATINUM-N	–0.000348	0.159606	–0.094631	0.0000089	0.0823188*	0.8716580*	0.1791900	1.2098900*
p-value	0.295	0.630	0.777	0.188	0.004	0.000	0.186	0.001
PLATINUM-S	–0.000348	0.159606	–0.094631	0.0000052	0.0697297*	0.908495*	0.0837301	1.1305600*
p-value	0.295	0.630	0.777	0.138	0.003	0.000	0.583	0.004
PLATINUM-GED	–0.000348	0.159606	–0.094631	0.0000068	0.0750439*	0.8926430*	0.1328730	1.1577800*
p-value	0.295	0.630	0.777	0.151	0.003	0.000	0.319	0.001
PALLADIUM-N	0.000204	–0.023916	0.102124	0.0000072	0.0675569*	0.9256710*	0.4478040*	0.9432450*
p-value	0.682	0.846	0.408	0.121	0.002	0.000	0.008	0.000
PALLADIUM-S	0.000204	–0.023916	0.102124	0.0000056	0.0765990*	0.927371*	0.5034770*	0.9248120*
p-value	0.682	0.846	0.408	0.132	0.001	0.000	0.005	0.000
PALLADIUM-GED	0.000204	–0.023916	0.102124	0.0000063	0.0695717*	0.9264560*	0.4642090*	0.9415910*
p-value	0.682	0.846	0.408	0.120	0.001	0.000	0.007	0.000

Source: own calculations

Table 5. Information criteria for ARMA(1,1)-GARCH(1,1) models

Metal	Error distribution	*LLF*	*AIC*	*BIC*	*HQC*
GOLD	Normal	4773.58	–9937.16	–9510.56	–9927.26
	Student	4852.20	–9694.40	–9667.79	–9684.49
	GED	4845.67	–9681.33	–9654.73	–9671.42
SILVER	Normal	3891.97	–7775.94	–7754.66	–7768.01
	Student	3981.16	–7952.32	–7925.71	–7942.41
	GED	3987.43	–7964.85	–7938.25	–7954.95
PLATINUM	Normal	4579.73	–9151.47	–9130.18	–9143.54
	Student	4594.08	–9178.15	–9151.55	–9168.25
	GED	4590.60	–9171.21	–9144.60	–9161.30
PALLADIUM	Normal	4011.20	–8014.39	–7993.11	–8006.47
	Student	4029.54	–8049.09	–8022.48	–8039.18
	GED	4025.42	–8040.84	–8014.23	–8030.93

Source: own calculations

Table 6. Information criteria for ARMA(1,1)-APARCH(1,1) models

Metal	Error distribution	*LLF*	*AIC*	*BIC*	*HQC*
GOLD	Normal	4774.33	–9536.66	–9504.74	–9524.77
	Student	4852.51	–9691.02	–9653.78	–9677.15
	GED	4845.68	–9677.36	–9640.11	–9663.49
SILVER	Normal	3892.64	–7773.29	–7741.36	–7761.40
	Student	3982.08	–7950.16	–7912.91	–7936.29
	GED	3988.19	–7962.39	–7925.14	–7948.52
PLATINUM	Normal	4582.49	–9152.99	–9121.06	–9141.10
	Student	4595.76	–9177.52	–9140.27	–9163.65
	GED	4592.53	–9171.07	–9133.82	–9157.19
PALLADIUM	Normal	4018.03	–8024.06	–7992.14	–8012.17
	Student	4036.15	–8058.29	–8021.05	–8044.42
	GED	4031.26	–8048.52	–8011.27	–8034.65

Source: own calculations

Taking into account the results provided by the information criteria, the lowest values, regardless of the type of metal, were obtained for the student or GED distribution, for both ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-APARCH(1,1) models. The theoretical models with corresponding real returns of SILVER and PALLADIUM are presented in Figures 4–5.

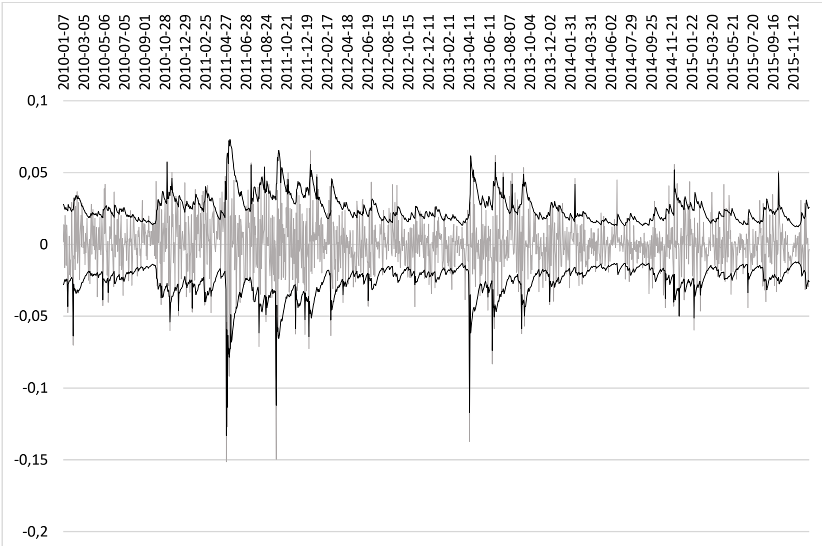

Figure 4. ARMA(1,1)-GARCH(1,1)-Normal model for SILVER

Source: own calculations

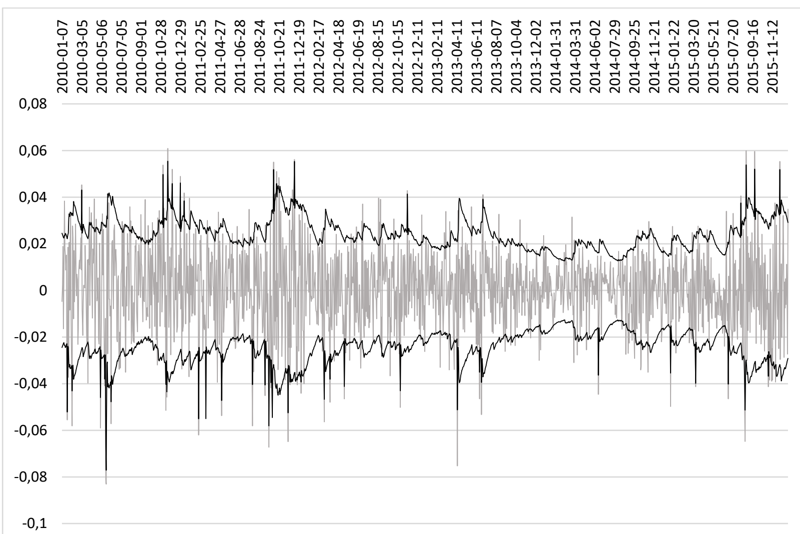

Figure 5. ARMA(1,1)-APARCH(1,1)-Student model for PALLADIUM

Source: own calculations

In the final part of the research, all the estimated models were analysed in terms of forecasting VaR. According to the formulas (14)–(16), the corresponding risk measures were calculated and presented in Table 7.

Table 7. Information criteria for ARMA(1,1)-APARCH(1,1) models

Risk measure	GOLD	SILVER	PLATINUM	PALLADIUM
$VaR_{0.95}$	0.015966	0.030080	0.018192	0.028266
$ES_{0.95}$	0.022678	0.040541	0.023528	0.037183
$MS_{0.95}$	0.020999	0.038872	0.022191	0.034241
$VaR_{0.99}$	0.027600	0.046788	0.025740	0.041081
$ES_{0.99}$	0.031517	0.053502	0.031705	0.050374
$MS_{0.99}$	0.031095	0.052211	0.030645	0.051040

Source: own calculations

The results presented in Table 7 indicate that the highest values of risk measure were obtained for investments in SILVER, whereas the lowest for investments in GOLD. These results strongly correspond to the basic descriptive statistics for these metals. Is worth mentioning again that taking into account the properties of risk measure VaR is not sufficient for measuring risk, so it is better to look at ES and MS. However, if we compare two coherent risk measures ES and MS, it is clear that MS has an advantage over ES, as the median in general is a robust measure, especially in terms of outliers.

At the end of the study, the hypothesis which states that the number of observations exceeding VaR complies with the expected one at the significance level is tested. The results of backtesting VaR using the POF test are shown in Tables 8 and 9.

Table 8. Backtesting VaR using ARMA(1,1)-GARCH(1,1) models

$VaR_{0.95}$	NORMAL		STUDENT		GED	
	No**	LR_{POF}	**No****	LR_{POF}	**No****	LR_{POF}
GOLD	97	5.88*	91	3.11	95	4.86*
SILVER	98	6.42*	90	2.73	94	4.39*
PLATINUM	96	5.36*	91	3.11	92	3.51
PALLADIUM	93	3.94*	89	2.37	94	4.39*
$VaR_{0.99}$	No	LR_{POF}	No	LR_{POF}	No	LR_{POF}
GOLD	26	6.51*	24	4.47*	24	4.47*
SILVER	26	6.51*	23	3.58	25	5.45*
PLATINUM	27	7.64*	25	5.45*	23	3.58
PALLADIUM	24	4.47*	25	5.45*	24	4.47*

* statistical significance at the level 0.05.
** No – the number of observations exceeding VaR.

Source: own calculations

Table 9. Backtesting VaR using ARMA(1,1)-APARCH(1,1) models

$VaR_{0.95}$	NORMAL		STUDENT		GED	
	No	LR_{POF}	No	LR_{POF}	No	LR_{POF}
GOLD	97	5.88*	91	3.11	95	4.86*
SILVER	98	6.42*	90	2.73	94	4.39*
PLATINUM	96	5.36*	91	3.11	92	3.51
PALLADIUM	93	3.94*	89	2.37	94	4.39*
$VaR_{0.99}$	No	LR_{POF}	No	LR_{POF}	No	LR_{POF}
GOLD	27	7.64*	22	2.77	22	2.77
SILVER	26	6.51*	24	4.47*	23	3.58
PLATINUM	27	7.64*	25	5.45*	26	6.51*
PALLADIUM	25	5.45*	22	2.77	24	4.47*

* statistical significance at the level 0.05.

Source: own calculations

The critical value verifying the hypothesis about the number of observations exceeding VaR is . The results presented in Tables 8 and 9 show that, regardless of the level of quantile, the model with the error term normally distributed is not correct. The values of Kupiec's test results are statistically significant. The study shows that the best model in this case is the one with residuals described by the student distribution (or rarely by the GED one). As we can see from the results of backtesting VaR, only for the student distribution of error term, the null hypothesis is not rejected.

7. Conclusions

The analysis of volatility observed in prices and returns of financial time series requires from researchers a somewhat deeper look into the background of theoretical models. This paper shows some results of the application of selected models and risk measures to describe the phenomenon of volatility observed in the precious metals market. The assets selected for this research are: GOLD, SILVER, PLATINUM, and PALLADIUM. Precious metals are an alternative investment area for classical investments undertaken in the capital market, especially during financial crises. Comparing the properties of financial time series of returns in the precious metals market to those observed for stock returns, it is worth emphasising the similarity of tools used for risk analysis in both markets.

In this paper, a class of extended ARCH-type models is proposed – the asymmetric power ARCH models (APARCH). As presented in section 2, ARCH an GARCH models are special cases of APARCH models. This class of models was not proposed accidentally. The APARCH model contains two additional parameters. The first one, δ, plays the role of a Box-Cox transformation of the conditional standard deviation σ_t, whereas the parameter γ_i reflects the leverage effect. As mentioned, the positive (negative) value of the parameter γ_i means that the past negative (positive) shocks have a deeper impact on current conditional volatility. Looking at the results, it can be seen that the ARMA(1,1)-GARCH(1,1) model is statistically significant only for GOLD and SILVER, regardless of conditional error distribution. Taking into account the APARCH model, the majority of statistically significant parameters observed for SILVER are for the normally distributed error term. Additionally, if the APARCH model is used, the parameters describing the leverage effect are statistically significant only for PALLADIUM. The assessment of quality of the model with the use of information criteria indicates that the lowest values, regardless of the type of metal, were obtained for the student or GED distribution, so these error terms in the models should be used for volatility modelling.

The analysis of volatility was enriched with risk measurement using VaR and measures based on it: Expected Shortfall and Median Shortfall. These two last measures belong to the family of coherent risk measures. In addition to the estimation of risk measures, the hypothesis which states that the number of observations exceeding VaR complies with the expected ones at the significance level was verified using Kupiec's test. The analysis shows that the best model in this case is the one with residuals described by the student distribution.

Summing up all the remarks and conclusions presented in this paper, a need to use more advanced models in the assessment of volatility in the market of precious metals than classical GARCH-type models should be emphasised. It is also important to select an appropriate risk measure that will be robust to the presence of outliers.

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Wybrane modele klasy GARCH na rynku metali – testowanie wsteczne Value-at-Risk

Streszczenie: Analiza ryzyka na rynku finansowym wymaga poprawnej oceny zmienności zarówno cen, jak i stóp zwrotu interesujących inwestora aktywów. Szumy informacyjne, sytuacja gospodarcza oraz polityczna, a także zwykła spekulacja powodują istotne trudności w stawianiu trafnych prognoz. Z punktu widzenia inwestora kluczowym zagadnieniem jest minimalizacja ryzyka dużych strat. W artykule podjęto próbę zastosowania wybranych modeli zagnieżdżonych klasy ARMA-GARCH oraz ARMA-APARCH do oceny zmienności stóp zwrotu wybranych aktywów notowanych na rynku metali. Do oceny ryzyka inwestycji wykorzystano wartość zagrożoną VaR, natomiast jakość tej oceny z faktycznie zaobserwowanymi stratami zweryfikowano za pomocą wybranych testów przekroczeń.

Słowa kluczowe: zmienność, modele klasy GARCH, ryzyko, Value-at-Risk, rynek metali

JEL: G01, G11, G31

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