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Investment Risk Measurement Based on Quantiles and Expectiles

Abstract: In the presented research, we attempt to examine special investment risk measurement. We use quantile regression as a model by describing more general properties of the response distribution. In quantile regression, we assume regression effects on the conditional quantile function of the response. In regression modelling, the focus is on extending linear regression (OLS), and in this paper we seek to apply expectile regression. The purpose of using both approaches is investment risk measurement. Both regression models are a version of least weighted squares model. The families of risk measures most commonly used in practice are the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR), which can be estimated by quantiles or expectiles in the tail of the response distribution.

Keywords: quantile, expectile, VaR, CVaR, least asymmetrically weighted squares

JEL: C14, C20, C21

1. Introduction

The question how to measure risk is important for the majority of market participants. For investors: risk assessment is an integral part of making an investment decision. Most investors will generally not be risk neutral and the decision whether to buy an asset will depend on its return as well as on its riskiness. For risk managers: the choice of measures to asses risk will substantially influence their decisions, e.g.: on how much capital to set aside to prepare for extreme market events. For clearing houses: the calculation of margin requirements for the clearing members depends on how risk is measured. For regulators: the determination of risk reserves that market participants will be required to hold has become an important part of modern financial market regulations.

All market participants seek the best risk description and the best investment risk measurement. In this paper, we attempt to examine the measures based on quantiles and expectiles. Additionally, we aim to analyse properties of these measures. We show, through an empirical study, the performance of the procedures, and also present specific applications for the sector of assets from the Warsaw Stock Exchange.

2. Quantiles and expectiles

The concept of expectiles is a least squares analogue of quantiles. Both expectiles and quantiles were embedded in the more general class of M-quantiles as the minimisers of an asymmetric convex loss function. It has been proven very recently that the only M-quantiles that are coherent risk measures are expectiles. Moreover, expectiles define the only coherent risk measure that is also elicitable.

Both expectiles and quantiles are found to be useful descriptors of the higher and lower regions of the data points in the same way as the mean and median are related to their central behaviour. Koenker and Bassett (1978) elaborated an absolute error loss minimisation framework to define quantiles which successfully extends the conventional definition of quantiles as left-continuous inverse functions. Later, Newey and Powell (1987) substituted the 'absolute deviations' in the asymmetric loss function of Koenker and Bassett with 'squared deviations' to obtain the population expectile of order $\tau \in (0, 1)$. Based on this definition, we have solved two problems:

Quantile $q_{i,\tau}$ can be estimated as:

$$\min\left\{\sum_{i=1}^{n} w_{i,\tau}\left|y_i - q_{i,\tau}\right|\right\}, \text{ where } w_{i,\tau} = \begin{cases} 1-\tau, \mathit{for}\ y_i < q_{i,\tau} \\ \tau, \quad \mathit{for}\ y_i \geq q_{i,\tau} \end{cases}. \tag{1}$$

Expectile $e_{i,\tau}$ can be estimated as:

$$\min\left\{\sum_{i=1}^{n} w_{i,\tau}\left(y_i - e_{i,\tau}\right)^2\right\}, \text{ where } w_{i,\tau} = \begin{cases} 1-\tau, \textit{ for } y_i < e_{i,\tau} \\ \tau, \quad \textit{for } y_i \geq e_{i,\tau} \end{cases}. \tag{2}$$

Generally, quantiles provide a natural interpretation even beyond the 0.5 quantile, the median. A comparable simple interpretation is not available for expectiles beyond the 0.5 expectile, the mean. Expectiles are unique, not quantiles. The estimation of expectiles receives attention from the perspective of extreme values: "this estimator can be interpreted as a maximum likelihood estimator when the disturbances arise from a normal distribution with unequal weight placed on positive and negative disturbances" (Aigner, Amemiya, Poirier, 1976). Expectile estimation is thereby a special form of M-quantile estimation (Breckling, Chambers, 1988) and expectile regression has aroused increasing interest in recent years (Sobotka, Kneib, 2012).

In the statistical literature, it is usually argued that typically expectiles are closer to the centre of the distribution than the corresponding quantiles, but it depends on the tail of distribution, as in the following example (Figure 1).

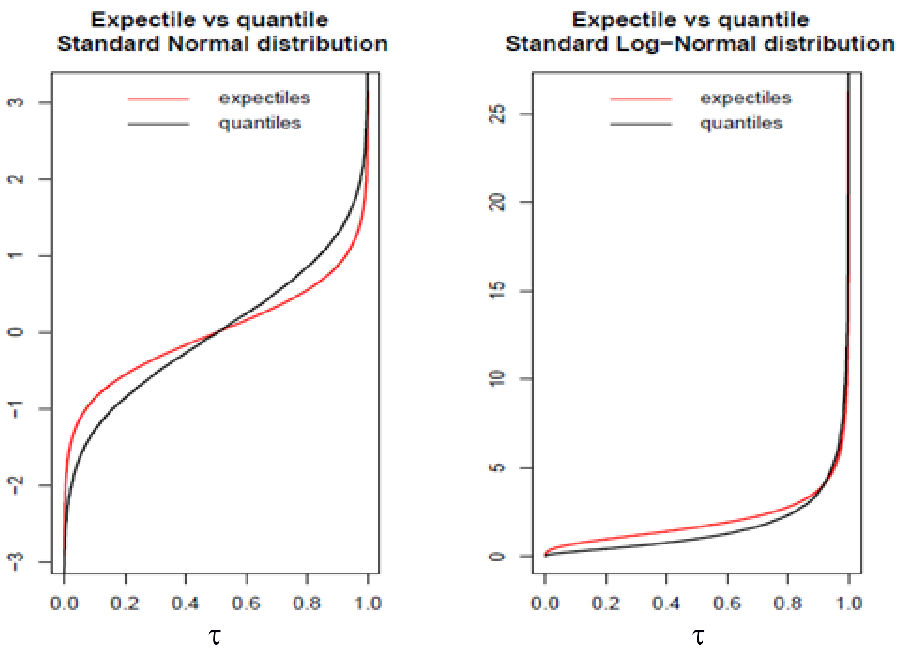

Figure 1. Expectile and quantile for standard normal and standard log-normal distribution

Source: own calculation

While quantile regression can be seen as a generalisation of median regression, expectiles as an alternative are a generalised form of mean regression.

The quantile regression model for the τ ∈ (0, 1) is specified as:

$$y_i = q_{i,\tau} + \varepsilon_{i,\tau} \tag{3}$$

with y_i as the response variable, for $i = 1, \ldots, n$, each quantile which may depend on covariates x_i[1], say, e.g., through the linear model:

$$q_{i,\tau} = \beta^q_{0\tau} + x_i \beta^q_{1\tau} \,. \tag{4}$$

Asymmetric least squares regression was proposed as an alternative to quantile regression, and the underlying regression model is now as follows[2]

$$y_i = e_{i,\tau} + \varepsilon_{i,\tau} \tag{5}$$

with y_i as the response variable, for $i = 1, \ldots, n$, each quantile which may depend on covariates x_i, say, e.g., through the linear model

$$e_{i,\tau} = \beta^e_{0\tau} + x_i \beta^e_{1\tau} \,. \tag{6}$$

Quantile and expectile regression are both versions of weighted least squares model. Quantile regression can be seen as a generalisation of median regression, expectiles as an alternative are a generalised form of mean regression. Quantiles are present in the L^1, while expectiles are rooted in the L^2. Expectile estimation is thereby a special form of M-quantile estimation. Quantiles need linear programming for estimation, while expectiles are fitted using quadratic optimisation.

3. Risk function and risk measurements

The formal definition of a risk measure is a mapping from Ω that contains all possible loss scenario – such that L^2 will hold – to the positive real numbers (Föllmer, Schied, 2002):

$$R : L^2 \to R_+ . \tag{7}$$

[1] Unlike classical regression where a zero mean is assumed for the residuals, in quantile regression one postulates that the τ-quantile of the residuals $\varepsilon_{i,}\tau$ is zero (Koenker, 2005).

[2] Under the assumption that the τ-expectile $e_{i,}\tau$ of the error terms is zero.

In the literature, we have an axiomatic definition of coherence risk measures (Artzner et al., 1998). Now we seek to add new properties – elicitable measures (Gneiting, 2011). A risk measure R is elicitable if it can be defined as the minimiser of a suitable expected scoring function.

Definition. T is an *elicitable* function if there exists a scoring function $S: R \times R \to [0, \infty)$ such as

$$T(Y) = \arg\min_{x \in R} \left\{ \int_R S(x, y) \, dF(y) \right\} = \arg\min_{x \in R} \left\{ E\left[S(x, Y) \right] \textit{ where } Y \sim F \right\} \qquad (8)$$

We can write some examples:

– Mean: T(Y) = E[Y] is elicited by $S(x, y) = \|x - y\|_{L^2}^2$.
– Median: T(Y) = Me[Y] is elicited by $S(x, y) = \|x - y\|_{L^1}$.
– Quantile: T(Y) = $q\tau$[Y] is elicited by $S(x, y) = \tau (x - y)_+ + (1 - \tau)(x - y)_-$.
– Expectile: T(Y) = $e\tau$ [Y] is elicited by $S(x, y) = \tau (x - y)_+^2 + (1 - \tau)(x - y)_-^2$.

We can now go back to the notation of risk measures in general. Elicitability is a very desirable property for computational efficiency, forecasting and testing algorithms. Elicitability corresponds to the existence of a natural backtesting methodology. Hence, we can consider the following risk function[3]:

$$R_\tau^q(u) = u \times (\tau - \mathrm{I}(u < 0)), \text{ with } R_{1/2}^q(u) \; \mu \; |u| = \|u\|_{L^1} \qquad (9)$$

and

$$R_\tau^e(u) = u^2 \times (\tau - \mathrm{I}(u < 0)), \text{ with } R_{1/2}^e(u) \; \mu \; |u| = \|u\|_{L^2}^2 \qquad (10)$$

Then

$$q_\tau(y) = \arg\min_m \left\{ E(R_\tau^q(Y - m)) \right\}, \qquad (11)$$

which is the median when $\tau = 1/2$

$$e_\tau(y) = \arg\min_m \left\{ E(R_\tau^e(Y - m)) \right\}, \qquad (12)$$

which is the expected value when $\tau = ½$.

Most importantly, from the point of view of the axiomatic theory of risk measures, Bellini and Bignozzi have proven that the only M-quantiles that are coherent risk measures are expectiles (Bellini, Bignozzi, 2013). They have also established

[3] Where I(z) = 1 if z is true or I(z) = 0 if z is false.

that expectiles are robust in the sense of lipschitzianity with respect to the Wasserstein metric. It has proven (Ziegel, 2016) that expectiles are the only coherent law-invariant measure of risk which is also elicitable. The property of elicitability corresponds to the existence of a natural backtesting methodology. It has been shown that the CVaR, the most popular coherent risk measure, is not elicitable (Gneiting, 2011), but jointly elicitable with the VaR (Fissler, Ziegel, 2016).

4. Quantiles and expectiles in market risk estimation

Empirical analysis of quantile and expectile regression model was attempted for companies listed under WIG BUD – a selected index from the Warsaw Stock Exchange. The analysis was focused on 15 companies which had the best return in the chosen period, and the observation period was from 1.03.2015 to 27.02.2017. Hence, we focus on the assets of: Budimex, Elektrobudowa, Lentex, MDI Energia, Mirbud, Mostostal Płock, Mostostal Zabrze, PBG, Projprzem (PJP), Rafako, Torpol, Trakcja PRKiI, Unibep, VISTAL, and Ropczyce. Preliminary analysis of daily rates of return on the analysed assets was shown in Table 1. To further calibrate models of market rate of return selected by the classical Markowitz portfolio with the criterion of max return, five companies Budimex, MDI Energia, PBG, Projprzem and Trakcja PRKil were analysed (the chosen portfolio has nonnegative return 0.001396103 with standard deviation 0.160: 5 out of 15 assets). For completeness of statistical analysis, the Kolmogorov-Smirnov test of normality (the Lilliefors test version) of chosen variables was carried out, which confirmed they did not come from the normal distribution (Table 2).

Table 1. Basic statistics for assets from WIG BUD and Markowitz portfolio

Asset	Mean of returns	Variance	Standard deviation of returns	Semivariance of returns	Mean absolute semideviation of returns*	Ratio in portfolio
Budimex	0.001055	0.000328	0.018109	0.000166	0.006629	0.2
Elektrobudowa	0.000359	0.000357	0.0189	0.000167	0.006637	0
Lentex	0.000688	0.000178	0.013342	0.000752	0.004785	0
MDI Energia	0.001105	0.001087	0.032969	0.000457	0.009112	0.2
Mirbud	0.000223	0.000872	0.029533	0.000382	0.009006	0
Mostostal Płock	–0.00084	0.000536	0.023157	0.000238	0.008362	0
Mostostal Zabrze	0.00230	0.002196	0.046859	0.000654	0.013831	0
PBG	0.000983	0.000593	0.024348	0.000221	0.007248	0.2
Projprzem	0.000299	0.000328	0.018098	0.000155	0.006093	0.2
Rafako	0.000464	0.000396	0.01991	0.000221	0.006604	0

Asset	Mean of returns	Variance	Standard deviation of returns	Semivariance of returns	Mean absolute semideviation of returns*	Ratio in portfolio
Torpol	0.00049	0.000401	0.020033	0.000222	0.006619	0
Trakcja PRKiI	0.000867	0.000451	0.021226	0.00021	0.007409	0.2
Unibep	0.001538	0.000371	0.019271	0.000158	0.006902	0
VISTAL	0.00066	0.000417	0.020417	0.000195	0.007535	0
Ropczyce	–0.00017	0.000373	0.019303	0.000183	0.006006	0

* The weighted average of absolute deviations for the set of values of daily rates of return which are below the market average.

Source: own calculation

Table 2. Results of Kolmogorov-Smirnov test of normality

		bdx	elb	sma	ltx	mdi	mrb	msz
N		501	501	501	501	501	501	501
Max distance	+	0.058	0.089	0.095	0.079	0.204	0.148	0.116
	–	–0.062	–0.068	–0.123	–0.044	–0.170	–0.126	–0.062
Test value		0.062	0.089	0.123	0.079	0.204	0.148	0.116
P value		0.000	0.000	0.000	0.000	0.000	0.000	0.000
		pjp	**rfk**	**tor**	**trk**	**uni**	**vtl**	**rpc**
N		501	501	501	501	501	501	501
Max distance	+	0.114	0.105	0.089	0.099	0.067	0.104	0.094
	–	–0.110	–0.085	–0.060	–0.065	–0.049	–0.104	–0.123
Test value		0.114	0.105	0.089	0.099	0.067	0.104	0.123
P value		0.000	0.000	0.000	0.000	0.000	0.000	0.000

Source: own calculation

Preliminary analysis of daily rates of return indicates the occurrence of outliers and extreme observations for selected companies in the analysed period (Figure 2). We can apply non linear regression – quantile regression. In Table 3, we can note the coefficient of quantile regression[4] for chosen quantiles ($\tau = 0.25; 0.5, 0.75$). In the tables, we present estimated parameters for selected regression models calibrated for the analysed time intervals. Figure 3 shows results of quantile regression for all the range of $\tau \in (0, 1)$. We can observe and comment on the influence of all the quantiles on the response variable, especially the influence of tail quantiles.

[4] R package – cf. Sobotka, Kneib (2012).

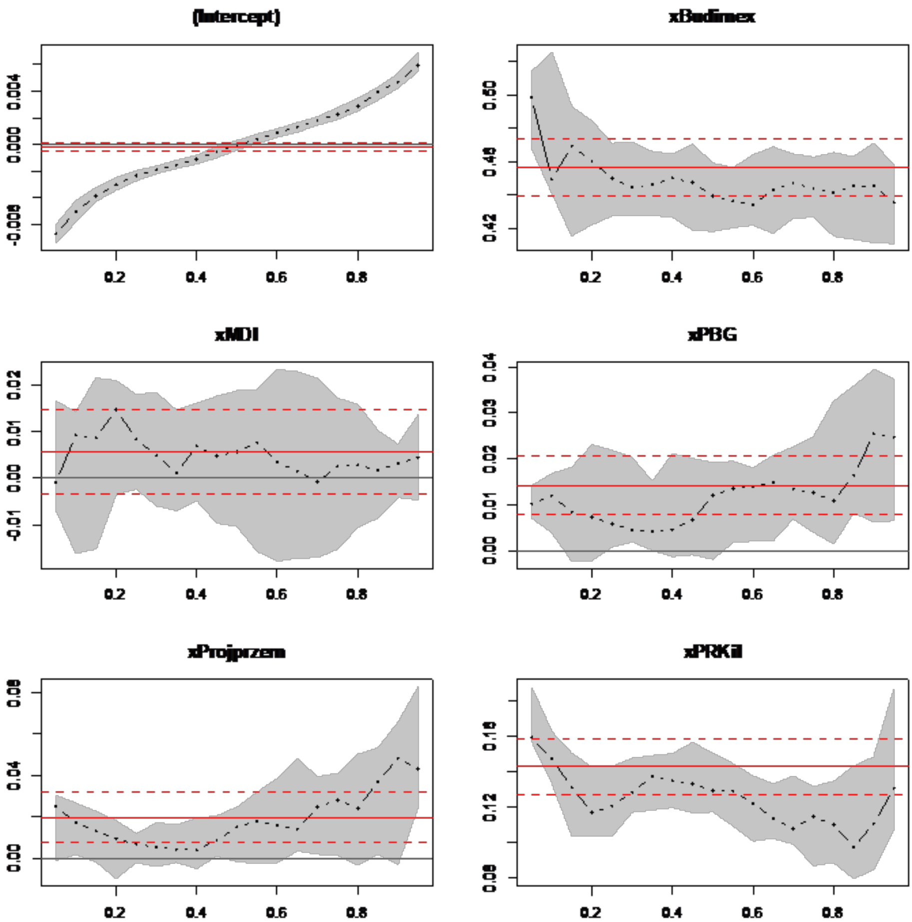

Figure 2. Normal QQ plot for the set of assets

Source: own calculation

Table 3. Results of estimation of quantile regression coefficients

tau = 0.25	coefficients	lower bd	upper bd
(Intercept)	–0.00231	–0.00269	–0.00198
Budimex	0.44989	0.42752	0.47079
MDI	0.00844	–0.00222	0.01785
PBG	0.00580	0.00085	0.02178
Projprzem	0.00669	–0.00193	0.01159
PRKil	0.12049	0.10385	0.14267

tau = 0.5	coefficients	lower bd	upper bd
(Intercept)	–0.00002	–0.00035	0.00028
Budimex	0.43920	0.41827	0.45891
MDI	0.00568	–0.01028	0.01869
PBG	0.01204	–0.00179	0.01908
Projprzem	0.01511	–0.00155	0.02449
PRKil	0.12932	0.11747	0.14991
tau = 0.75	**coefficients**	**lower bd**	**upper bd**
(Intercept)	0.00225	0.00192	0.00275
Budimex	0.44372	0.42692	0.46246
MDI	0.00258	–0.01512	0.01706
PBG	0.01261	0.00407	0.02478
Projprzem	0.02802	0.00143	0.04079
PRKil	0.11465	0.08722	0.13132

Source: own calculation

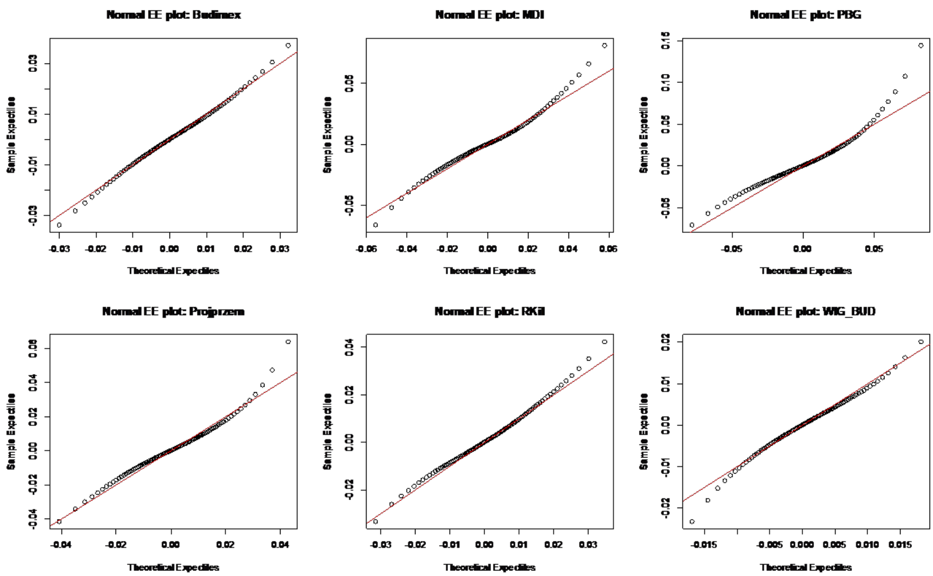

Figure 3. Quantile regression for the portfolio

Source: own calculation

In the confidence interval, we can observe almost all tail quantiles of covariates i.e. assets (equation 3 and 4). For a more detailed description of market behaviour, we additionally use downside risk measures: VaR (Value-at-Risk) and CVaR (Conditional Value-at-Risk). This is a more complete approach which treats risk as a probability of deviating the achieved results from the expected state.

The next step of the analysis of daily rates of return on the analysed assets proved once more the presence of outliers (Figure 4) and extreme observations for all the companies over the observation period. We can apply also expectile regression. In Table 4, we can see the results of estimation of expectile regression coefficients (equation 5 and 6) for selected expectiles (τ = 0.25; 0.5, 0.75).

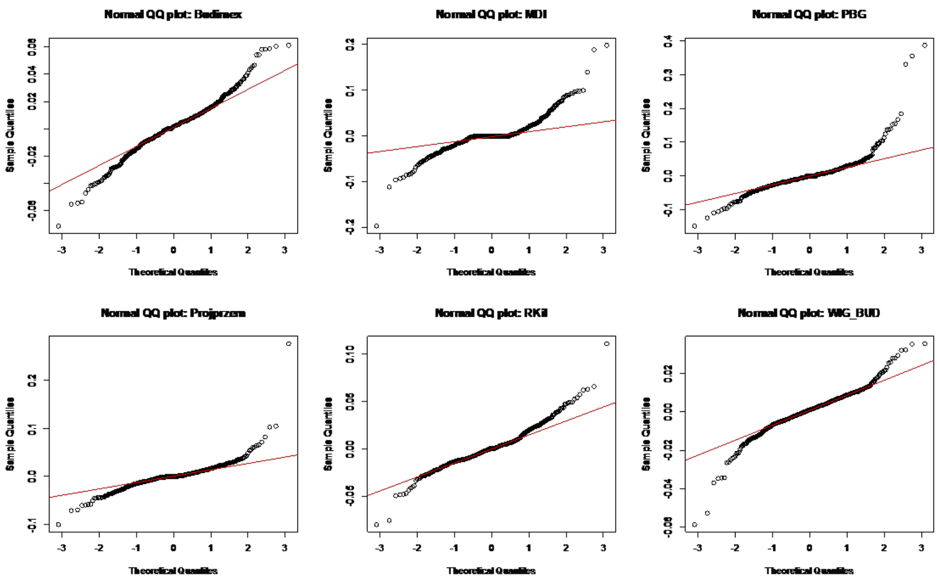

Figure 4. Normal EE plot for the set of assets

Source: own calculation

Table 4. Results of estimation of expectile regression coefficients

	tau = 0.25	tau = 0.5	tau = 0.75
(Intercept)	–0.00185519	–0.00014726	0.00151114
Budimex	0.46944980	0.45579350	0.44703930
MDI	0.00570424	0.00524984	0.00456373
PBG	0.01368684	0.01408298	0.01510063
Projprzem	0.01368684	0.01979301	0.02437974
PRKil	0.15241630	0.14212700	0.13475050

Source: own calculation

Now we can apply risk measures based on quantiles and expectiles. Additionally, we attempt to analyse properties of these measures. Portfolio analysis based on quantile coherent risk measures such as the CVaR can be applied using different estimators (Rockafellar, Uryasev 2000; 2002; Trzpiot 2007a; 2007b; 2008; 2009a; 2009b; Trzpiot, Krężołek, 2009; Trzpiot, Majewska, 2010; Trzpiot, 2010). First, we set values from the tail: quantiles and expectiles (Tables 5 and 6).

Table 5. Results of estimation of the tail: quantiles

tau	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
WIG_BUD	0.0106	0.0111	0.0117	0.0123	0.0131	0.0140	0.0162	0.0193	0.0219	0.0281
Budimex	0.0213	0.0236	0.0253	0.0259	0.0286	0.0291	0.0325	0.0362	0.0433	0.0544
MDI	0.0309	0.0395	0.0405	0.0448	0.0541	0.0606	0.0667	0.0767	0.0893	0.0977
PBG	0.0379	0.0398	0.0438	0.0474	0.0531	0.0586	0.0826	0.0957	0.1250	0.1550
Projprzem	0.0230	0.0241	0.0247	0.0280	0.0294	0.0308	0.0349	0.0378	0.0540	0.0659
PRKil	0.0250	0.0268	0.0287	0.0302	0.0319	0.0356	0.0390	0.0429	0.0483	0.0539

Source: own calculation

Table 6. Results of estimation of the tail: expectiles

tau	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
WIG_BUD	0.0084	0.0089	0.0094	0.0099	0.0106	0.0115	0.0126	0.0141	0.0163	0.0201
Budimex	0.0163	0.0173	0.0183	0.0195	0.0208	0.0224	0.0243	0.0268	0.0305	0.0371
MDI	0.0311	0.0333	0.0358	0.0385	0.0417	0.0457	0.0507	0.0569	0.0658	0.0809
PBG	0.0432	0.0465	0.0504	0.0550	0.0608	0.0680	0.0769	0.0888	0.1074	0.1442
Projprzem	0.0202	0.0215	0.0229	0.0247	0.0268	0.0295	0.0331	0.0385	0.0473	0.0639
PRKil	0.0193	0.0203	0.0214	0.0226	0.0241	0.0259	0.0281	0.0310	0.0352	0.0423

Source: own calculation

In the second step, we compare the results from the tail: quantiles and expectiles with general statistics properties (Figure 5). It turns out that the empirical distributions (the tail quantiles and expectiles) intersect, which is a result of numerous outliers and extreme observations over the observation period.

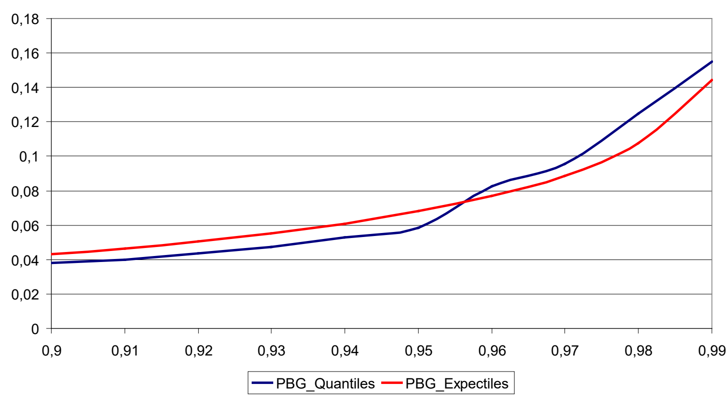

Figure 5. Empirical tail expectiles and quantiles – PBG

Source: own calculation

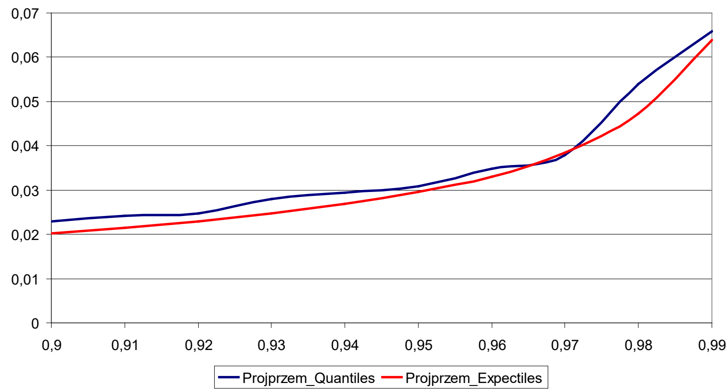

Figure 6. Empirical expectiles and quantiles – Projprzem

Source: own calculation

In the second step, we count risk measures CVaR and EVaR (expectile-VaR). The results are presented in Tables 7 and 8. We can observe by comparing results that, despite the intersection of tail quantiles and expectiles for some assets, using the value of EVaR for evaluation of portfolio requires a lower level of capital collateral on the part of the investor.

Table 7. Value of CVaR for the set of assets

	$CVAR_{0,9}$	$CVAR_{0,95}$	$CVAR_{0,99}$
WIG_BUD	0.017627	0.022517	0.03185
Budimex	0.034673	0.042783	0.05780
MDI	0.072709	0.098333	0.14835
PBG	0.102445	0.150817	0.27150
Projprzem	0.057236	0.08340	0.17145
PRKil	0.043027	0.055117	0.08245

Source: own calculation

Table 8. Value of EVaR for the set of assets

	$EVAR_{0,9}$	$EVAR_{0,95}$	$EVAR_{0,99}$
WIG_BUD	0.014309	0.018367	0.02785
Budimex	0.026773	0.033717	0.04915
MDI	0.061718	0.083083	0.13970
PBG	0.102627	0.14550	0.26595
Projprzem	0.055036	0.08155	0.17045
PRKil	0.034664	0.04560	0.07670

Source: own calculation

At the end of the study, we can comment on the results of our experimental analysis by comparing chosen risk measures to other risk measures (Table 9). We can see that the worse choice is variance, all the other measures have a bigger set of good properties. We can also compare measures based on quantiles and expectiles.

Table 9. Properties of risk measures

Property	Variance	VaR	CVaR	EVaR
Coherence			X	X
Comonotonic Additivity		X	X	
Robust, Weak Topology		X		
Robust, Wasserstein	X	X	X	X
Elicitability		X		X
Conditional Elicitability	X	X	X	X

Source: based on: Emmer, Kratz, Tasche (2013), Trzpiot (2016)

5. Final remarks

It has been recently suggested by many authors that elicitability might be a very important property in risk management, mainly for two reasons: it provides a natural methodology to perform backtesting, as well as a natural way of comparing different procedures for computing a risk measure. In this article, we looked at quantiles and expectiles, next we examined risk measures based on both so we would obtain the relationship between them and describe how it could be used in practice. We observed and examined properties of extreme quantiles and expectiles and discussed the crossing issue of quantile and expectile regression. One of the most important elements of risk management is its measurement. Knowing the risk value enables one to make the optimal portfolio choice. The evaluation of the selected portfolio allows us to conclude that the use of EVaR instead CVaR requires a lower level of equity securities on the part of the investor.

References

Aigner D., Amemiya T., Poirier D. (1976), *On the estimation of production frontiers: Maximum likelihood estimation of the parameters of a discontinuous density function*, "Journal of Economic Review", vol. 17(2), pp. 377–396.

Artzner P., Delbaen F., Eber J.M., Heath D. (1998), *Coherent measures of risk*, https://people.math.ethz.ch/~delbaen/ftp/preprints/CoherentMF.pdf [accessed: 12.09.2018].

Bellini F., Bignozzi V. (2013), *Elicitable risk measures*, Working Paper, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2334746 [accessed: 12.09.2018].

Breckling J., Chambers R. (1988), *M-quantiles*, "Biometrika", vol. 75, pp. 761–772.

Emmer S., Kratz M., Tasche D. (2013), *What is the best risk measure in practice? a comparison of standard measures*, http://arxiv.org/abs/1312.1645 [accessed: 20.05.2018].

Fissler T., Ziegel J.F. (2016), *Higher order elicitability and Osband's principle*, "Annals of Statistics", vol. 4, pp. 1680–1707.

Föllmer H., Schied A. (2002), *Convex measures of risk and trading constraints*, "Finance and Stochastics", vol. 6, issue 4, pp. 429–447.

Gneiting T. (2011), *Making and evaluating point forecasts*, "Journal of the American Statistical Association", vol. 106(494), pp. 746–762.

Koenker R., Bassett G. (1978), *Regression quantiles*, "Econometrica", vol. 46(1), pp. 33–50.

Koenker R. (2005), *Quantile regression*, Cambridge University Press, Cambridge.

Newey W.K., Powell J.L. (1987), *Asymmetric least squares estimation and testing*, "Econometrica", vol. 55(4), pp. 819–847.

Rockafellar R.T., Uryasev S. (2000), *Optimization of conditional value-at-risk*, "The Journal of Risk", vol. 2(3), pp. 21–41.

Rockafellar R.T., Uryasev S. (2002), *Conditional Value-at-Risk for General Loss Distributions*, "Journal of Banking and Finance", vol. 26, pp. 1443–1471.

Sobotka F., Schnabel S., Schulze Waltrup L., Eilers P., Kneib T., Kauermann G. (2011), *Expectreg: Expectile and quantile regression*, R package version 0.25.

Sobotka F., Kneib T. (2012), *Geoadditive expectile regression*, "Computational Statistics and Data Analysis", vol. 56, pp. 755–767.

Trzpiot G. (2007a), *Decomposition of Risk and Quantile Risk Measures*, [in:] *Dynamiczne Modele Ekonometryczne*, "Prace Naukowe Uniwersytetu Mikołaja Kopernika w Toruniu", pp. 35–42.

Trzpiot G. (2007b), *Regresja kwantylowa a estymacja VaR*, "Prace Naukowe Akademii Ekonomicznej we Wrocławiu", vol. 1176, pp. 465–471.

Trzpiot G. (2008), *Implementacja metodologii regresji kwantylowej w estymacji VaR*, "Studia i Prace", no. 9, Uniwersytet Szczeciński, Szczecin, pp. 316–323.

Trzpiot G. (2009a), *Application weighted VaR in capital allocation*, "Polish Journal of Environmental Studies", vol. 18, no. 5B, pp. 203–208.

Trzpiot G. (2009b), *Estimation methods for quantile regression*, "Studia Ekonomiczne. Zeszyty Naukowe Akademii Ekonomicznej w Katowicach", vol. 53, pp. 81–90.

Trzpiot G. (2016), *Semi-parametric risk measures*, "Studia Ekonomiczne. Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach", vol. 288(5), pp. 108–120.

Trzpiot G. (red.) (2010), *Wielowymiarowe metody statystyczne w analizie ryzyka inwestycyjnego*, Polskie Wydawnictwo Ekonomiczne, Warszawa.

Trzpiot G., Krężołek D. (2009), *Quantiles ratio risk measures for stable distributions models in finance*, "Studia Ekonomiczne. Zeszyty Naukowe Akademii Ekonomicznej w Katowicach", vol. 53, pp. 109–120.

Trzpiot G., Majewska J. (2010), *Estimation of Value at Risk: Extreme value and robust approaches*, "Operation Research and Decisions", vol. 20, no. 1, pp. 131–143.

Ziegel J.F. (2016), *Coherence and elicitability*, "Mathematical Finance", vol. 26, pp. 901–918.

Pomiar ryzyka inwestycyjnego z wykorzystaniem kwantyli i oczekiwań

Streszczenie: W badaniach starano się przyjrzeć szczegółowemu pomiarowi ryzyka inwestycyjnego. Użyto regresji kwantylowej jako modelu, opisując bardziej ogólne właściwości rozkładu stopy zwrotu. W regresji kwantylowej przyjęto efekty regresji względem warunkowych kwantyli regresorów. W modelu regresji skoncentrowano się na rozszerzeniu regresji liniowej (OLS), wykorzystując regresję oczekiwań. Celem zastosowania obu podejść jest pomiar ryzyka inwestycyjnego. Obydwa modele regresji są wersją ważonego modelu najmniejszych kwadratów. Najczęściej stosowanymi rodzinami miar ryzyka, poza miarami zmienności, są miary zagrożenia, a w praktyce wartość zagrożona (VaR) i warunkowa wartość zagrożona ryzykiem (CVaR). Można je oszacować przez kwantyle lub oczekiwania wyznaczone w ogonie rozkładu odpowiedzi.

Słowa kluczowe: kwantyle, oczekiwania, VaR, CVaR, asymetryczny model najmniejszych kwadratów

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