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## SOME CHALLENGES FOR SPATIAL ECONOMETRICIANS

### 1. INTRODUCTION

Though before World War II some seminal works on (theoretical) spatial economics were published (see Paelinck and Nijkamp 1975, chapter 2), its real upswing began in the fifties of last century. Only at their end econometrics proper was applied to that discipline, but early econometric exercises were of the most classical type, relating only variables possessing the same regional index. An example is Thompson and Mattila 1959 (see the comments on this study in Paelinck and Klaassen 1979, p.6; for an unexpected result, see Paelinck 2012), Paelinck 1967 noticed there that such models were inadequate to represent the correct spatial workings of the economy, which would then be reflected in the policy outcomes. To quote from that study, pp.57–58:

“...les résultats de l'économétrie régionale, telle qu'elle est souvent pratiquée, sont fortement affectés par la négligence de deux facteurs essentiels:

- la localisation relative des régions faisant l'objet de l'étude;
- la localisation intra-régionale des activités sur lesquelles porte l'analyse.

De statique et indifférenciée, l'économétrie doit devenir *dynamique et différenciée*; des modèles adaptés au *caractère spécifique de l'analyse régionale* doivent être mis au point.”

The original French text was quoted here for its historical value, its translation into English going as follows:

“...the results of regional econometrics, such as it is often practiced, are heavily affected by the neglect of two essential factors:

- the relative location of the regions under study;
- the intra-regional location of the activities subjected to the analysis.

From static and undifferentiated, econometrics should become *dynamic and differentiated*; models adapted to the *specific nature of regional research* should be developed.”

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One can reasonably ask the question why no attention was being given to activities that were known to be spatially entwined. Sure, some stepping stoned had already been laid down:

- time series analysis knew the notion of serial correlation, in terms of the stochastic terms or lagged variables;
- input-output analysis was starting to develop multi-regional variants, though typical topological variables were still absent;
- in studies of inter-regional and international movements (inter-regional migration or inter-zone commuting, international trade) gravity models were in use, which implied the use of certain distance measures.

The full answer to the question raised above lies probably in the fact that the bridge between spatial analysis and econometrics proper had still to be built; how progressively that has happened will now be commented on.

What was needed indeed was a complete integration of theoretical spatial economics and econometrics proper; this was what “Spatial Econometrics” (1979) intended to do. The initial vision on what spatial econometrics should be, was already expressed in the General Address of the author to the Dutch Statistical Association, on the occasion of its Annual Meeting on May 2<sup>nd</sup> 1974, held at the city of Tilburg, The Netherlands.

The integration just mentioned was presented in the form of five principles which are listed hereafter.

The first one was already introduced by the previous considerations, to wit spatial interdependence; the new focus however was to derive that interdependence from the workings of spatial economies, for instance spatial income generation (with spatial consumption propensities, to be estimate; see: Paelinck and Klaassen 1979, *varii loci*), or sectoral location (locational choices as a function of expected profits, e.g.). Of course classical econometric problems, such as specification, dimensional homogeneity, identification, estimation, testing should not be neglected, as they are covering every application of econometrics.

The second principle was that of spatial asymmetry in the measures of spatial interdependence; this immediately implies a larger number of parameters to be estimated in whatever spatial econometric model is set up (for an nbexam0pole, see Paelinck and Klaassen 1979, pp. 118–126, where urban and non-urban regions have been distinguished vin a spatialized consumption model).

Principle three was called “allotropy”, from the Greek words *αλλος* and *τοπος*, meaning respectively “other” and “site”; it alludes to the influence at a distance of exogenous variables, the Weberian location model being a perfect example of this.

Choice problems in space lead more than often to non-linear solutions, so this too should be reflected in our model specifications, especially if the model intends to picture ex-ante choices; this is the case of locational models (see

e.g. the European FLEUR-model: Ancot and Paelinck 1983), though resulting ex-post behavior (for instance, transport flows) could still be treated linearly.

Finally, topological variables (locations, distances, densities,...) should not be absent; it should be explicitly said here that the choice of an appropriate distance measure is a strategic moment in the model specification, too little attention being often given to the problems of metric topology involved.

As spatial econometrics *is* about economics, much stress should be laid on the specification of the underlying model, the first moments of the distributions so to say (see Paelinck and Klaassen 1979, conclusions to chapter 2, pp. 42–43); this is indeed one of the themes of our further developments.

Shortly afterwards a remarkable evolution took place of articles, volumes and special journal issues devoted to spatial econometrics took place, also in the complementary field of spatial statistics (for references, see Griffith and Paelinck 2007 and 2009); Figure 1, taken from Paelinck 2009, presents the evolution of volumes and special journal issues over 28 years since 1979.

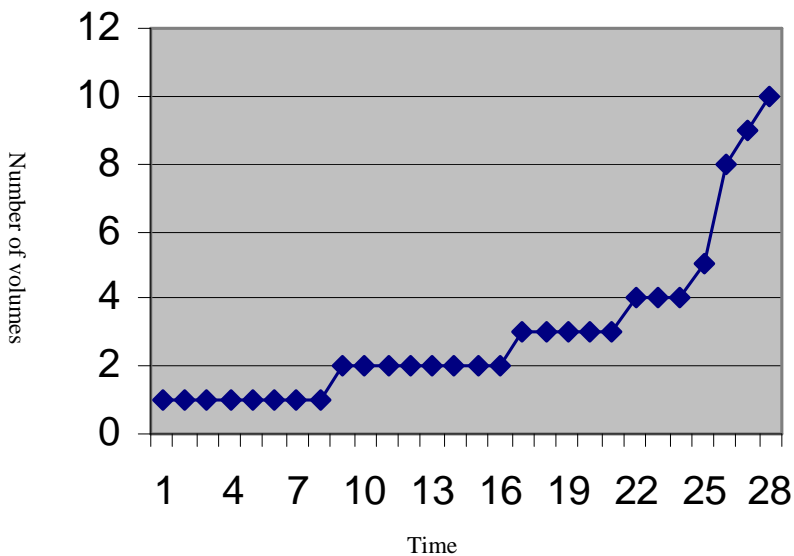


Figure 1. Trend in spatial econometrics publications

Source: author's investigation.

In next sections will be developed two important challenges that will confront spatial econometricians in the future, on the one hand specification and multiple regimes, on the other spatial bias and estimation. Other problems will be mentioned in the conclusions, references following.

## 2. SPECIFICATION AND MULTIPLE REGIMES.

In theoretical and applied physics space-time models have been classical tools of investigation; it comes to mind to try and *appropriately* apply some of them to spatial econometrics. Those space-time representations have since long been expressed in terms of partial differential equations; considering only one space variable,  $x$ , and time,  $t$ , a partial differential equation – abbreviated as PDE – for some function  $g(x,t)$  is a relation of the form:

$$h(x, t; g; g_x, g_t; g_{xx}, g_{xt}, g_{tt}; \dots) = 0 \quad (1)$$

where, in general:  $h$  is a given function of the independent variables,  $x$  and  $t$ , of the still unknown function  $g$ , and of a finite number of its partial derivatives. One well-known member of that family is the wave equation specified as:

$$f(x, t) = \alpha^2 f''(x, t) \quad (2)$$

the double dot meaning the second time-derivative (acceleration), the double prime the second  $x$ -derivative (curvature). Equation (2), as many other ones commonly studied, especially in theoretical (non-quantum) physics, is an expression of *local* interaction; but as in spatial economics, as, for that matter, in quantum physics, "non-locality" is the rule, to express spatial interaction, equation (2) should be generalized to:

$$f(x, t) = \alpha^2 \int_{-1}^{+1} w(x, \xi) f''(x, t) d\xi \quad (3)$$

where:  $w(x, \xi)$  is a so-called "spatial discount function", its convolution with some variable representing a *potential* over a line  $(-1, +1)$ ; so equation (3) should be called a PPDE, a *potentialized partial differential equation*.

To start with its study, Kaashoek and Paelinck (1994, 1996, 1998, 2001) analyzed various aspects of PPDE's: two-dimensional spatial cases, the effects of varying the potentializing function, and the possibility of controlling the space-time process, the latter problem resulting from the fact that the realizations of the process happen to be *chaotic*, but, being generated from "exact" equations, they belong to the family of so-called "exact" chaotic processes.

Figure 2 hereafter pictures one such process taken from Kaashoek and Paelinck 1998. One will notice the presence of sharp "peaks" which have been dubbed "pseudo-solitons", as genuine solitons are in fact infinitely dense local peaks (Dirac functions), but pseudo-solitons, like the true ones, can travel over space, as figure 2 clearly shows.

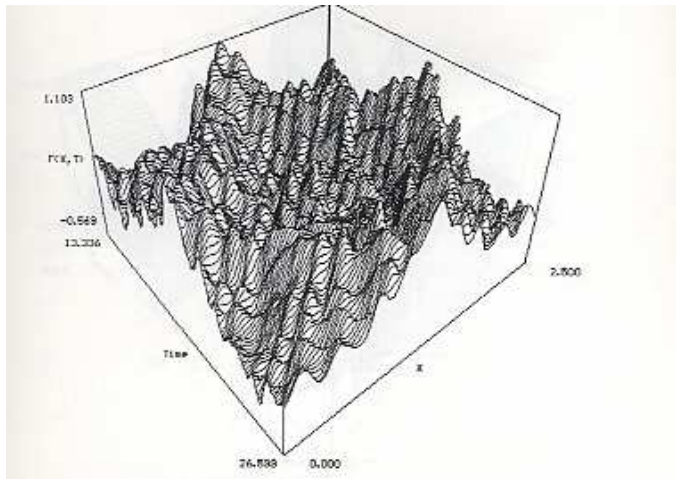


Figure 2. Realization of a PPDE  
Source: co-author's simulation.

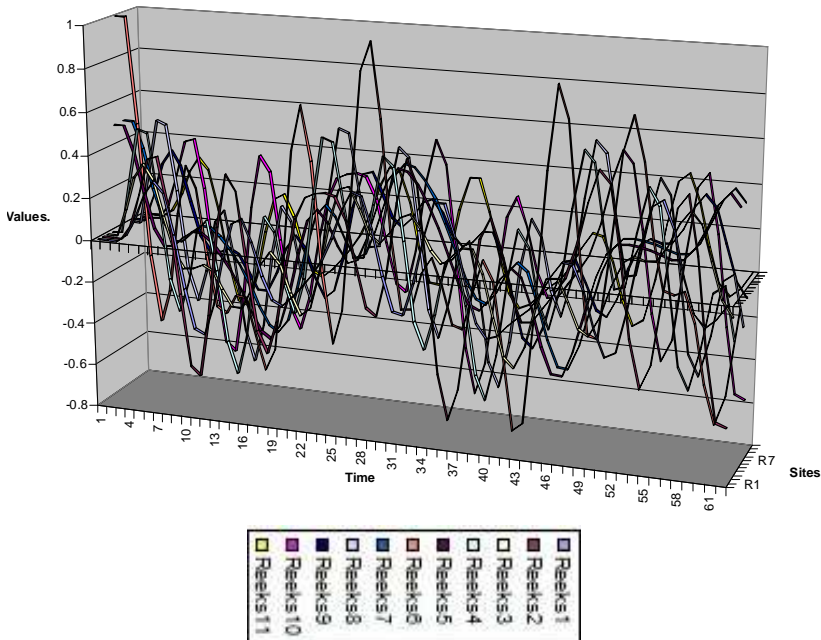


Figure 3. Simulation of a PPFDE  
Source: author's simulation.

One now has to rewrite equation (3) in a finite difference specification, which results in:

$$\Delta_t^2 f(x, t) = \alpha^2 \sum_{-1}^{+1} w(x, \xi) \Delta_x^2 f(x, \xi) \quad (4)$$

the summation depending on the spatial interaction process selected. Figure 3 hereafter reproduces a simulation of an estimated test model over 62 time-units; the graph has again all the characteristics of Figure 2, and illustrates the fact that potentialized processes can produce very complex patterns (Paelinck 2000a).

Model (4) has been applied to the most populated region in France after Ile-de-France, the Rhône-Alpes region (Coutrot, Paelinck, Rutter and Sallez 2010); it has been used to analyze the development of knowledge-based industries by means of employment in activities close to the concept of knowledge-based industries over 3 periods in 39 towns of Rhône-Alpes,

The model was first applied to all 39 regional units together, but as simulations suggested, there is evidence for the existence of at least two regimes (Griffith and Paelinck 2011, Chapter 13); the equation to be estimated is then the following:

$$\begin{aligned} \Delta^2 \ln(n_{i0i}) &= \lambda(a\Delta \ln(n_{0i}) + b\Delta \ln(n_{1i}) + c\Delta \ln(n_{3i})) \\ &+ (1-\lambda)(a^* \Delta \ln(n_{0i}) + b^* \Delta \ln(n_{1i}) + c^* \Delta \ln(n_{3i})), \end{aligned} \quad (5)$$

the  $\lambda$ -s being binary switching variables, one more instance of non-convexity, typical of spatial economic analysis; the model has been estimated by minimizing absolute discrepancies to neutralize outliers, but given the result below, any estimator would have done. The result is indeed remarkable from different points of view, as table 1 shows, *A* referring to model (4), and *B* to model (5); note that *A* has been computed from natural values, *B* from natural logarithms. First, the regimes are each other's reverse in terms of signs; second, the fit is almost an interpolation, so all coefficients should be significant, and, as said, the estimation method problem could be side-stepped, remarkable being also the fact that this happened for a double second-order difference specification.

Apart from these theoretical-econometric considerations, the empirical content of the results should be viewed; there is empirical evidence to add robustness to the conclusions, as the B-class of table 1 includes the three main activity centers of the region, to wit Lyons, Grenoble and Saint-Etienne, which moreover have a positive constant, so positive autonomous “acceleration”.

Table 1. Two regression results

Parameters	A	B
$A$	-0.0041	0.012176
$B$	-0.0049	0.007219
$c$	0.0002	-0.001544
$a^*$	-	-0.005856
$b^*$	-	-0.000270
$c^*$	-	0.000829
(Pseudo-)R <sup>2</sup>	0.5156	0.9990

Source: author's computations.

The conclusion should be, first that partial difference models do seem to be a very suitable tool to analyze large sets of small spatial units, but second, and most important, that some of the larger units can behave in a different way from the bulge of the set; the latter fact is one of the many instances of multiple regimes, characteristic of spatial econometric practice.

### 3. SPATIAL BIAS AND ESTIMATION

The starting point is Paelinck 2000b, where the so-called *MAUP* (Modifiable Spatial Unit Problem) is taken up from an econometric point of view, to wit the aggregation problem. That study (see pp. 158-159) starts with the simplifying assumption that complete spatial homogeneity is present, the latter being defined by the identity of all reaction parameters and exogenous variables, but over them a spatial unit specific bias would arise. the following example illustrates this point.

Suppose detailed underlying data – of an additive nature – are located next to each other on a circle or a torus; assume that only first- and second-order contiguities are relevant. For underlying micro-regions 3 and 4, the linear equation then is as follows, with only one exogenous variable being taken into account

$$y_3 = ax_3 + b(x_2 + x_4) + c(x_1 + x_5) + d \quad (6)$$

and

$$y_4 = ax_4 + b(x_3 + x_5) + c(x_2 + x_6) + d \quad (7)$$

Accordingly, the meso-regional equation becomes, after aggregation over the two meso-regions:

$$y_2^* = (a + b)x_2^* + (0.5b + c)(x_1^* + x_3^*) + 2d. \quad (8)$$

The second term on the right hand side of equation (8) would have to be changed if inequality of the exogenous variables is present (i.e., the factor 0.5 would have to be replaced by  $(x_1+x_5)/(x_1^*+x_3^*)$ ), hence a generally present area-specific bias.

Table 2: Linear regression parameters

Parameters	Values	F- and t-values
$R^2$	0.9968	386.7782
$x^*$ -intrareg.	1.6249	16.7149
$z^*$ -intrareg.	0.0524	0.5232
$x^*$ -contig.	0.3942	4.5931
$z^*$ -contig.	-0.4923	-4.6871

Source: author's computations.

This result gives a clue to how to specify a possible bias correction, leading up to a model in which the endogenous variable is corrected (details in Griffith and Paelinck 2011, chapter 18). Table 2. summarizes the results obtained. These regression results do not invalidate the assumption that spatial homogeneity is present after correction; however the sample was too small to test that model against an alternative, e.g. a min-algebraic model which would have 10 parameters so further investigation with larger empirical samples is in order.

It appears that filtering for spatial bias can reveal an underlying simple interregional model, so the hint is that bias is responsible for part of the specification complexity needed to represent the data; but this complexity is also affected by spatial heterogeneity, so complexity in spatial econometrics has a double dimension.

A last point to be made is that we again advise spatial scientists to start an exercise in spatial econometric modeling with a complexity analysis of the data. Obvious candidates for simple exogenous variables are their space-time coordinates. An example can be found in Getis and Paelinck 2004, in which regional product data for the Netherlands are analyzed. A model specification implies the choice of exogenous variables, and possibly endogenous ones in interdependent models or lagged endogenous variables in dynamic models so that they too should be implied in a complexity approach.

#### 4. CONCLUSIONS

Other openings have already been encountered: adequate estimators, hybrid identification, non-convexities. So, empirical work on spatial econometric modeling reveals nearly every time a problem specific to that discipline; no doubt the future will bring along new challenges to be faced in order to obtain a better



understanding of the workings of spatial economies. But also new insights are available, that could improve spatial econometric work, such as isomorphisms and hybrid dynamical systems, but those topics are still being worked on, so only a mention of it will be made here.

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### **SOME CHALLENGES FOR SPATIAL ECONOMETRICIANS**

After in 1979 (Paelinck and Klaassen) were isolated five principles that should guide spatial econometric modeling, this paper takes up some challenges derived from recent research on estimation, identification, multiple regimes, non-convexities, spatial bias and specification.

### **PEWNE WYZWANIA WYNIKAJĄCE Z MODELOWANIA DANYCH PRZESTRZENNYCH**

W publikacji Paelincka oraz Klassena określono pięć zasad modelowania danych przestrzennych. Niniejsze opracowanie stanowi charakterystykę pewnych wyznań wynikających z aktualnych badań dotyczących estymacji oraz specyfikacji przestrzennych modeli ekonometrycznych.