A C T A U N I V E R S I T A T I S L O D Z I E N S I S
FOLIA OECONOMICA 3(302), 2014

Marta Małecka[*]

GARCH CLASS MODELS PERFORMANCE IN CONTEXT OF HIGH MARKET VOLATILITY

Abstract. In the presented paper *GARCH* class models were considered for describing and forecasting market volatility in context of the economic crisis. The sample composition was designed to emphasize models performance in two groups of markets: well-developed and transition. As a preview to our results, we presented the procedure of model selection form the *GARCH* family. We distinguished three subperiods in the time series in a way that the dependencies between forecast outcomes and a scale of market volatility were emphasized. The comparison of the forecast errors revealed a serious problem of volatility prediction in times of high market instability. The crisis impact was particularly apparent in transition markets. Our findings showed that *GARCH* models allowed risk control, with risk understood as a relation of forecast error to the level of predicted volatility.
Keywords: nonlinear GARCH, volatility forecasting, forecast error

I. INTRODUCTION

Since the inception of the first *GARCH* model in 1982 by Engle, and its generalization of 1986 by Bollerslev, a constant development in the field of *GARCH* processes modelling has been observed. A widespread interest in *GARCH* models properties stems from a wide scope of applications, ranging from portfolio analysis and options pricing, through risk premium valuation, to verification of capital market models like CAPM or APT. *GARCH* models enter as an important ingredient in financial markets description, as they incorporate widely recognized facts about financial time series, namely volatility clustering and its asymmetry. *GARCH* models proved also to perform well in describing other stylized facts about financial series like excess kurtosis and fat tails. Since 90s of XX century many modifications have been made to the basic *GARCH* model, including nonlinear specifications, in order to incorporate empirical properties of financial variables. The main representatives of that line of enquiry are: *GARCH-in-mean* (Engle, Lilien and Robins, 1987), *SA-ARCH* (*simple asymmetric ARCH*, Engle 1990), *E-GARCH* (*exponential ARCH,* Nelson, 1991),

[*] M.Sc., Department of Statistical Methods, University of Łódź.

P-ARCH (*power ARCH,* Higgins and Bera 1992), *A-GARCH* and *AP-GARCH* (*asymmetric* and *asymmetric power ARCH*, Ding, Granger, and Engle 1993), *GJR-GARCH* (Glosten, Juganatan i Runkle, 1993), *N-ARCH* and *NP-ARCH* (*nonlinear* and *nonlinear power ARCH,* Bollerslev et al., 1994) and *Q-GARCH* (*quadratic GARCH*, Sentana, 1995).

Very significant in-sample parameter estimates brought the *GARCH* family a wide recognition within academic research. On the other hand, *GARCH* models have been subject to criticism relating to their performance in forecasting future volatility. Out-of-sample forecasts exhibit little effectiveness, which boosts a discussion about their practical utility. Addressing this issue, it was shown that the failure to produce accurate forecasts is not due to the models, but to inappropriate measures of forecasts quality and their interpretation (Andersen and Bollerslev, 1998, McMillan and Speight, 2004, Ulu, 2007). The recent crisis of 2008/2009 posed a question about models performance in times of high instability in the markets. The rank of this question arises from the fact that volatility forecasts in highly volatile periods may lead to major errors in securities pricing, posing a danger of loss of risk control.

The aim of this paper was to present a comparative analysis of *GARCH* models performance in highly volatile periods and in times of stability. In order to supply a comprehensive view on models properties in context of the economic crisis, three situations were distinguished: models performance in times of stability, forecasts in times of entering a highly volatile period and finally forecasts in situation of passing form the financial crisis to a relatively quiet recovery period. The first section of the paper introduces *GARCH* model specification. This is followed by the presentation of methodology used in the study and the description of the data set. Then we turn to the estimation process and the related testing procedure. A detailed analysis of model performance in terms of volatility forecasting in different market conditions is presented in the fourth section of the paper. The final section provides summary and conclusion.

II. MODEL REPRESENTATION

The general form of the equation for the conditional expected value of the return from the financial instrument in the GARCH model is $r_t = E(r_t \mid \Omega_{t-1}) + \varepsilon_t$, where Ω_t is the information set at time t, $\varepsilon_t = z_t \cdot \sqrt{h_t}$, z_t is i.i.d. over time, with zero mean and unit variance, $t = 1, 2, \ldots$. Under the above notation, h_t denotes the conditional variance of the process. The variance equation in the basic $GARCH(p,q)$ model of Bollelslev (1986) is given as

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \,, \tag{1}$$

where $\omega > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0$, $i = 1,2,...,q,$ $j = 1,2,...,p,$ $q \in N,\ \mathrm{p} \in N$. Addressing implications form the capital markets equilibrium theory (eg. CAPM or APT), in which a direct interaction between risk and return is postulated, the *GARCH-in-mean* model was introduced (Engle, Lilien i Robins, 1987). The feedback between risk and return was achieved by inclusion of a variance function $g(h_t)$ in the mean equation. In most applications $g(h_t)$ is an identity or square root function, i.e. $g(h_t) = h_t$ or $g(h_t) = \sqrt{h_t}$. In the first case, the expected value equation is given as $r_t = ah_t + \varepsilon_t$, where the term ah_t is often interpreted as a risk premium.

Most often cited nonlinear representations, being modifications of the basic variance model, are: $SA - GARCH(p,q)$ of Engle (1990)

$$h_t = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-i} \right) + \sum_{j=1}^{p} \beta_j h_{t-j} \,, \tag{2}$$

where $\gamma_i < 0$, $i = 1,...,q$, are additional parameters representing asymmetry; exponential model $E - GARCH(p,q)$ of Nelson (1991)

$$\ln(h_t) = \omega + \sum_{i=1}^{q} \left[\alpha_i z_{t-i} + \gamma_i (| z_{t-i} | - E | z_{t-i} |) \right] + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) \,, \tag{3}$$

where $z_t = \dfrac{\varepsilon_t}{\sqrt{h_t}}$ and which, under the normality assumption, takes the form

$$\ln \ln(h_t) = \omega + \sum_{i=1}^{q} \left[\gamma_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} + \alpha_i \left(\frac{| \varepsilon_{t-i} |}{\sqrt{h_{t-i}}} - \sqrt{\frac{2}{\pi}} \right) \right] + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) \tag{4}$$

and $GJR - GARCH(p,q)$ of Glosten, Juganatan and Runkle (1993) given as

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 I[\varepsilon_{t-1} < 0] + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{5}$$

with dummy variables relating to positive or negative shocks.

Further enquiry into *GARCH* models properties resulted in a postulate to add elasticity by the parameter φ in the exponent of ε_{t-i}, instead of imposing the value of two. That gave the $P-GARCH(p,q)$ model of Higgins and Bera (1992):

$$\sqrt{h_t}^{\varphi} = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^{\varphi} + \sum_{j=1}^{p} \beta_j \sqrt{h_{t-j}}^{\varphi}, \tag{6}$$

where $\varphi > 0$ is the additional parameter. Ding, Granger and Engle (1993) proposed generalizations of previous models in forms of $A-GARCH(p,q)$ and $AP-GARCH(p,q)$, respectively as:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i} \right)^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{7}$$

and

$$\sqrt{h_t}^{\varphi} = \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i} \right)^{\varphi} + \sum_{j=1}^{p} \beta_j \sqrt{h_{t-j}}^{\varphi}. \tag{8}$$

The model (7) is identical, in describing asymmetry property, to the $GJR-GARCH(p,q)$ specification. Models $N-GARCH(p,q)$[1] and $NP-GARCH(p,q)$ of Bollerslev et al. (1994) provide a slightly different view on asymmetry, respectively:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \left(\varepsilon_{t-i} - \kappa \right)^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{9}$$

$$\sqrt{h_t}^{\varphi} = \omega + \sum_{i=1}^{q} \alpha_i \left| \varepsilon_{t-i} - \kappa \right|^{\varphi} + \sum_{j=1}^{p} \beta_j \sqrt{h_{t-j}}^{\varphi}, \tag{10}$$

where $\kappa > 0$ is another parameter, representing asymmetry, introduced instead of $\gamma_i < 0$. The model (9) can be reduced to the representation

[1] Precisely, the presented specification is cited in the literature as $N-GARCH(p,q)$ *with single shift*. Additional elasticity is possible when different κ parameters are allowed for different periods

$SA-GARCH(p,q)$ form 1990, but has the advantage of a clear graphical interpretation, where κ parameter is a translation of a graph that relates h_t estimate to the past shock ε_{t-1}, against the $\varepsilon_{t-1}=0$ axis. The model $Q-GARCH(p,q)$ of Sentana (1995) is given as

$$h_t=\omega+\sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2+\sum_{i=1}^{q}\gamma_i\varepsilon_{t-i}+\sum_{j=1}^{p}\beta_j h_{t-j}\,. \tag{11}$$

It can be shown that the above model is identical to $SA-GARCH$ and $N-GARCH$ representations of a relevant order.

Together with a dynamic development in the class of GARCH models, many diagnostic tests have been proposed to check properties of alternative representations: the ARCH effects test LM^A, the higher order GARCH test LM^H, for nonlinearity checking – the sign and size bias test LM^S, quadratic asymmetry test LM^Q and the remaining ARCH test LM^{RA}. A comprehensive discussion on GARCH tests can be found in Franses, Dijk (2008).

III. DATA AND METHODOLOGY

The research was conducted on a daily basis, on close to close logarithmic stock index returns, over the period January 1, 2005 to October 30, 2010. 12 indexes were included in the sample, 6 from well-developed markets and 6 form transition countries: NIKKEI225 – Japan, FTSE100 – United Kingdom, DAX – Germany, CAC40 – France, BEL20 – Belgium, IBEX35 – Spain, RTS STD – Russia, WIG20 – Poland, PX – Czech Republic, SBI20 – Slovenia, BUX – Hungary, OMXB10 – Baltic republic index, Estonia, Lithuania and Latvia. Index choice was aimed at comparability of results among surveyed countries – therefore indexes including most liquid shares were considered. The purpose of a sample composition was also to obtain a comparability with previous studies on *GARCH* models performance.

The process of prediction and evaluation of *GARCH* forecasts effectiveness was preceded by the estimation and testing procedure aimed at model choice. In the initial stage, models for the conditional mean were fit with the restriction of homoskedasticity, residuals of which were tested for *ARCH(p)* process. A general mean specification was in the form of an autoregressive process with the structural factor of the DJCA (Dow Jones Composite Average) index $r_t=a_0+a_1r_{t-1}^{DJ}+\sum_{i=1}^{p}b_ir_{t-i}+\varepsilon_t$, where r_t^{DJ} denotes the return on the DJCA index at time t, $t = 1, 2, \ldots$, a and b_i, $i = 1, \ldots, p$, are model parameters. In *GARCH-in-mean* models the additional variable h_t was included with the parameter *c*.

Subsequently main *GARCH* family models were fit, among which were: the basic *GARCH* model, nonlinear specifications – *N-GARCH, E-GARCH, GJR-GARCH* and their modifications – *NP-GARCH* and *AP-GARCH*. *GARCH-in-mean* terms inclusion was considered for all specifications. The focus on the three above asymmetric representations was motivated by a clear parameter interpretation (see point II). Estimation was made by quasi maximum likelihood. Each stage in the estimation process was followed by the diagnostic testing procedure, in order to check subsequent specifications on their statistical properties, omitted variables and remaining or higher order *GARCH* effects. The above procedure was designated to facilitate the choice of a representation form the *GARCH* family.

According to the aim of the paper, which was to supply a comprehensive discussion on *GARCH* models performance in context of high market instability, three subperiods were distinguished:

1) Low price fluctuations in the financial market, before the crisis of 2008/2009 (January, 2005 – March, 2008),

2) Entering into the period of high volatility (August, 2005 – December, 2008),

3) Shifting from high instability into the recovery period (January, 2007 – March, 2010).

In each subperiod rolling regression was adopted with a window length of 700 observations. Each estimation step was used to make a one-session-ahead forecast. The estimation was then moved one day into the future, by deleting the first observation and adding one observation, which gave 100 forecasts in each subperiod. Forecast effectiveness was assessed on the root mean squared error:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(e_t^2 - \hat{h}_t\right)^2}\ , \tag{12}$$

where T denotes the number of observations, e_t – the realized volatility and $\hat{h}_t$ – the variance forecast. As a complementary measure, the heteroskedasticity adjusted root mean squared error was calculated, according to the formula:

$$HRMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\frac{e_t^2 - \hat{h}_t}{\hat{h}_t}\right)^2}\ . \tag{13}$$

The above measure was suggested as a means to verify forecasts quality in relation to the degree of uncertainty inherent in the particular observation (Ulu, 2007, pp. 673-674). Realized variance was approximated by the squared deviation from the expected value in the return series. The resulting error was compared to the forecast errors made on sample variance basis.

IV. ESTIMATION RESULTS

ARCH tests conducted on return series in the first step of the study suggested presence of heteroskedasticity, indicating the *GARCH* process in the error term structure (Table 1). Initial estimation results showed a vast improvement in statistical properties and in-sample errors gained with a shift to a *GARCH(1,1)* specification from a homoskedastic model.

Table 1. ARCH tests

Index	LM^A	*p*
NIKKEI225	438.91	0.00*
FTSE100	415.63	0.00*
DAX	309.33	0.00*
CAC40	310.77	0.00*
BEL20	438.27	0.00*
IBEX35	284.78	0.00*
RTS	303.74	0.00*
OMXB10	270.10	0.00*
WIG20	209.95	0.00*
PX	538.30	0.00*
LJSE	609.72	0.00*
BUX	371.70	0.00*

*Significant at 10% level

GARCH approach adoption eliminated serial correlation in nine out of twelve indexes and heteroskedasticity in eight cases[2]. Inclusion of variance equations had the effect of substantial lags order reduction in the equations for expected values, which confirmed misspecification errors at the first stage of estimation process.

[2] Correlation test results are not presented for the sake of brevity, but are available upon request from the author.

The subsequent testing procedure on residuals from *GARCH(1,1)* model indicated the presence of nonlinear effects in the error term processes (Table 2 LM^S, LM^Q). Asymmetric reactions of volatility were detected by the *Q-GARCH* test in all cases within the group of well-developed countries, which showed the necessity for nonlinear *GARCH* models adoption. Asymmetry effects were not captured by the tests in the group of transition economies. For prevailing majority of indexes, remaining and higher order *GARCH* tests did not reveal misspecification errors, proving no need for inclusion of higher order of lags (Table 2, LM^{RA}, LM^H).

Table 2. Remaining ARCH, higher order GARCH and asymmetry tests

Indeks	LM^S	p	LM^Q	p	LM^{RA}	p	LM^H	p
NIKKEI225	22.90	0.29	33.55	0.03*	7.70	0.99	4.60	1.00
FTSE100	11.78	0.92	39.71	0.01*	23.29	0.27	9.73	0.97
DAX	17.28	0.63	50.13	0.00*	12.98	0.88	3.84	1.00
CAC40	17.90	0.59	59.83	0.00*	25.42	0.19	11.52	0.93
BEL20	7.70	0.99	44.00	0.00*	16.80	0.67	3.71	1.00
IBEX35	11.24	0.94	34.03	0.03*	54.68	0.00*	14.07	0.83
RTS	2.53	1.00	9.66	0.97	8.75	0.99	5.54	1.00
OMXB10	7.01	1.00	2.08	1.00	7.38	1.00	3.92	1.00
WIG20	15.93	0.72	11.08	0.94	20.12	0.45	7.17	1.00
PX	5.82	1.00	11.08	0.94	6.62	1.00	3.33	1.00
LJSE	11.88	0.92	8.36	0.99	6.29	1.00	4.65	1.00
BUX	7.17	1.00	14.59	0.80	8.87	0.98	3.49	1.00

*Significant at 10% level

Estimation outcomes for nonlinear models:[3] *N-GARCH, E-GARCH, GJR-GARCH* and their modifications – *NP-GARCH* and *AP-GARCH* – confirmed *Q-GARCH* test results, which revealed asymmetry effects in the error term fluctuations. With one exception (of OMXB10), parameters on asymmetric variables were statistically significant at a very high level of confidence in both groups of countries. For indexes FTSE100, DAX, CAC40, BEL20, IBEX35, high asymmetry parameters resulted in a fairly different specifications relating to positive or negative past shocks, giving

[3] Parameter estimates for these models are not presented here in full in the interests of brevity, but are available upon request from the author. In Table 3 we present the *E-GARCH* models estimates.

the evidence of a very strong asymmetry in well-developed markets.[4] Inclusion of *GARCH-in-mean* terms gave non-significant estimates in most cases, hence the *in-mean* approach was rejected in a final model choice. In case of transition economies both results: week evidence of asymmetric behavior in popular tests and week statistical significance of risk premium parameters in *GARCH-in-mean* models were in line with previous studies, regarding countries with relatively short capital markets tradition (e.g., Fiszeder, 2009).

A comparative analysis of all nonlinear specifications, in terms of in-sample errors, showed that *E-GARCH* and *AP-GARCH* provide superior volatility forecasts to other considered models. For further analysis an *E-GARCH* model was chosen (Table 3), on account of additional advantages: smaller number of parameters and nonnegativeness of variance, implied by the exponential form of the specification.

Table 3. E-GARCH(1,1) models estimates

Index	a_1	p	b_1	p	b_2	p	ω	p	γ	p	α	p	β	p
NIKKEI225	0.62	0.00	–0.16	0.00			0.01	0.01	–0.11	0.00	0.22	0.00	0.97	0.00
FTSE100	0.28	0.00	–0.20	0.00			0.00	0.18**	–0.11	0.00	0.13	0.00	0.99	0.00
DAX	0.33	0.00	–0.18	0.00			0.01	0.00	–0.13	0.00	0.14	0.00	0.97	0.00
CAC40	0.37	0.00	–0.23	0.00			0.01	0.00	–0.16	0.00	0.12	0.00	0.98	0.00
BEL20	0.28	0.00	–0.12	0.00			0.01	0.02	–0.13	0.00	0.17	0.00	0.97	0.00
IBEX35	0.30	0.00	–0.16	0.00			0.01	0.00	–0.12	0.00	0.15	0.00	0.98	0.00
RTS	0.39	0.00					0.05	0.00	–0.05	0.00	0.20	0.00	0.97	0.00
OMXB10	0.23	0.00	0.19	0.00			0.02	0.00	–0.01	0.06	0.23	0.00	0.98	0.00
WIG20	0.29	0.00					0.02	0.00	–0.05	0.00	0.13	0.00	0.98	0.00
PX	0.38	0.00					0.03	0.00	–0.09	0.00	0.31	0.00	0.97	0.00
LJSE	0.12	0.00	0.34	0.00	–0.11	0.00	–0.03	0.00	–0.13	0.00	0.46	0.00	0.90	0.00
BUX	0.35	0.00					0.04	0.00	–0.06	0.00	0.23	0.00	0.97	0.00

**Not significant at 10% level.

[4] For a study on asymmetry in well developed countries see also Charles, 2010.

V. FORECAST EVALUATION IN CONTEXT OF FINANCIAL MARKET INSTABILITY

The *E-GARCH* model estimates were further used to evaluate forecasting properties of *GARCH* class models, in context of the degree of instability in the financial market. Three situations were considered: (1) relative stability of stock prices, before the crisis of 2008/2009, (2) entering into the high volatility period and (3) shifting to the recovery period after the extraordinary price movements of 2008/2009. Comparison of root mean squared errors from the three subperiods (Table 4) gave the overall view on the scale of errors in times characterized by different levels of prices instability. Largest inaccuracy in volatility forecasts was observed in the period when markets were entering the most volatile period, where estimation sample exhibited relatively low volatility and was used to make forecasts for times of extraordinary price fluctuations. The crisis impact was especially apparent in transition markets, where forecast errors were in several cases around ten times bigger than in the earlier period.

Table 4. Absolute forecasts errors RMSE in the three subperiods

Index	1st subperiod		2nd subperiod		3rd subperiod	
	E-GARCH	Sample variance	*E-GARCH*	Sample variance	*E-GARCH*	Sample variance
NIKKEI225	4.93	5.31	22.91	25.7	1.4	3.75
FTSE100	3.65	3.9	11.48	12.33	1.63	2.6
DAX	5.91	6.2	19.24	21.49	1.86	2.91
CAC40	5.43	5.71	13.91	16.82	1.99	3.1
BEL20	5.07	5.49	13.78	17.39	1.97	2.8
IBEX35	7.82	8.29	13.38	16.31	4.38	4.83
RTS	6.04	6.08	65.51	70.12	1.77	2.65
OMXB10	2.61	2.8	16.96	19.4	5.08	5.32
WIG20	5.37	5.65	11.15	12.58	2.85	3.7
PX	7.17	7.56	35.24	39.47	1.58	4.11
LJSE	6.32	6.82	11.68	14.68	0.36	2.06
BUX	3.54	3.68	21.28	24.65	3.73	4.34

Comparison of forecast errors form the *E-GARCH* model and from sample variance approach in times of relative market stability (Table 4, 1st subperiod) showed that adoption of the *GARCH* approach did not bring much improvement in terms of *RMSE*. Major difference in forecast error was evident however in the other two periods (Table 4, 2nd and 3rd subperiod). Errors were lower for model approach for all indexes, supporting the view that *GARCH* models prevail over historical measures in times of changes in the level of market stability, no matter whether the market enters the high instability period or recovers.

Addressing that postulate that the accuracy of a particular variance forecast should be measured relative to the inherent uncertainty in predicting that particular observation, further error analysis included the heteroskedasticity adjusted errors. *HRMSE* results showed a distinct advantage of a model approach in the first two considered periods, namely stable market and entering the time of instability (Table 5).

Table 5. Heteroskedasticity adjusted forecasts errors HRMSE in the three subperiods

Index	1st subperiod		2nd subperiod		3rd subperiod	
	E-GARCH	Sample variance	*E-GARCH*	Sample variance	*E-GARCH*	Sample variance
NIKKEI225	2.24	3.74	2.39	9.23	1.27	0.84
FTSE100	2.13	4.83	3.03	8.55	1.35	0.82
DAX	3.29	6.49	2.44	10.93	0.98	0.84
CAC40	2.58	5.92	2.37	8.38	1.15	0.83
BEL20	1.44	6.69	2.18	9.53	1.38	0.89
IBEX35	4.03	9.44	2.11	8.08	1.52	1.36
RTS	1.75	2.16	4.4	13.67	1.66	0.84
OMXB10	2.39	3.7	2.08	12.31	1.54	1.59
WIG20	1.36	2.67	2.1	4.22	1.12	0.84
PX	1.33	5.34	2.6	13.7	0.86	0.81
LJSE	3.32	8.73	2.39	9.4	0.53	0.87
BUX	1.68	1.84	2.38	8.79	1.44	0.87

Analysis of *HRMSE* from the *E-GARCH* models gave the observation that the relative error was fairly stable and hovered around two, independent of a subperiod. By contrast, *HRMSE* estimates for sample variance forecasts were large even in times of stable price level. In case of two market indexes (IBEX35 and LBSE) *HRMSE* for sample variance approached 10, showing that the forecast error was nearly ten times as big as predicted variance (Table 5, 1st subperiod). In situation of high price fluctuations, sample variance gave errors up to fourteen times higher than the predicted level of variance (CAC40, RTS, OMXB10 and PX), while in case of *GARCH* forecasts that relation was constantly around two (Table 5, 2nd subperiod). In the third period, sample variance estimates seemed to give better results in terms of relative errors (Table 5, 3rd subperiod), which can be explained by large sample variance estimates from the preceding period, resulting in systematically overstated predictions. *HRMSE* with the predicted variance in the denominator, gave therefore the impression of small relative errors, which cannot be interpreted in favor of sample variance estimates.

Further problem addressing the question of *GARCH* forecasts effectiveness is that a proper interpretation of sample forecast errors is not feasible when relevant population moments are not known, which is most often a fact in practice. Forecast evaluation requires a population value for the evaluation criteria under the correctly specified model. In the light of the above arguments, *GARCH* models performance seems difficult to be measured precisely on basis of daily observations. However, the comparative analysis presented in the paper, gave the opportunity to capture the effects of market volatility on forecast errors scale and the results strongly supported *GARCH* usefulness in risk management.

VI. SUMMARY AND CONCLUSION

In the presented analysis *GARCH* class models were considered for describing and forecasting market volatility in context of the economic crisis. The process of estimation was complemented by a diagnostic testing procedure aimed at detecting misspecification errors and model choice. The analysis of subsequent stages of estimation and testing resulted in the *E-GARCH* specification choice. A core stage of the presented analysis addressed the recent discussion in the literature referring to *GARCH* forecasts effectiveness. The main focus was on models performance in times of high instability in financial markets. Three subperiods were distinguished form the time series, in a way that the dependencies between forecast outcomes and a scale of market volatility were emphasized.

Comparison of absolute forecast errors in the three subperiods showed a large problem of volatility prediction in times of financial market instability. The crisis impact was especially apparent in transition markets, where forecast errors were in several cases around ten times bigger than in the earlier period. Absolute forecast errors showed that *GARCH* models prevailed mainly in the two subperiods: when markets were entering a high volatility period and when the economy was recovering after the crisis. It suggested therefore that considered models are especially advisable for risk management in times of changes in economy.

GARCH models advantage over sample variance approach was clear from relative errors analysis. Heteroskedasticity adjusted errors from the *E-GARCH* model hovered around two, independent of subperiod, while in case of the sample variance approach, they soared to more than ten during crisis. The results showed *GARCH* models superiority in risk control, with risk understood as a relation of forecast error to the level of predicted volatility.

Comparison of errors obtained for developed and transition economies showed that variance forecasts were more accurate for countries with longer capital market tradition. This was especially evident during the period of high market instability. It is difficult to pass a clear-cut judgment, to what extend large errors in transition economies are implied by higher variance of estimators, or result from less predictable market behavior.

REFERENCES

Andersen T. G., Bollerslev T. (1998), *Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts*, "International Economic Review" 39, s. 885–904.

Bera A. K., Higgins M. L., Lee S. (1992), *Interaction between Autocorrelation and Conditional Heteroscedasticity: A Random-Coefficient Approach*, Journal of Business & Economic Statistics, American Statistical Association 10, 133–42.

Bollerslev T. (1986), *Generalised Autoregressive Heteroscedasticity*, "Joumal of Econometrics" 31, s. 307–327

Bollerslev T., Engle R. F., Nelson D. B. (1994), *Arch Models*, w: Engle, R. F. i McFadden, D. L. (red.), Handbook of Econometrics 4, 2959–3038, Londyn, Nowy Jork, Elsevier North-Holland.

Charles A. (2010), *The Day-of-the-Week Effects on the Volatility: the Role of the Asymmetry*, "European Journal of Operational Research" 202, s. 143–152.

Ding Z. C., Granger C. W. J., Engle R. F. (1993), *A Long Memory Property of Stock Market Returns and a New Model*, Journal of Empirical Finance 1, 83–106.

Engle R. F., (1982) *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation*, Econometrica, 50, 987–1008

Engle R. F., (1990), *Discussion: Stock Market Volatility and the Crash of 87*, Review of Financial Studies 3, 103–106.

Engle R. F., Lilien D. M., Robins R. P. (1987), *Estimating Time-varying Risk Premia in the Term Structure: The ARCH-M Model*, Econometrica 55, 391–408.

Fiszeder P. (2009), *Modele klasy GARCH w empirycznych badaniach finansowych*, UMK, Toruń.

Franses P. H., Dijk D. (2008), *Non-Linear Time Series Models in Empirical Finance, Cambrdge University Press*, Cambrigde

Glosten L. R., Jagannathan R., Runkle D. E. (1993), *On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks*, Journal of Finance, 48, 1779–1801.

Higgins M. L., Bera A. K. (1992), *A Class of Nonlinear Arch Models*, International Economic Review 33, 137–158.

McMillan D. G., Speight A. E. H. (2004), *Daily Volatility Forecasts: Reassessing the Performance of GARCH Models*, "Joumal of Forecasting" 23, s. 449–60.

Nelson D. B. (1991), *Conditional Heteroscedasticity in Asset Retums: A New Approach*, "Ecometrica" 59, s. 347–370.

Sentana E. (1995), *Quadratic ARCH Models*, Review of Economic Studies 62, 639–61.

Ulu Y. (2007), *Sampling Properties of Criteria for Evaluating GARCH Volatility Forecasts*, Applied Financial Economics 17, 671–681.

Marta Małecka

**ZASTOSOWANIE MODELI GARCH
W KONTEKŚCIE WYSOKIEJ ZMIENNOŚCI RYNKOWEJ**

W pracy rozważano zastosowanie modeli klasy GARCH do opisu i prognozowania zmienności rynkowej w kontekście kryzysu gospodarczego. Wybrano szeregi czasowe pochodzące z dwóch grup krajów: wysokorozwiniętych i transformacji. We wstępnej części pracy zaprezentowana zastała procedura wyboru odpowiedniej specyfikacji modelu. W badaniu empirycznym wyróżniono trzy podokresy w taki sposób, by podkreślić zależność między wynikami prognoz a poziomem zmienności rynkowej. Analiza wyników ukazała skalę problemu związanego z niedokładnością prognoz zmienności w okresie dużych fluktuacji cen rynkowych. Wpływ kryzysu był szczególnie widoczny w krajach transformacji. Badanie pokazało, że zastosowanie modeli GARCH pozwoliło na kontrolę ryzyka, w tym sensie, że uzyskano stabilną relację błędu prognozy do poziomu przewidywanej zmienności.