A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
FOLIA OECONOMICA 5 (307), 2014

Edyta Łaszkiewicz[*]

MULTIPARAMETRIC AND HIERARCHICAL SPATIAL AUTOREGRESSIVE MODELS: THE EVALUATION OF THE MISSPECIFICATION OF SPATIAL EFFECTS USING A MONTE CARLO SIMULATION

1. INTRODUCTION

For spatial econometrics, one of the basic regression models is the spatial autoregressive model (SAR), predestined for the processes with spatial autocorrelation that are captured as the spatially lagged dependent variable (see e.g.: Anselin 1988). In multilevel modelling, the traditional hierarchical (multilevel) model (MLM) with random effects for higher levels (see e.g.: Goldstein 2011) plays a significant role. It might be applied to processes with spatial heterogeneity, among others (Getis, Fischer 2010: 507-511). Although both models are widely applied in economic, social, health and educational studies, we can suppose that the simple data generating process associated with the SAR or MLM model is not always enough to capture all spatial effects.

For example, Lottmann (2013) considered spatial dependence in the job creation process and argued that there is more than one channel of spatial interactions. Spatial dependence based on geographic proximity is the first because when the spatial mobility of the labour force decreases, the distance to the workplace increases. Additionally, economic and transport connectedness are also of great importance. Hence, both geographic distances (mostly expressed in the spatial matrix **W**) and regional interactions based on economic distance were found as significant for the job creation process. López-Hernández (2013) also proved for unemployment in Andalusia that the SAR model might be not enough when the spatial dependence consists of global and local parts.

Besides the above, spatial dependence might be combined with spatial heterogeneity to form a spatial hierarchical process. It is when individuals

[*] University of Lodz, Faculty of Economics and Sociology, Department of Spatial Econometrics.

are nested within groups and spatial interactions occur between individuals as well as groups. The housing price model, which evaluates the willingness to pay for residential characteristics (see: Chasco, Le Gallo 2012; Dong, Harris 2014), should be considered. The spatial heterogeneity of housing prices might be observed due to differences in the willingness to pay for houses located in different districts. Moreover, we expect the value of the residence can be estimated according to the prices of other properties, which results in spatial dependence between individuals. The value of properties in a given district might be also spatially correlated due to the spatial proximities of similar taxes (see: Dong, Harris 2014).

More advanced models can be applied to processes with both spatial autocorrelation and heterogeneity as well as with spatial relationships that are multidimensional in structure. For the first one, the hierarchical spatial autoregressive model (HSAR) can be applied as it allows for spatially correlated random effects and spatial dependence among individuals (Dong, Harris 2014; Baltagi et al. 2014). For the second one, the multiparametric spatial autoregressive model (m-SAR)[1] with more than one spatial weight matrix is common (Hepple 1995; Olejnik 2009; Hays et al. 2010). Only the HSAR model is able to capture spatial heterogeneity, but both models allow for more than one form of spatial dependence with different ways of incorporating it.

In the m-SAR model, spatial interactions occur only on the individual level and spatial homogeneity (lack of higher levels) is assumed. In contrast, the HSAR model allows for spatial dependence on the individual level as well as at a higher level. The existence of this higher level means heterogeneity in space. The specific situation is when one of the spatial weight matrices in m-SAR model contains a group-wise spatial dependence (see: Corrado, Fingleton 2012), but spatial heterogeneity is omitted. In such a case, it might be expected that the full structure of the spatial interactions is captured, but estimates of the spatial parameter for group-wise dependences might be affected in the m-SAR model by an incorrect assumption about spatial homogeneity. Analogously, the estimated parameter for spatial interaction and random effect variance might be biased when the spatially homogenous process is treated as spatially heterogeneous by using the HSAR model. Unfortunately, in the existing body of research there is no empirical evaluation of this supposition. Because it is crucial to correctly recognise the spatial process for applying m-SAR and HSAR models, it is worth discussing the potential consequences of the misspecification of spatial effects.

[1] Elhorst et al. (2012) defined a higher-order spatial econometric model as a model with higher-order polynomials in spatial weights matrices or higher-order spatial autoregressive processes. In this paper, we mean by this term a richer spatial dependence structure that is not capable of being captured by a single weights matrix.

The aim of this paper is to evaluate the effects of the incorrect classification of a higher-order spatial autoregression and the omission of spatial hierarchy in m-SAR model. In the HSAR model, we checked for the consequences of a valid assumption about the existence of spatial heterogeneity and spatial autocorrelation at a fictional higher level. To make the higher-order spatial autoregressive process in the m-SAR model more similar to the truth, we used a spatial weight matrix with group-wise dependence. A Monte Carlo simulation was used in this study. The effects of the misspecification were expressed by the relative bias of an estimator for parameters and by the rate of convergence. The results from this study might be potentially useful for understanding the role of spatial effects diagnosis in spatial and multilevel modelling.

2. A BRIEF DESCRIPTION OF THE MULTIPARAMETRIC AND HIERARCHICAL SPATIAL AUTOREGRESSIVE MODELS

In our study, the HSAR model proposed by Dong and Harris (2014) was used. The general formula for a HSAR model can be written as:

$$\begin{aligned} &\mathbf{Y} = \rho \mathbf{WY} + \boldsymbol{\beta}\mathbf{X} + \boldsymbol{\Delta}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \\ &\boldsymbol{\theta} = \lambda \mathbf{M}\boldsymbol{\theta} + \boldsymbol{\mu} \\ &\varepsilon \sim N(0, \mathbf{I}_N \sigma_\varepsilon^2) \\ &\mu \sim N(0, \mathbf{I}_J \sigma_\mu^2) \end{aligned} \quad , \tag{1}$$

with the $N \times J$ block-diagonal design matrix $\boldsymbol{\Delta}$:

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{l}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{l}_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathbf{l}_J \end{bmatrix}, \tag{2}$$

where: $\mathbf{Y}$ – $N \times 1$ vector of dependent variable; $\mathbf{X}$ – $N \times K$ matrix of control variables; $\mathbf{M}$, $\mathbf{W}$ – $J \times J$ and $N \times N$ row-standardised spatial weight matrices; ρ, λ – scalar with an estimated parameter of spatial interactions; $\boldsymbol{\beta}$ – $K \times 1$ vector of coefficients; $\boldsymbol{\varepsilon}$ – $N \times 1$ vector of error terms $\boldsymbol{\mu}$ – $J \times 1$ vector of random effects; σ_ε^2 – error term variance; σ_μ^2 – random effect variance; N – the number of observations; J – number of groups; $\mathbf{0}$ – n_j×1 vector of zeroes; $\mathbf{l}_j$ – n_j×1 vector of ones; n_j – the number of observations in the group j.

In the HSAR Model 1, the higher-order spatial autoregressive process is modelled by the spatial parameters ρ and λ, while spatial heterogeneity is captured by the estimated random effects variance σ_μ^2. The data generating process associated with Model 1 is as follows:

$$\mathbf{Y} = \left(\mathbf{I}_N - \rho\mathbf{W}\right)^{-1}\left(\mathbf{X\beta} + \boldsymbol{\varepsilon} + \boldsymbol{\Delta}\left(\mathbf{I}_J - \lambda\mathbf{M}\right)^{-1}\boldsymbol{\mu}\right). \qquad (3)$$

We also used m-SAR model in which spatial homogeneity was assumed (σ_μ^2=0); instead of the matrix **M** for the higher level spatial interactions, the additional N×N spatial matrix $\mathbf{W}_2$ is used. Assuming that the matrix **W** is the same as $\mathbf{W}_1$ and ρ is the equivalent of ρ_1, the m-SAR model can be written as:

$$\mathbf{Y} = \rho_1\mathbf{W}_1\mathbf{Y} + \rho_2\mathbf{W}_2\mathbf{Y} + \mathbf{X\beta} + \boldsymbol{\varepsilon}. \qquad (4)$$

The data generating process for the m-SAR Model 4 is:

$$\mathbf{Y} = \left(\mathbf{I}_N - \rho_1\mathbf{W}_1 - \rho_2\mathbf{W}_2\right)^{-1}\left(\mathbf{X\beta} + \boldsymbol{\varepsilon}\right). \qquad (5)$$

The quantum leaps for both models are the specifications of the spatial weight matrices, especially $\mathbf{W}_2$ and **M**, which might bring both models closer. To demonstrate the relations between matrix **M** and its "*equivalent*" – the matrix $\mathbf{W}_2$ with group-wise dependence, the following example should be considered. Let us take the regular spatial grid with N=54, J=6, all n_j=9 and specify the elements of matrix **M**=[m_{ij}] using the k-nearest neighbouring algorithm with k=1 (see: Figure 1).

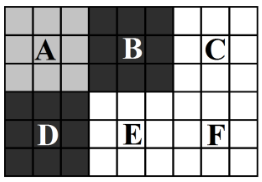

Figure 1. Regular spatial grid

Source: own elaboration.

Due to Figure 1, the nearest neighbours (calculated using the distance between centroids) for the "*A*" group of spatial units are groups "*B*" and "*D*"; the neighbours of "*B*" are "*A*", "*C*" and "*E*"; etc. The result is the following configuration of the spatial weight matrix M (before standardisation):

$$\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}. \tag{6}$$

The elements of the group-wise spatial dependence matrix $\mathbf{W}_2=[w_{ij}]$ will be equal to 1 if the spatial units i and j are nested in the groups which were found as neighbours or 0 if not. That gives the following structure of the spatial weight matrix $\mathbf{W}_2$ before standardisation:

$$\mathbf{W}_2 = \begin{bmatrix} \mathbf{w} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{w} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{w} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{w} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{w} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{w} \end{bmatrix}, \tag{7}$$

where: $\mathbf{1}$ – $n_j \times n_j$ matrix of ones, $\mathbf{w}$ – $n_j \times n_j$ matrix of ones with diag($\mathbf{w}$)=0.

The formulation of the higher-level interactions as a group-wise dependence in the m-SAR model seems to be enough when the spatial process is homogenous and the spatial matrix $\mathbf{W}_1$ differs from matrix $\mathbf{W}_2$. Despite this, the spatial heterogeneity of the groups in the presence of a higher-order spatial autoregression makes the HSAR model with the spatial matrix **M** more appropriate.

The Bayesian Markov Chain Monte Carlo (MCMC) method was used to estimate both models. Non-informative priors were used for parameters, while the initial values were drawn randomly from prior distributions for both the m-SAR and HSAR models. The inferences were based on one MCMC chain that each consisted of 10 000 iterations with a burn-in period of 5 000 for each model. A detailed discussion about the MCMC algorithm for implementing the HSAR model has been provided by Dong and Harris (2014) and it is omitted in this work. The MCMC samplers for the HSAR and m-SAR models were coded using R language.[2]

---

[2] The author would like to thank R. Harris and G. Dong for providing the R code for the HSAR model. The m-SAR model was obtained by the author as a modification of the HSAR.

3. SIMULATION DESIGN

In this simulation, a regular spatial grid with N=900 spatial units and J=100 groups was used to specify the spatial weight matrices $\mathbf{W}_1$, $\mathbf{W}_2$ and $\mathbf{M}$. The elements of the matrix $\mathbf{W}_1$ (and $\mathbf{W}$ respectively) were specified using an inverse exponential function. For the matrices $\mathbf{W}_2$ and $\mathbf{M}$, the k=1 nearest neighbours algorithm was applied to obtain a more local structure for the spatial interactions and to differ the matrices from the previous one.

16 combinations of the target value for the parameters ρ, λ, σ_μ^2={0.20, 0.40, 0.60, 0.80} were used to specify the data-generating process associated with the HSAR model to check whether the value of the parameter determined any bias. In each combination, the vector of the parameters for the control variables was the same $\boldsymbol{\beta}$=[0.3, 0.7] as was the error term variance σ_ε^2=0.20. For the data generating process associated with the m-SAR model in each of the 12 combinations, the value of the ρ_1 and ρ_2 was differentiated. Values for the control variables were drawn in each replication (for both processes) from the multivariate normal distribution $\mathbf{X}$~MVN($\mathbf{0}$,$\mathbf{1}$), while the random effects and error term were drawn from the normal distribution.

For each condition, 50 replications were applied, which is relatively small number but for the calculation of the bias measure we used at least 200 replications. In each replication the MCMC method of estimation for the m-SAR and HSAR models were applied. The accuracy of the estimates was evaluated using two measures:
1) the relative bias of an estimator for a parameter, and
2) the rate of coverage.

The first measure was used to check the parameter estimates, while the second one was to evaluate the standard error estimates. The rate of convergence was calculated using a 95% credible interval, which was set up as a quartile 0.025 and 0.975 from *a posteriori* distribution for the parameter. For further details about the calculation of these measures, (see: Łaszkiewicz 2013). Different conditions were compared using ANOVA.

4. RESULTS AND DISCUSSION

In the first simulation, we used the data-generating process associated with the m-SAR model to estimate the HSAR model. The matrix $\mathbf{M}$ was used to capture the real spatial structure from $\mathbf{W}_2$, while the elements of matrix

W were equal to $\mathbf{W}_1$. In the second experiment, the data-generating process associated with the HSAR model was used to estimate the m-SAR model. In this case, the real spatial structure represented in matrix $\mathbf{W}_2$ was captured in the HSAR model by matrix **M**.

4.1. m-SAR as the HSAR model

The incorrect specification of the HSAR model does not affect the parameter estimates for the control variables and their standard errors (Table 1). The relative biases for the β_1 and β_2 estimates are less than 5%, which suggests a lack of bias. Additionally, the coverage of the 95% credibility interval for the fixed effect parameters is high, which means that the misspecification does not negatively affect the estimates and standard errors for the fixed effect parameters. Surprisingly, it was found that the error variance estimates and their standard errors were unbiased. The relative bias for the error variance is only -0.06%, while the rate of coverage is as high as for the estimates for the fixed effects parameter.

In contrast to above, the estimates for the spatial effects are biased. The random effect variance estimates is higher than zero in the HSAR model, suggesting incorrectly the spatial heterogeneity of the process. Additionally, the 95% credibility interval (CI) is too small to capture the real value of the random effect variance. Besides the overestimation of this parameter, a high underestimation of the ρ parameter for the spatial interaction was observed. The relative bias for this estimate was -31%, while the rate of coverage was only 31%. Although both the random effect variance and the spatial interaction parameter are biased, the relative bias for the λ parameter is low and equal to 6%.

Table 1. Relative biases and the coverage of the 95% credible interval

Parameter	Relative bias	Rate of the coverage
β_1	-0.00	0.92
β_2	0.00	0.96
ρ	-0.31	0.31
λ	0.06	0.68
σ_μ^2	0.47	0.00
σ_ε^2	-0.00	0.96

Source: own calculations in R Cran.

Such a small overestimation was unexpected because, in the estimation of the spatial parameter λ, an incorrect value of the random effect variance was used. However, the relative bias for the λ estimates is only by 1% over the acceptable value, which is 5% according to Hoogland and Boomsm (1998). The rate of coverage for λ is significantly lower than for the fixed effects and error variance, and biased. It seems that the misspecification of the spatial effects in the HSAR model mostly affects the coverage of the 95% CI for λ. The results support the observation that the structure of the spatial connections from matrix $\mathbf{M}$ might be successfully modified vis-à-vis spatial group-wise dependences, as is the case in $\mathbf{W}_2$.

The underestimation of the ρ parameter and the overestimation of the error variance suggest that when there is no spatial heterogeneity in the process, a part of the spatial variability that is connected with the lower level spatial interactions is captured by the random effect variance, making it significantly higher than zero. Hence, the spatial dependence with the structure of the spatial connections as is the case in matrix $\mathbf{W}_1$ is wrongly estimated in the HSAR model.

Next, the ANOVA analysis was used to check whether the relative bias for the parameters that were found to be biased in the HSAR model depends on the value of the spatial interaction parameters ρ_1 and ρ_2. The results are shown in the Table 2. The value of the ρ_1 parameter, which might be treated as the equivalent of the ρ in the HSAR model, significantly modifies the relative bias for all of the spatial effects' parameters. Overestimation of the random effect variance increases when ρ_1 grows. Additionally, the underestimation of ρ_1 is higher for the higher values for this parameter. This is because for all combinations of the target value of ρ_1, the ρ estimate is equal to 0.20 and does not rise as it should. It seems that the growth of the spatial interaction parameter is captured incorrectly by the random effect variance for which thesystematic increase was noticed. The λ estimates were found as the least susceptible for the ρ_1 parameter changes. For all of the ρ_1 values, except the lowest, the relative bias for the λ parameter suggests a lack of bias.

In contrast to ρ_1, the value of the ρ_2 parameter does not influence the relative bias for the spatial effect parameters. The relative bias for the ρ parameter is the same for each value of ρ_2. Also, the λ estimates which correspond with the ρ_2 parameter is not susceptible to ρ_2 changes. The relative bias for the random effect variance is the same for two of the three values of ρ_2 and higher than for the value equal to 0.20. Although the difference was found as significant in ANOVA, it has a rather minor impact on ρ_2 on the random effect variance estimates.

Table 2. The influence of the target value on relative biases

Parameter	Target value and p-value*	ρ	λ	σ_{μ}^{2}
ρ_1	0.20	-0.01	0.06	0.21
	0.40	-0.21	0.05	0.42
	0.60	-0.41	0.04	0.64
	0.80	-0.61	0.03	0.81
	p-value	0.00	0.00	0.00
ρ_2	0.20	-0.30	0.06	0.36
	0.40	-0.31	0.04	0.50
	0.60	-0.31	0.06	0.50
	p-value	0.61	0.90	0.00

* p-value from the ANOVA.

Source: own calculations in R Cran.

According to the estimation results for the HSAR model, the misspecification of the spatial process negatively affects only the estimated parameters for spatial interaction and spatial heterogeneity. When the HSAR model is used for the homogenous spatial process with a higher-order spatial autocorrelation, the underestimation of the spatial interaction parameter for lower level spatial dependences might be observed. The non-zero value of the random effect variance might wrongly suggest the existence of spatial heterogeneity. It seems that in the case of spatial homogeneity, the unnecessary parameter for the random effect variance captures part of the variability connected with spatial dependence.

4.2. HSAR as the m-SAR model

Next, the m-SAR model was estimated using a data-generating process associated with the HSAR model to check how misspecification of spatial processes influences estimates and standard errors. The estimation results for the m-SAR model with the omitted spatial heterogeneity and with the group-wise spatial dependence are presented in Table 3. As in the previous section, the estimates for the fixed effects were not affected by the misspecification. The relative biases for β_1 and β_2 were small and not exceed -2%. The rate of coverage for both is high (96% and 88%, respectively), which suggests no negative effect for using an inappropriate data-generating process.

Although the higher level spatial interactions were represented in the m-SAR model by using the group-wise spatial dependence matrix $\mathbf{W}_2$, the value of the relative bias for the spatial parameter ρ_2 suggests only a small underestimation (by 7%). In contrast, the rate of coverage is low for only 23% of the 800 replications; the 95% credible interval covers the target value of the parameter. This means the standard error for the spatial parameter ρ_2 is too low. The results from the m-SAR model for the higher-level spatial interaction parameter are similar to those obtained from the HSAR model, especially in the case of relative bias. When the spatial homogenous process with the higher-level spatial autocorrelation is treated as a process with spatial heterogeneity, the estimates for the spatial interaction parameter (λ) are quite overestimated. The opposite situation results in a small underestimation of the parameter ρ_2, which represents the same structure of spatial relations. The relative bias in both cases is almost the same (the difference between them are only 1 p.p.), but with the opposite sign.

Table 3. Relative biases and the coverage of the 95% credible interval

Parameter	**Relative bias**	**Rate of the convergence**
β_1	-0.01	0.96
β_2	-0.02	0.88
ρ_1	0.33	0.00
ρ_2	-0.07	0.27
σ_ε^2	4.79	0.00

Source: own calculations in R Cran.

The omitted spatial heterogeneity represented by the σ_μ^2>0 mostly affects estimates for the spatial parameter ρ_1 as well as error variance. Both of them are overestimated, while the 95% CI does not cover the true value of the parameter. The relative bias for the spatial parameter ρ_1, which captures the spatial autocorrelation at the lower level, is equal to 33%. It seems that part of the variability connected with the omitted spatial heterogeneity is captured by the ρ_1 estimates. Also, the results from the HSAR model support this conclusion. Analogously to the parameter for the higher-level spatial dependence, it was noticed that the ρ_1 parameter was overestimated for the model with the omitted spatial heterogeneity and underestimated when we wrongly assumed that spatial heterogeneity occurs. Again, the relative bias for the estimates is almost the same, but with different signs in both models.

It was observed that omitted spatial heterogeneity affects error variance estimates in the m-SAR model. Despite this, the error variance estimates in the HSAR model are not affected by the unnecessary assumption about spatial heterogeneity (see: Table 1). This might suggest that when the process is spatially homogenous, the wrong specification of the model does not cause the error variance to be biased.

The relationships between the target value of the parameters and the relative bias for the estimates which were found to be biased were tested using ANOVA. The results are presented in the Table 4. It was noticed that the relative bias for both the spatial interaction parameters and error variance depended on the values of ρ and σ_μ^2. The higher the random effect variance (σ_μ^2) and the spatial correlation at the lower level (ρ) the higher the relative bias for the error variance. In contrast, the inverse dependencies were observed for both spatial interaction parameters.

Table 4. The influence of the target value on the relative biases

Parameter	**Target value and p-value***	ρ_1	ρ_2	σ_ε^2
ρ, σ_μ^2	0.20	0.46	-0.14	0.29
	0.40	0.43	-0.07	0.84
	0.60	0.30	-0.05	2.88
	0.80	0.12	-0.02	15.16
	p-value	0.00	0.00	0.00
λ	0.20	0.33	-0.05	0.38
	0.40	0.33	-0.05	0.89
	0.60	0.33	-0.09	2.97
	0.80	0.33	-0.10	14.93
	p-value	0.84	0.01	0.00

* p-value from ANOVA.

Source: own calculations in R Cran.

The value of the spatial parameter λ affected only the relative bias for the error variance and the ρ_2 parameter. It was noticed that the relative bias for both parameters increased when the value of λ rose. The bias for the parameter of the lower level spatial dependence (ρ_1) was stable and equal to 33% for each value of λ. This result is consistent with the observation from the previous simulation (see: Table 2). The value of the parameter for the higher level spatial dependence (λ in the data-generating process associated with the HSAR model) or its equivalent, the group-wise spatial dependence (ρ_2 in the data-generating process associated with the m-SAR model), does not affect the relative bias for lower-level spatial dependences (assigned by ρ in the HSAR model and by ρ_1 in the m-SAR model).

5. CONCLUSIONS

In this paper, the m-SAR and HSAR models were evaluated by using two data-generating processes with spatial heterogeneity and multiparametric structure for spatial autocorrelation. The misspecification of spatial effects affected the estimates in both models was considered. Additionally, the relationships between the group-wise spatial dependence matrix $\mathbf{W}_2$ and the spatial weight matrix $\mathbf{M}$ were examined (in $\mathbf{M}$, the interactions between the higher-level spatial units were incorporated). Monte Carlo simulations were used for this purpose.

It was proven that the estimated parameters for the control variables were unbiased when spatial heterogeneity was omitted in a m-SAR model as well as when the spatially homogenous process was applied in the HSAR model. Overestimation of error variance was observed when spatial heterogeneity was not captured, but for the estimated error variance was unbiased for spatially homogenous processes.

Moreover, the misspecification of spatial homogeneity/heterogeneity mostly affects the estimated parameter for spatial interactions at the individual level. The application of a HSAR model for spatially homogenous processes might result in the underestimation of this parameter, while using the m-SAR model for spatially heterogeneous processes causes the overestimation of the spatial interaction parameter.

According to our results, a small relative bias (6% and -7%) for the parameter of higher-level spatial interactions was observed when the group-wise spatial dependence matrix $\mathbf{W}_2$ was used instead of the spatial matrix **M**. This suggests the strong similarities between both spatial weight matrices.

REFERENCES

Anselin L. (1988), *Spatial Econometrics: Methods and Models*, Vol. 4. Springer.

Baltagi B. H., Fingleton B., Pirotte A. (2014), *Spatial lag models with nested random effects: An instrumental variable procedure with an application to English house prices*, "Journal of Urban Economics", 80, pp. 76-86.

Chasco C., Le Gallo J. (2012), *Hierarchy and spatial autocorrelation effects in hedonic models*, "Economics Bulletin", 32 (2), pp. 1474-1480.

Corrado L., Fingleton B. (2012), *Where is the economics in spatial econometrics?*, "Journal of Regional Science", 52(2), pp. 210-239.

Dong G., Harris R. J. (2014), *Spatial Autoregressive Models for Geographically Hierarchical Data Structures*, "Geographical Analysis".

Elhorst J. P., Lacombe D. J., Piras G. (2012), *On model specification and parameter space definitions in higher order spatial econometric models*, "Regional Science and Urban Economics", 42 (1), pp. 211-220.

Getis A., Fischer M. M. (2010), *Handbook of Applied Spatial Analysis: Software Tools, Methods and Applications*, Springer.

Goldstein H. (2011), *Multilevel statistical models*, vol. 922, John Wiley & Sons.

Hays J. C., Kachi A., Franzese Jr. R. J. (2010), *A spatial model incorporating dynamic, endogenous network interdependence: A political science application*, "Statistical Methodology", 7 (3), pp. 406-428.

Hepple L. W. (1995), *Bayesian techniques in spatial and network econometrics: 2. Computational methods and algorithms*, "Environment and Planning A", 27(4), pp. 615-644.

Hoogland J., Boomsma A. (1998), *Robustness studies in covariance structure modeling: An 14 overview and a meta-analysis*, "Sociological Methods and Research", 26(3).

Łaszkiewicz E. (2013), *Sample size and structure for multilevel modelling: Monte Carlo investigation for the balanced design*, "Metody Ilościowe w Badaniach Ekonomicznych", XIV-2", pp. 19-28.

López-Hernández F. A. (2013), *Second-order polynomial spatial error model. Global and local spatial dependence in unemployment in Andalusia*, "Economic Modelling", 33, pp. 270-279.

Lottmann F. (2013), *Spatial dependence in German labor markets*, Doctoral dissertation, Humboldt-Universität zu Berlin, Wirtschaftswissenschaftliche Fakultät.

Olejnik A. (2009), *Metodologia i zastosowania modeli przestrzenno-autoregresyjych w badaniach rozwoju regionalnego*, Doctoral dissertation, University of Lodz.

ABSTRACT

The aim of this paper is to evaluate the spatial and hierarchical models for data generating processes with spatial heterogeneity and spatial dependence at the higher level. The simulation for the m-SAR and HSAR models was used to discuss the consequences of spatial misspecification. We noticed that the misspecification of spatial homogeneity or heterogeneity in both models affects i.a. the estimated parameter for spatial interactions at the individual level. Applying a m-SAR model for spatially heterogeneous processes causes the overestimation of the spatial interaction parameter.

WIELOPARAMETRYCZNE I HIERARCHICZNE MODELE PRZESTRZENNEJ AUTOREGRESJI. EWALUACJA SKUTKÓW BŁĘDNEJ SPECYFIKACJI EFEKTÓW PRZESTRZENNYCH NA PODSTAWIE SYMULACJI MONTE CARLO

ABSTRAKT

Artykuł ma na celu przetestowanie modelu przestrzennego i hierarchicznego, przeznaczonych do analiz procesów przestrzennych cechujących się przestrzenną heterogenicznością i autoregresją, pod kątem skutków błędnej specyfikacji efektów przestrzennych. W badaniu wykorzystano symulację Monte Carlo, którą przeprowadzono dla modelu m-SAR i HSAR. Wyniki badania wskazują, że błędne rozpoznanie przestrzennej homogeniczności lub heterogeniczności procesu wpływa negatywnie m.in. na oszacowania parametru interakcji przestrzennych na poziomie indywidualnym. Zastosowanie modelu m-SAR do analizy procesu z przestrzenną heterogenicznością skutkuje przeszacowaniem parametru interakcji przestrzennych.