Folia Oeconomica
Acta Universitatis Lodziensis
ISSN 0208-6018 e-ISSN 2353-7663

www.czasopisma.uni.lodz.pl/foe/
5(338) 2018

DOI: http://dx.doi.org/10.18778/0208-6018.338.07

Anna Staszewska-Bystrova
University of Łódź, Faculty of Economics and Sociology, Chair of Econometric Models and Forecasts, emfans@uni.lodz.pl

# Monte Carlo Analysis of Forecast Error Variance Decompositions under Alternative Model Identification Schemes[1]

**Abstract:** The goal of the paper is to investigate the estimation precision of forecast error variance decomposition (FEVD) based on stable structural vector autoregressive models identified using short-run and long-run restrictions. The analysis is performed by means of Monte Carlo experiments. It is demonstrated that for processes with roots close to one, selected FEVD parameters can be estimated more accurately using recursive restrictions on the long-run multipliers than under recursive restrictions on the impact effects of shocks. This finding contributes to the discussion of pros and cons of using alternative identification schemes by providing counterexamples for the notion that short-run identifying restrictions lead to smaller estimation errors than long-run restrictions.

**Keywords:** forecast error variance decomposition, structural vector autoregressive model, long-run restrictions, short-run restrictions

**JEL:** C32

[1] Financial support from the National Science Center (NCN) through MAESTRO 4: DEC–2013/08/A/HS4/00612 is gratefully acknowledged.

# 1. Introduction

The most popular tools used in structural vector autoregressive (VAR) analysis are the impulse response functions (IRFs) and the forecast error variance decompositions (FEVDs). Estimation precision of the parameters of IRFs and FEVDs depends, among others, on the identification scheme used to obtain structural shocks. In particular, it was emphasized that it might be difficult to estimate accurately the long-run effects of structural shocks using SVARs identified with long-run restrictions (e.g., Faust, Leeper, 1997; Christiano, Eichenbaum, Vigfusson, 2006; Francis et al., 2014; see also Kilian, Lütkepohl, 2017 for an overview). These problems can be more severe if the model is misspecified, e.g. it contains too few lags of the modelled variables. This implies that VARs based on long-run restrictions might be unreliable in empirical studies.

Lütkepohl, Staszewska-Bystrova and Winker (2017) compared, by means of Monte Carlo experiments, the relative estimation accuracy for impulse responses associated with popular short-run and long-run identifying restrictions for structural VAR models. To allow direct comparison, such data generating processes (DGPs) and such short-run and long-run identifying restrictions were devised which implied exactly the same structural impulse responses. The estimation accuracy measure was given by the mean squared error (MSE). It was shown that short-run restrictions lead, as expected, to more precise estimates of the impact effects of shocks, but long-run responses for persistent processes may be estimated more accurately when long-run identification scheme is followed. In the VAR context, estimation uncertainty is usually depicted using confidence intervals or bands (see e.g. Staszewska, 2007; Lütkepohl, Staszewska-Bystrova, Winker, 2015a; 2015b). Larger estimation accuracy implies narrower and hence more informative confidence regions.

In this paper, the relative estimation precision of forecast error variance decompositions under selected types of short-run and long-run identifying restrictions is analyzed using some suitably designed Monte Carlo experiments. The aim is to verify concerns from the literature related to poor estimation accuracy associated with using long-run identification schemes. The findings of Lütkepohl, Staszewska-Bystrova and Winker (2017) indicate that it should be possible to present cases when a long-run approach to identification outperforms a short-run identification scheme in terms of estimation precision of the FEVD parameters.

The experimental design involves DGPs which imply the same FEVD when matched with the short-run recursive identification scheme and recursive restrictions concerning long-run effects of shocks. The former type of restrictions are common in applications (see, for example Kilian, 2009; Li, İşcan, Xu, 2010 or Bruno, Shin, 2015). The latter were used e.g. by Blanchard and Quah (1989) (hence they are often referred to as Blanchard-Quah type of restrictions), Chaudourne, Fève and Guay (2014) or Lin and Liu (2016). The specific setup of the simulations makes it possible to directly compare the impact of using these alternative identification schemes on the estimation accuracy of FEVDs. Furthermore, the DGPs under study are two- or three-dimensional, exhibit various degrees of persistence and have the form of finite- and infinite-order VARs. Therefore, they share characteristics of many models used in empirical studies.

The structure of the paper is as follows. In Section 2 the structural vector autoregressive model and the forecast error variance decomposition are presented. Sections 3 and 4 discuss respectively, design of the Monte Carlo experiments and the results obtained. Finally, Section 5 concludes.

# 2. The model

The general form of the reduced-form VAR model under consideration is given by:

$$y_t = \nu + \sum_{i=1}^{\infty} A_i y_{t-i} + u_t, \tag{1}$$

where $y_t = \left(y_{1t}, \ldots, y_{Kt}\right)'$, the $A_i$, $i = 1, 2, \ldots$, are coefficient matrices with dimensions $K \times K$, $\nu$ represents a $K \times 1$ vector of constants and $u_t = \left(u_{1t}, \ldots, u_{Kt}\right)'$ is a white noise process such that $u_t \sim \left(0, \Sigma_u\right)$ and $\Sigma_u$ is positive definite. Equation (1) is general since it allows for an infinite lag order. In selected cases considered below, a finite number of lags in the data generating process will be considered, i.e. the parameters corresponding to lags greater than $p$ will be assumed to be 0.

The analysis concerns VARs which are stable and stationary. Such processes, meeting the following condition,

$$\det A(z) = \det\left(I_K - \sum_{i=1}^{\infty} A_i z^i\right) \neq 0 \text{ for } z \in \mathbb{C}, |z| \leq 1, \tag{2}$$

can be represented as a vector moving average (VMA)

$$y_t = A(1)^{-1} \nu + A(L)^{-1} u_t = \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i}. \tag{3}$$

The parameters of (1) and (3) are related in the following way $\mu = A(1)^{-1} \nu$, $\Phi_0 = I_K$ and $\sum_{i=0}^{\infty} \Phi_i L^i = A(L)^{-1}$ (see e.g. Lütkepohl, 2005 or Kilian, Lütkepohl, 2017).

To identify the structural shocks, $\varepsilon_t$, and hence to obtain the structural model, some assumptions have to be made. It is common to assume that $\varepsilon_t = B^{-1} u_t$, where

$BB' = \Sigma_u$ and $\varepsilon_t \sim (0, I_K)$, i.e. that the structural shocks are related to the reduced form errors in a linear way, they are not correlated and have unit variances. In general, in order to uniquely identify the matrix $B$ some additional restrictions have to be imposed (for the discussion of some exceptions see e.g. Lanne, Meitz, Saikkonen, 2017). These most often refer to the short-run or long-run responses to structural shocks which can be obtained from equation (3) by replacing reduced form errors with $B\varepsilon_t$:

$$y_t = \mu + \sum_{i=0}^{\infty} \Phi_i B \varepsilon_{t-i} = \mu + \sum_{i=0}^{\infty} \Theta_i \varepsilon_{t-i}, \tag{4}$$

where $\Theta_i = \Phi_i B$. The impact effects of shocks are contained in $B = \Theta_0$, while the total cumulated effects can be computed by adding all the $\Theta_i$ matrices. This sum is given by $\Xi_\infty = A(1)^{-1} B$.

In this paper, two identification schemes are considered. According to the first one, which involves short-run restrictions, $B$ is specified to be lower triangular leading to a recursive model. Such a matrix, denoted in what follows by $B^s$, can be obtained from a Cholesky decomposition of the variance matrix $\Sigma_u$:

$$B^s = \mathrm{chol}(\Sigma_u). \tag{5}$$

The second approach to identification imposes long-run restrictions on the total effects matrix $\Xi_\infty$, which is assumed to be lower-triangular. This implies a unique $B^l$ matrix which can be computed as

$$B^l = A(1)\,\mathrm{chol}\left(A(1)^{-1} \Sigma_u A(1)^{-1'}\right). \tag{6}$$

The matrices $B^s$ or $B^l$ can be used in the impulse response analysis or to conduct the forecast error variance decomposition which is the focus of this paper. The FEVD consists in computing proportions of the $h$-step forecast error variance of each variable accounted for by the structural shocks. The contribution of $k$-th shock to the forecast error variance of variable $j$ at horizon $h$ for $h = 0, 1, \ldots$, is given by

$$\omega_{jk,h} = \sum_{i=0}^{h-1} \left(e_j{}'\Theta_i e_k\right)^2 / \sum_{i=0}^{h-1}\sum_{k=1}^{K} \theta_{jk,i}^2, \tag{7}$$

where $e_k$ represents the $k$-th column of an identity matrix of order $K$ and $\theta_{jk,i}$ stands for the $jk$-th element of $\Theta_i$. For systems which are stationary it is possible to consider this decomposition for finite horizons but also for horizon infinity.

Lütkepohl, Staszewska-Bystrova and Winker (2017) show that $B^s$ and $B^l$ matrices are identical if $A(1)$ is lower triangular and $A(1)^{-1} B^s$ has positive diagonal elements. In such cases, the structural impulse responses $\Theta_i$ implied by the specific types of short-run and long-run restrictions are also the same. Since $\omega_{jk,h}$'s depend, in a nonlinear fashion, only on the $\Theta_i$ values, they are also identical if the above conditions are met. It should be emphasized that the equivalence concerns the true parameter values. In empirical work, these quantities have to be estimated and the estimates obtained using short-run restrictions will be different than those corresponding to long-run restrictions. The discrepancy arises since the estimated $A(1)$ matrix will not be lower triangular.

Derivation of conditions required for the equality of the parameters of FEVDs obtained using Blanchard-Quah type of restrictions and short-run recursive restrictions, makes it possible to set up simulation experiments for analyzing the relative estimation accuracy corresponding to these identification schemes. The design of such a Monte Carlo study is provided in the next section.

## 3. Monte Carlo experiments

The DGPs used in simulations meet the criteria necessary to ensure that the true parameters of the forecast error variance decomposition under short-run and long-run identification schemes are exactly the same. Both finite and infinite VAR processes are investigated. In the latter case the model used for the analysis can only approximate the true DGP which can have an impact on the results.

Estimation is performed using multivariate least squares and its precision is measured by the mean squared error. To allow quick comparisons, the results are presented as MSE ratios computed for the FEVD parameters, where the value in the numerator corresponds to identification based on short-run restrictions and the denominator value is associated with long-run restrictions. Thus, values greater than one mean that long-run identifying restrictions lead, on average, to more accurate estimates of FEVD parameters than short-run restrictions, while values smaller than one imply the opposite conclusion.

The first DGP (DGP1) is trivariate and has the form:

$$y_t = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0.3 & 0.7 & 0 \\ 0.2 & 0.2 & 0.5 \end{bmatrix} y_{t-1} + u_t, \quad u_t \sim i.i.d.N\left(0, \begin{bmatrix} 1 & \sigma_{12} & 0.2 \\ \sigma_{12} & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix}\right),$$

where $\alpha_{11} \in \{-0.9, -0.5, 0, 0.5, 0.9\}$ and $\sigma_{12} \in \{0.2, 0.5\}$. This DGP describes series of various degrees of persistence, controlled by the parameter $\alpha_{11}$, and hence captures the features of many observed economic variables (measured in levels or growth rates). The system is most persistent for $\alpha_{11} = 0.9$ and $\alpha_{11} = -0.9$. The parameter $\sigma_{12}$ measures correlation between the error terms. DGP1 is typical for Monte Carlo analyses of stable vector autoregressions where it is quite common to study bivariate or trivariate processes with one lag, varying degree of persistence and alternative correlations between the errors (see e.g. Kilian, 1998 or Kim, 2014). Models with two or three variables are also popular in applications. Similar DGP (labeled as DGP2) was used by Lütkepohl, Staszewska-Bystrova and Winker (2017) who considered identical values of the autoregressive parameters but slightly different variance matrix of the errors which was fixed. In this paper two different forms of the covariance matrix are considered to study sensitivity of the results with respect to strength of contemporaneous correlations between the error terms.

For DGP1,

$$A(1)=\begin{bmatrix} 1-\alpha_{11} & 0 & 0 \\ -0.3 & 0.3 & 0 \\ -0.2 & -0.2 & 0.5 \end{bmatrix}$$

and

$$A(1)^{-1}B^s=\begin{bmatrix} \frac{1}{1-\alpha_{11}} & 0 & 0 \\ \frac{1}{1-\alpha_{11}}+3.33\sigma_{12} & 3.33\sqrt{1-\sigma_{12}^2} & 0 \\ \frac{0.8}{1-\alpha_{11}}+1.33\sigma_{12}+0.4 & 1.33\sqrt{1-\sigma_{12}^2}+\frac{0.4(1-\sigma_{12})}{\sqrt{1-\sigma_{12}^2}} & 2\sqrt{0.96-\frac{0.04(1-\sigma_{12})}{1+\sigma_{12}}} \end{bmatrix}.$$

Since $A(1)$ is lower triangular and the elements on the main diagonal of $A(1)^{-1}B^s$ are positive, then it follows that $B^s=B^l$ which can be easily checked. This equality implies that the true parameter values of the forecast error variance decomposition are the same, independently of whether short-run or long-run restrictions are used.

The next DGP (DGP2) is a 4-dimensional system with two lags:

$$y_t = \begin{bmatrix} 0.7 & 0.3 & 0 & 0.2 \\ 0.2 & 0.5 & 0.1 & 0 \\ 0.4 & 0.2 & 0.5 & 0.2 \\ 0.3 & 0.3 & 0.2 & 0.3 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0.1 & -0.3 & 0 & -0.2 \\ 0.1 & 0.2 & -0.1 & 0 \\ -0.2 & 0.3 & 0.1 & -0.2 \\ 0.4 & 0.2 & 0.1 & 0.3 \end{bmatrix} y_{t-2} +$$

$$u_t,\ u_t \sim i.i.d.N\left(0, \begin{bmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.3 & 0.3 \\ 0.3 & 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 0.3 & 1 \end{bmatrix}\right).$$

The stability condition (2) is met for this DGP, however the process is quite persistent as the absolute value of the dominant root is close to 1 and amounts to 1.0529.

Matrices $A(1)$ and $A(1)^{-1} B^s$ are as follows

$$A(1) = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ -0.3 & 0.3 & 0 & 0 \\ -0.2 & -0.5 & 0.4 & 0 \\ -0.7 & -0.5 & -0.3 & 0.4 \end{bmatrix},\ A(1)^{-1} B^s = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 6 & 3.18 & 0 & 0 \\ 10.75 & 4.53 & 2.32 & 0 \\ 25.06 & 7.92 & 2.18 & 2.28 \end{bmatrix},$$

and so the forecast error variance decomposition returns the same true values for short-run and long-run identification approach.

DGP2 is studied as persistent processes with a root close to one are very common in empirical macroeconomic studies.

DGP3 involves richer dynamics than DGP1 and DGP2 as its lag order amounts to 8:

$$y_t = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.3 \end{bmatrix} y_{t-1} + \begin{bmatrix} 0 & -0.2 \\ -0.3 & 0 \end{bmatrix} y_{t-2} + \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix} y_{t-3} +$$

$$\begin{bmatrix} -0.3 & -0.1 \\ 0 & 0 \end{bmatrix} y_{t-4} + \begin{bmatrix} 0 & 0 \\ 0 & -0.2 \end{bmatrix} y_{t-8} + u_t.$$

The process is considered, as greater lag length is likely to affect the accuracy of the implied FEVD parameter estimators. $u_t$ are zero mean errors with the variance matrix $\begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$. The smallest modulus of a root of the reverse characteristic polynomial from (2) is 1.1087, implying stability of this DGP. The form of matrices $A(1)$ and $A(1)^{-1}B^s$:

$$A(1) = \begin{bmatrix} 0.9 & 0 \\ -0.1 & 1.1 \end{bmatrix} \text{ and } A(1)^{-1}B^s = \begin{bmatrix} 1.11 & 0 \\ 0.28 & 0.89 \end{bmatrix},$$

indicates that FEVD for both approaches to identification yield the same results if true parameter values are used.

Finally, the most complex data generating process (DGP4) is VMA(1) which corresponds to a VAR with infinite lag order. Such processes are often considered in the literature as they are consistent with the popular dynamic stochastic general equilibrium (DSGE) models (see e.g. Giacomini, 2013). DGP4 has the form:

$$y_t = u_t + M_1 u_{t-1} = u_t + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0.3 & 0.7 & 0 \\ 0.2 & 0.2 & 0.5 \end{bmatrix} u_{t-1},$$

$$u_t \sim i.i.d.N\left(0, \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix}\right),$$

where $\alpha_{11} \in \{-0.9, -0.5, 0, 0.5, 0.9\}$. As typical for Monte Carlo analyses of VMA processes, the objective of considering different values of $\alpha_{11}$ is to obtain alternative magnitudes of the roots of the characteristic equation $\det(I_3 + M_1 z) = 0$ (see e.g. Galbraith, Ullah, Zinde-Walsh, 2002 or Bruder, 2015). In effect, DGP4 might characterize a large number of empirical processes.

$A(1)$ and $A(1)^{-1}B^s$, corresponding to DGP4, are

$$A(1)=\begin{bmatrix} \dfrac{1}{1+\alpha_{11}} & 0 & 0 \\ \dfrac{-0.18}{1+\alpha_{11}} & 0.59 & 0 \\ \dfrac{-0.11}{1+\alpha_{11}} & -0.08 & 0.67 \end{bmatrix}$$

and

$$A(1)^{-1}B^{s}=\begin{bmatrix} 1+\alpha_{11} & 0 & 0 \\ 0.64 & 1.67 & 0 \\ 0.54 & 0.44 & 1.45 \end{bmatrix}.$$

and so they also meet the conditions required for equality of $B^s$ and $B^l$.

Further settings for the simulation experiments are as follows. The number of Monte Carlo replications is given by 5000 (see e.g. Huh, 2013 or Ludvigson, Ma, Ng, 2017 who employ the same number of Monte Carlo trials in simulations similar to those performed in this study). Two sample sizes are analyzed, $T \in \{200, 400\}$. These values approximate numbers of observations typically used in empirical macroeconometric investigations. Sample sizes of 100 or smaller, which might be also of interest, are not considered given a considerable complexity of some of the DGPs under investigation. The true lag order is assumed to be known for DGP1, DGP2 and DGP3 or it is selected based on the Akaike information criterion (AIC). The lag length for DGP4 is only estimated using AIC. The maximum lag order for model selection is set to 10 if $T = 200$ and to 14 if $T = 400$. The VAR models include intercepts.

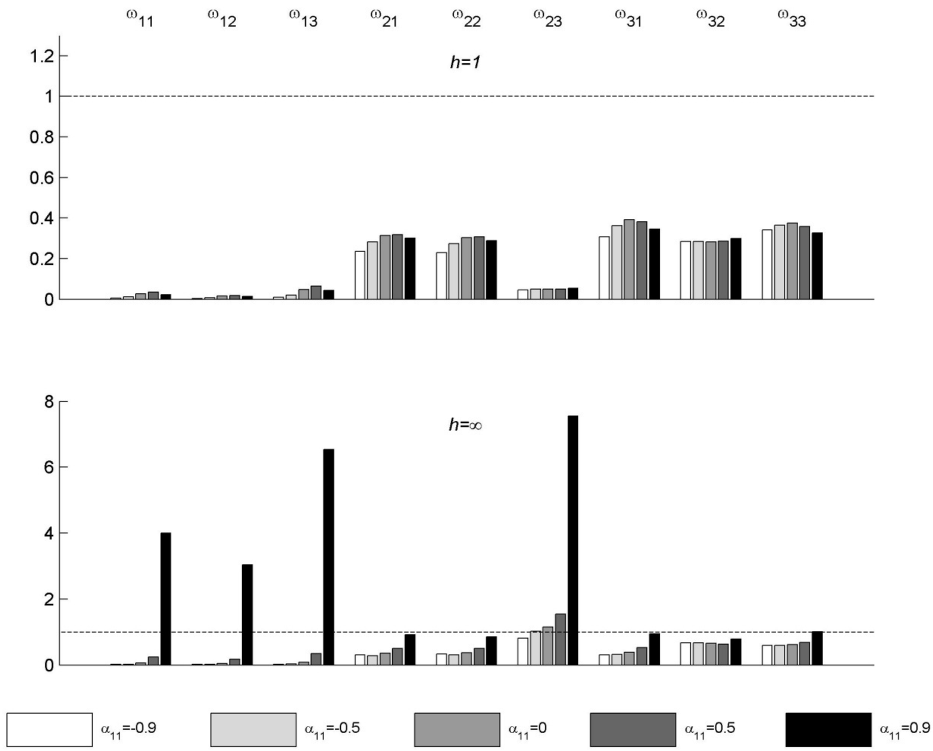


Figure 1. Relative MSEs for the estimation of forecast error variance decomposition for DGP1 with $\sigma_{12} = 0.2$, $T = 200$ and lag order selected using AIC

Source: own elaboration

# 4. Results

In this section, selected simulation results are presented. First, conclusions for DGPs with small number of lags are described (DGP1 and DGP2) and later, processes with richer dynamics (DGP3 and DGP4) are analyzed.

Figure 1 shows the MSE ratios for the forecast error variance decomposition for DGP1 with alternative values of $\alpha_{11}$ and $\sigma_{12} = 0.2$, estimated on the basis of 200 observations for horizons 1 and infinity. Infinity is approximated by a large horizon for which convergence of the estimated values is achieved (the convergence criterion is set to 0.001). The results were obtained under the more realistic assumption of unknown lag order of the VAR that generated the data, i.e. the lag length was chosen using AIC. The values marked with bars greater than 1 indicate that the MSE corresponding to long-run restrictions is smaller than that for short-run restrictions.

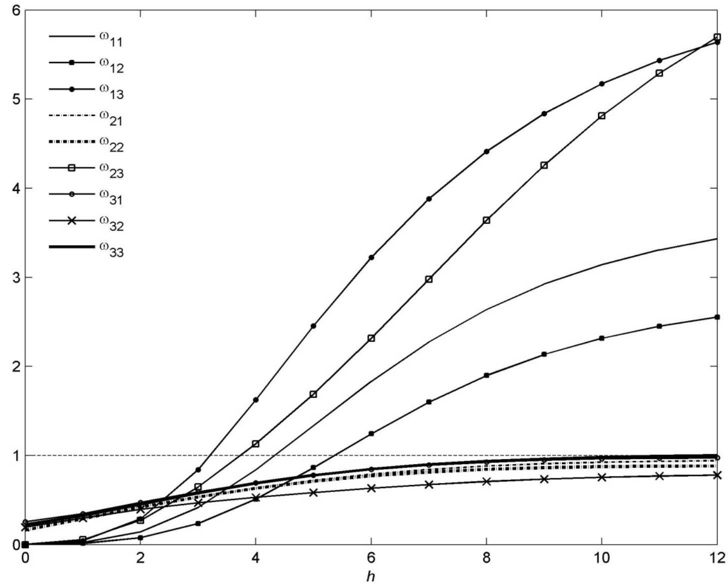


Figure 2. Relative MSEs for the estimation of forecast error variance decomposition for DGP1 with $\alpha_{11} = 0.9$, $\sigma_{12} = 0.2$, $T = 200$ and lag order selected using AIC for horizons $h = 0,1,\ldots,12$

Source: own elaboration

It is clear that for $h = 1$ all the parameters of the forecast error variance decomposition are estimated more precisely under short-run identifying restrictions. This result holds independently of the value of $\alpha_{11}$. The conclusions change however, when horizon infinity is considered. In this case, the estimation precision depends on the dynamic features of the DGP. For $\alpha_{11} = 0.9$, selected contributions of shocks to the forecast error variance of the variables (five out of nine) are estimated more accurately using the long-run identification scheme. In some cases, i.e. for $\omega_{11}$, $\omega_{12}$, $\omega_{13}$ and $\omega_{23}$, the gains are very substantial as the relative MSEs have values between 3.03 (for $\omega_{12}$) and 7.55 (for $\omega_{23}$). In the remaining cases for $\alpha_{11} = 0.9$ and horizon infinity, the MSEs corresponding to the estimation based on long-run restrictions are not very much larger than those obtained using the short-run identification scheme (the MSE ratios vary from 0.78 (for $\omega_{32}$) to 0.94 (for $\omega_{31}$)). This demonstrates that employing long-run restrictions is not always associated with worse estimation accuracy of the parameters of FEVDs as compared to using short-run restrictions implying the same true values of these parameters.

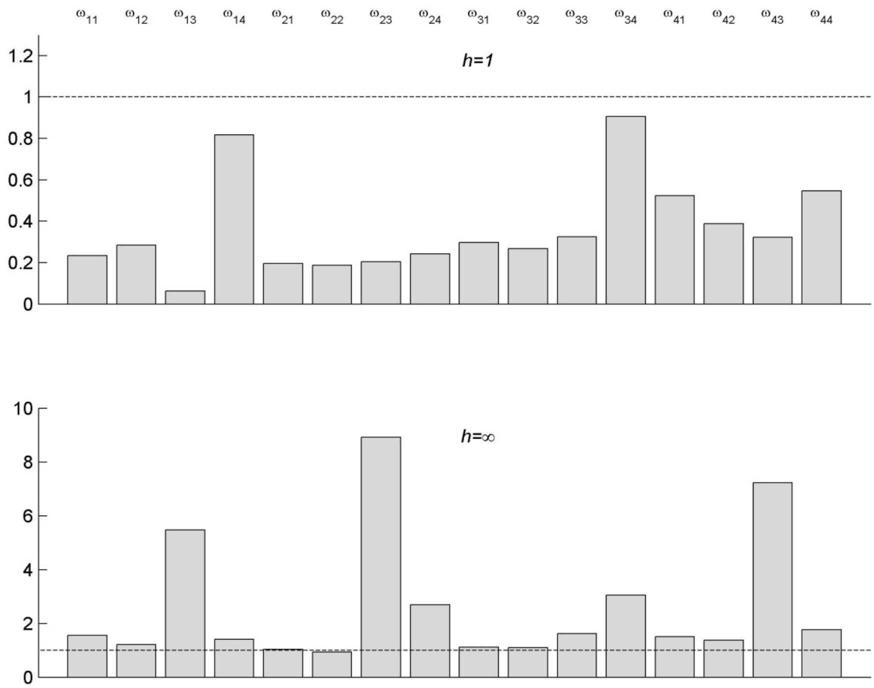


Figure 3. Relative MSEs for the estimation of forecast error variance decomposition for DGP2, *T* = 200 and lag order selected using AIC

Source: own elaboration

To analyze the effect of changing *h*, Figure 2 shows relative MSEs for DGP1 with $\alpha_{11} = 0.9$, $\sigma_{12} = 0.2$ and $T = 200$ for horizons $0,1,\dots,12$. It becomes apparent that the advantages associated with using long-run restrictions which could be noticed at $h = \infty$ for $\omega_{11}$, $\omega_{12}$, $\omega_{13}$, $\omega_{23}$ and $\omega_{33}$ manifest themselves much earlier, for horizons smaller than 12.

These graphs look very similar for *T* = 400 and for the true lag order used in the estimated VAR. Also, in these cases, the parameters $\omega_{11}$, $\omega_{12}$, $\omega_{13}$ and $\omega_{23}$ are estimated much more precisely using Blanchard-Quah type of restrictions for $\alpha_{11} = 0.9$ and $h = \infty$ (the values of the MSE ratios are even higher than for *T* = 200 and unknown lag length). Thus, some general conclusions for DGP1 are as follows. The forecast error variance decomposition can be estimated more precisely, as measured by the MSE, using short-run restrictions for initial horizons and process with roots further from 1, but it might be beneficial to use restrictions on the long-run effects of the shocks for persistent processes and intermediate or long horizons. The conclusions do not change if $\sigma_{12}$ is set to 0.5 instead of 0.2.

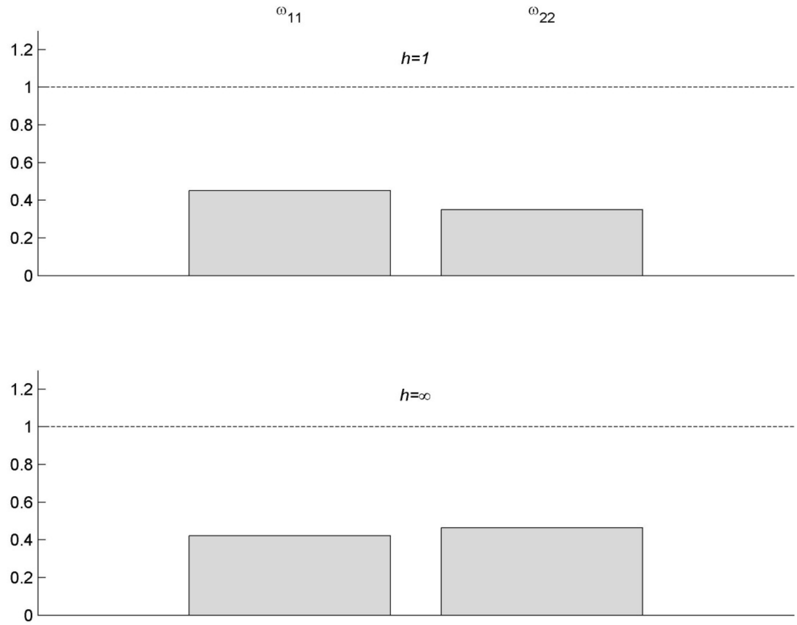


Figure 4. Relative MSEs for the estimation of forecast error variance decomposition for DGP3, $T = 200$ and lag order selected using AIC

Source: own elaboration

Figure 3 presents the MSE ratios (for $h=1$ and for $h=\infty$) computed for DGP2. Again, results for estimated lag order of the VAR are shown. The sample size is 200. Relative MSEs from top panel of this graph are all smaller than 1, indicating estimation gains associated with using short-run identifying restrictions. The conclusions concerning the relative estimation precision are again opposite for horizon infinity for which, for this quite persistent process, all the parameters apart from one ($\omega_{22}$) are estimated with larger accuracy using restrictions on the long-run effects of the shocks. As before, whenever long-run restrictions lead to improvements in estimation precision, this can be already observed for relatively small values of $h$. In 7 out of 15 cases, the MSE ratios become larger than one for $h \leq 5$, in further 7 cases they assume values greater than 1 for $6 < h \leq 11$ and in one case ($\omega_{21}$) in a further period.

The conclusions for DGP2 are qualitatively similar for the other experimental settings, i.e. using the actual lag length in the estimated model and larger sample size ($T = 400$).

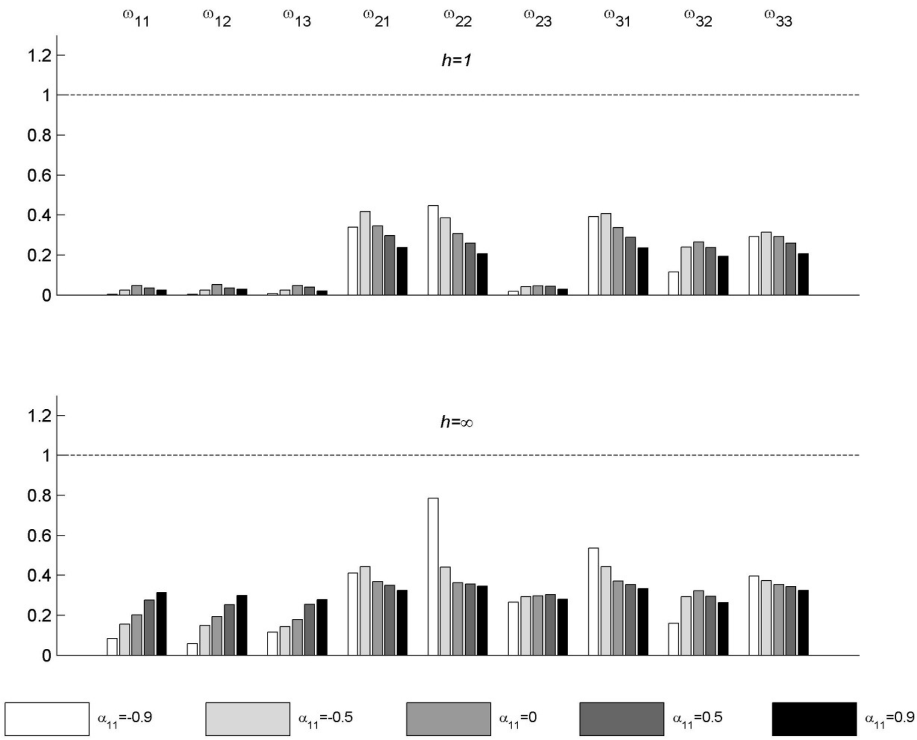


Figure 5. Relative MSEs for the estimation of forecast error variance decomposition for DGP4, $T$ = 200 and lag order selected using AIC

Source: own elaboration

Next, results for DGP3 are presented. These are given for two out of four $\omega_{ij}$ parameters as the relative estimation precision is identical for $\omega_{11}$ and $\omega_{12}$ and also for $\omega_{21}$ and $\omega_{22}$. The MSE ratios for $T$ = 200 and the case of unknown lag order are shown in Figure 4. It can be seen that for a DGP which is quite persistent but contains more lags, using short-run restrictions is more efficient than using long-run identifying assumptions for both $h=1$ and $h=\infty$. The same conclusion as to the relative estimation precision holds true for intermediate horizons, larger sample size or even under the assumption that the correct number of lags is used in the model.

The conclusion that the relative estimation precision depends on specific features of the data generating process is confirmed for DGP4. In this case, a VAR model with up to 10 or 14 lags (for 200 and 400 observations respectively) is only an approximation to the true DGP. For both sample sizes analyzed (Figure 5 presents the results for $T$ = 200), the estimation precision is greater if short-run identifying restrictions are employed. This holds for all horizons $h$, including horizon infinity and all values of $\alpha_{11}$.

# 5. Conclusions

The aim of this paper was to compare estimation accuracy for the forecast error variance decomposition in structural vector autoregressive models identified using short-run and long-run restrictions.

The analysis was performed by means of Monte Carlo simulations. To enable meaningful comparison, such data generating processes were used which imply exactly the same forecast error variance decomposition under two popular identification schemes from the VAR literature, i.e. short-run recursive identification scheme and long-run Blanchard-Quah type of restrictions.

The main conclusions from the study are as follows. For short horizons, the parameters of forecast error variance decomposition can be estimated more accurately under short-run identifying restrictions than under long-term restrictions. The results for intermediate as well as long horizons and horizon infinity depend on the features of the data generating process. For processes whose roots are far from 1 and those which can be only approximated by the model used, estimators based on short-run restrictions have smaller mean squared errors than those using long-run restrictions also for further horizons. The Blanchard-Quah type of restrictions may, however, have an advantage for persistent processes with roots close to 1 which can be well represented by the estimated model.

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**Badanie dekompozycji wariancji błędów prognozy przy różnych schematach identyfikacji modeli wektorowej autoregresji za pomocą metody Monte Carlo**

**Streszczenie:** Celem artykułu jest zbadanie dokładności estymacji parametrów dekompozycji wariancji błędów prognozy dla strukturalnych modeli wektorowej autoregresji zidentyfikowanych z użyciem restrykcji na parametry krótko- i długookresowe. W analizie wykorzystano eksperymenty Monte Carlo. Wykazano, że dla procesów o pierwiastkach, których wartość zbliżona jest do jedności, wybrane parametry dekompozycji wariancji błędów prognozy można oszacować z większą precyzją przy założeniu trójkątnej macierzy mnożników długookresowych niż przy restrykcji trójkątnej macierzy mnożników bezpośrednich. Uzyskane wyniki wnoszą wkład do dyskusji dotyczącej zalet i wad różnych schematów identyfikacji przez wskazanie kontrprzykładów dla hipotezy, że wykorzystanie restrykcji krótkookresowych prowadzi do mniejszych błędów szacunku niż zastosowanie restrykcji na parametry długookresowe.

**Słowa kluczowe:** dekompozycja wariancji błędów prognozy, strukturalne modele wektorowej autoregresji, restrykcje długookresowe, restrykcje krótkookresowe

**JEL:** C32

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| OPEN ACCESS |  |
| | Received: 2017-09-16; verified: 2018-02-05. Accepted: 2018-08-03 |