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ON THE METHOD OF DETECTING CHANGES IN TREND USING PERMUTATION TESTS

Abstract. The article presents a proposal of the test for detecting changes in trend. The proposed procedure refers to the permutation test. The use of this procedure allowed the adoption of fairly general assumptions. The proposed method can be applied , in particular to detect the appearance of the trend. In this case, it is possible to use this method to monitor industrial processes. The proposed method was compared with well- known in the literature methods of detecting disturbances in monitoring processes. For comparing procedures computer simulations were used.

Key words: trend, detecting changes, permutation tests, Monte Carlo study.

1. INTRODUCTION AND BASIC NOTATIONS

In practice of economic research we can often deal with the situation in which it is important to have information on changes or the appearance of trend. An early detection of changes in the trend in stocks and financial markets, gross domestic product at market prices, export totals in successive months, company profits in successive years and so on could be very important. In particular, the detection of the presence of a trend in manufacturing processes. The appearance of the trend in a monitored characteristic can inform about the deregulation of the process and give a signal to take an appropriate action. In the literature, various methods of detecting changes in trend are presentd (Shao, Zhang: 2010). There are papers which use the change point analysis in monitoring processes (Perry, Pignatiello 2006). Several tests to check stationary of time series are presented in Domański et al (2014). The method proposed in the article is also applicable when the basic assumptions of the analyzed time series are not met. This method can be used even if the errors in time series model are not normally distributed.

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Let us consider the stochastic process $\{Y_t, t = 1, 2, ...\}$. Let y_1, y_2, ..., y_T denote the time series which are the realization of this stochastic process. The main goal will be the detection of trend change in point k, where $1 < k < T$. The graphical representation of the trend (the dotted line) change in time k is presented in Figure 1. The first case shows the change of the direction in trend. The second one shows the change in the intensity of the linear trend. The next picture shows the disappearance and the last one the appearance of the linear trend.
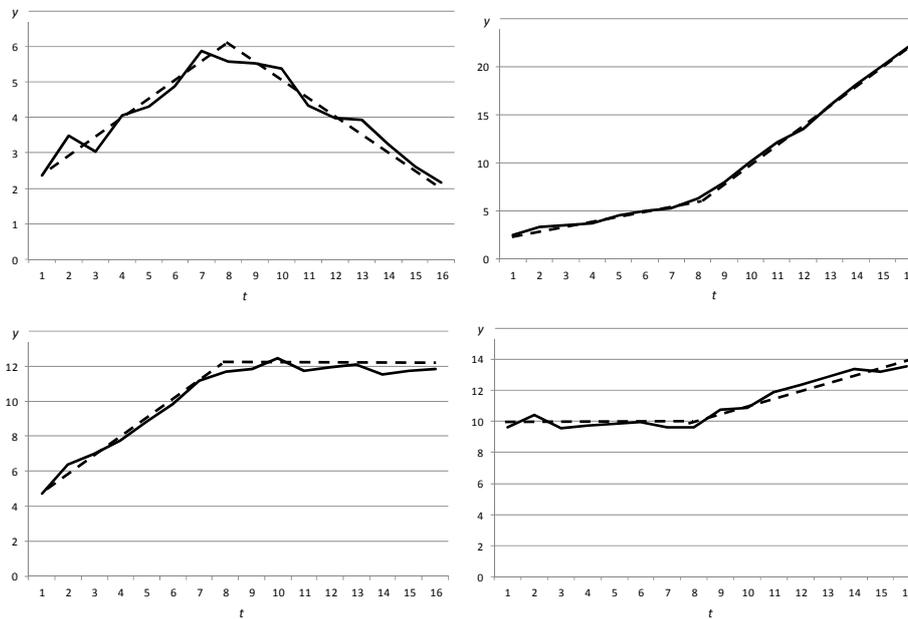

Fig. 1. Examples of changes in trend

Source: own elaboration.

The chain indices and the time series theoretical values which are calculated on the basis of the trend form will be used in the proposed method. The chain indices could be written in the following form

$$i_t = \frac{y_t}{y_{t-1}} \ \text{ for } (t = 2, 3, ..., T). \tag{1}$$

Let $\hat{y}_t = at + b$ by the linear trend obtained by the least square estimation method basing on the empirical data $y_1, y_2, ..., y_k$. The theoretical value of time series will be denoted by $\hat{y}_t$ (for t = 1, 2, …, k, …, T).

2. PERMUTATION TESTS

Permutation tests are computer-intensive statistical methods. These tests were introduced by R.A. Fisher and E.J.G. Pitman in 1930's (see Kończak 2012). In these tests the observed value of the test statistic is compared with the empirical distribution of this statistics under the null hypothesis. Permutation tests are generally asymptotically as good as the best parametric ones (Lehman 2009). The concept of these tests is simpler than of the tests based on normal distribution. The main application of these tests is a two-sample problem (Efron, Tibshirani 1993). When dealing with permutation tests thye following steps are taken (Good 2005):

1. Identify the null hypothesis and the alternative hypothesis.
2. Choose a test statistic (T).
3. Compute the test statistic (T_0).
4. Determine the frequency distribution of the statistic under the null hypothesis (T_1, T_2, …, T_N, where $N > 1000$).
5. Make a decision using this empirical distribution as a guide.

The ASL (*Average Significance Level*) has the following form:

$$ASL = P\left(\hat{\theta} \geq \hat{\theta}_0\right). \tag{2}$$

The estimated value of ASL could be determined by the following formula

$$ASL \approx \frac{card\{i : \hat{\theta}_i \geq \hat{\theta}_0)}{N}. \tag{3}$$

This notation applies, where the H_0 rejection area is right-sided. In the case of the left-sided rejection area in the above notation inequality should be changed. If the value of *ASL* is lower than the assumed level of significance α, then H_0 will be rejected.

The parametric tests are usually used for comparing parameters of population such as means, variances or proportions. These tests, except for the last one, require the samples to be taken from a population with a normal distribution. For large samples the limit distributions of statistics could be used.

In the case of small samples, if the normality assumption is not fulfilled, appropriate non-parametric tests such as *U* Mann-Whitney test or Kruskal-Wallis test should be used. Besides comparisons of parameters, it is often necessary to refer to the comparison of distributions in the two populations. In this case, Kolmogorov-Smirnov test is used most often. In this case the test statistic refers to the comparison of the empirical distribution function. In the next part of this paper the permutation test will be used to detect changes in linear trend. The considered time series will be divided into two series: $y_1, y_2, ..., y_k$ and $y_{k+1}, y_{k+2}, ..., y_T$.

3. TEST PROCEDURE

The hypothesis which says that at the point k ($1 < k < T$) there has been no change in the trend will be verified. Alternative hypothesis could be defined as the simple negation of H_0, but it is also possible to formulate the hypothesis claiming that the rate of change increased (or decreased). The following two statistics were considered in the permutation test:

$$T^{(1)} = \frac{1}{n-k-1} \sum_{t=k+1}^{n-1} i_t - \frac{1}{k} \sum_{t=1}^{k} i_t \qquad (4)$$

$$T^{(2)} = \frac{y_t}{\hat{y}_t} \qquad (5)$$

The permutation test procedure that is used for testing the hypothesis on the change in trend in the point $t = k$ has the following steps:

1. Assume the significance level α.

2. Calculate the value T_0 of the test statistics T for the sample data.

3. Perform the permutation of time series N-times, then calculate the value of the test statistics.

4. On the basis of empirical distribution of the test statistics T, the *ASL* value is obtained.

If $ASL < \sigma$, then H_0 is rejected, otherwise H_0 hypothesis cannot be rejected.

The number of replications in permutations was $N = 1000$.

4. TEST APPLICATION IN QUALITY CONTROL

When a time series is the result of the quality control process then the aim of the analysis may be to control this process. In statistical quality control the observations obtained from the monitored process are plotted on the control chart. These charts are the most frequently used statistical tools for monitoring processes. There are three horizontal lines in the control charts: upper control limit, central line and lower control limit. The levels of upper and lower control lines are related to the critical values in hypothesis testing. A point plotting outside the control limits is equivalent to rejecting the null hypothesis and the point plotting between control lines is equivalent to failing to reject this hypothesis in parametric test. There are usually two more lines plotting on the control chart. They are called the upper warning limit and the lower warning limit. The warning limits are plotted at the level two sigma above and below the center line.

There are some decision rules for detecting nonrandom patterns on control charts. Most often the following rules are used (Montgomery 1996, Kończak 2007):
  − one point plots outside the control limits,
  − two of three consecutive points plot beyond of the two-sigma warning limits,
  − eight consecutive points plot on the one side of the center line,
  − eight consecutive points plot in increasing (decreasing) trend.
These signals are shown schematically in Figure 2.

Computer simulation has been performed based on the following assumptions:
  − the process to the point k runs steadily
  − at point $k+1$ the trend appears With each measurement, the expected value increases by Δm ($\Delta m = 0.01\sigma$, 0.05σ, 0.1σ). The expected vale of the random variable Y_t can be written as follows

$$E(Y_t) = \begin{cases} m & \text{for} \quad t=1,2,...,k \\ m + \Delta mt & \text{for} \quad t=k+1,k+2,... \end{cases}$$

In each iteration of the simulation, 10 values were generated from the stable distribution and 100 in accordance with the trend. Then, the position of the first instance of following signals of deregulation was recorded:
  − one point plots outside the control limits (S_1),
  − eight consecutive points plot on the one side of the center line (S_2),
  − eight consecutive points plot in increasing (decreasing) trend (S_3),
  − using permutation test (statistic $T^{(1)}$ and statistic $T^{(2)}$).

Exceeding of upper control limit

2 of 3 points above (below) the warning limit

8 points in decreasing trend
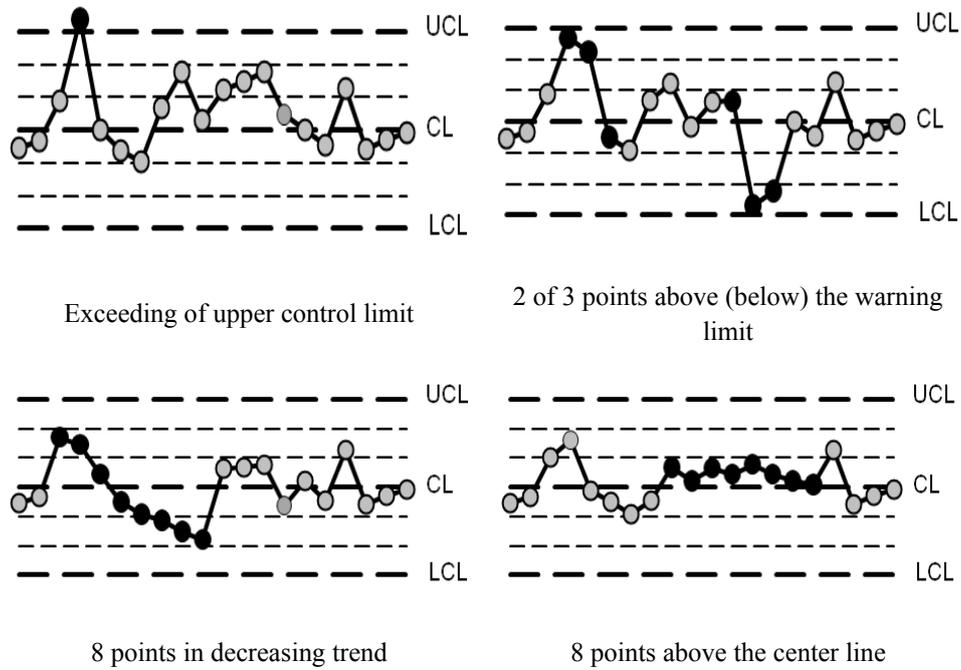
8 points above the center line

Fig 2. The typical rules used for detecting nonrandom patterns on control charts
Source: own elaboration.

The computer simulation has been made for the process shift Δm. For the permutation test the significance level $\alpha = 0.0013$ was assumed, which corresponds to the probability of exceeding one control limit in classical Shewhart control chart (corresponding to the method S_1). In Tables 1 and 2 the simulation results are presented. The results were obtained for each of the methods: classical Shewhart's and permutation tests.

Table 1. Average number of signals ($m = 10$, $\sigma = 0.5$)

Test	Δm		
	0.01σ	0.05σ	0.1σ
S_1	30.89	60.34	75.51
S_2	43.30	66.12	77.52
S_3	58.43	72.42	79.20
$T^{(1)}$	40.37	62.83	75.04
$T^{(2)}$	19.28	47.66	68.74

Source: Own elaboration.

Three variants of process disturbance were considered ($\Delta m = 0.01\sigma$, 0.05σ, 0.1σ). Table 1 shows the average number of detected signals for each of the analyzed methods. Analyzing these values, it can be seen that the greater the slope of this trend line, the more detections of the deregulation process are observed. Deregulation of the process most frequently was detected using the classical method and the statistics $T^{(1)}$. The values of average number of signals are presented in Figure 3.
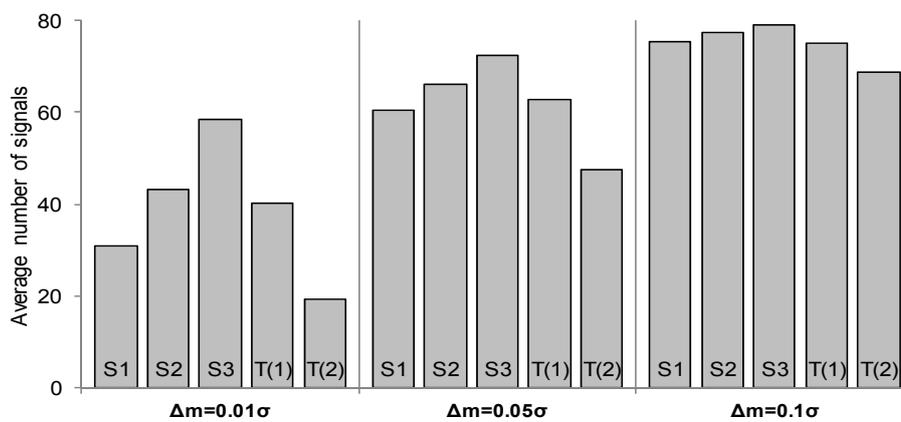

Fig. 3. Average number of signals for processes with trend
Source: own elaboration.

Table 2. Estimated ARL values ($m = 10$, $\sigma = 0.5$)

Test	Δm		
	0.01σ	0.05σ	0.1σ
S_1	30.82	18.67	10.90
S_2	27.97	17.21	10.80
S_3	24.95	16.51	11.39
$T^{(1)}$	45.58	26.11	15.14
$T^{(2)}$	33.93	20.73	12.55

Source: own elaboration.

Table 2 contains the estimated values of ARL for analyzed methods. In this case the deregulation is quickly detected by the classical method and the permutation test with statistic $T^{(1)}$. The values of average run length are presented in Figure 4.
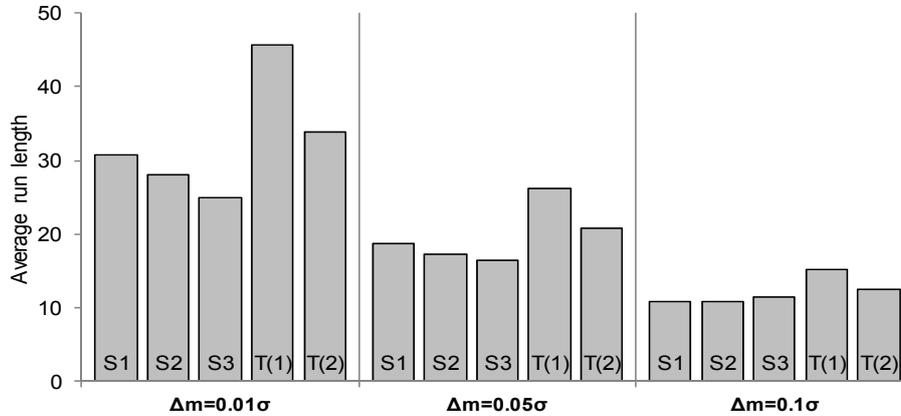
Fig. 4. Average run length for processes with trend
Source: own elaboration.

The comparison of the results for Shewhart method and the proposed method shows that the second one is a bit worse than the classical. However, the second (permutation) method has a significant advantage because it can also be used for the analysis in the case of small samples, and in the situation when the assumption about the origin of the sample from a normally distributed population is not satisfied.. In order to illustrate the results better , the values obtained in computer simulation are also presented in Figure 4.

5. CONCLUDING REMARKS

The proposed test procedure leads to detecting changes in time series trend. Due to the use of the permutation test it is not required to meet assumptions as in parametric tests. In particular, the proposed method can be used to detect the appearance of the trend, for instance in monitoring production processes. Simulation studies have shown that the procedure is effective even with small changes in the trend. They also show that the effectiveness of the method depends on the form of the adopted test statistics.

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WYKRYWANIE ZMIAN TRENDU Z WYKORZYSTANIEM TESTÓW PERMUTACYJNYCH

Streszczenie. W artykule przedstawiono propozycję testu pozwalającego na wykrywanie zmian trendu. Proponowana procedura odwołuje się do testu permutacyjnego. Zastosowanie takiej procedury testowej pozwoliło na przyjęcie dość ogólnych założeń. W szczególności proponowana metoda może być wykorzystywana do wykrywania pojawienia się trendu. W takim przypadku możliwe jest wykorzystanie metody do monitorowania procesów produkcyjnych. Proponowane rozwiązanie porównano ze znanymi z literatury rozwiązaniami z wykorzystaniem symulacji komputerowych.

Słowa kluczowe: trend, wykrywanie zmian, testy permutacyjne, symulacja Monte Carlo.