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EXTENDED **MR** WITH NESTING OF PREDICATE EXPRESSIONS AS A BASIC LOGIC FOR SOCIAL PHENOMENA

Abstract

In this article, we present the positional logic that is suitable for the formalisation of reasoning about social phenomena. It is the effect of extending the Minimal Realisation (**MR**) logic with new expressions. These expressions allow, inter alia, to consider different points of view of social entities (humanistic coefficient). In the article, we perform a metalogical analysis of this logic. Finally, we present some simple examples of its application.

Keywords: Logic for social sciences, positional logic, realisation operator, social phenomena.

1. Introduction: Quality vs quantity

Our work aims to develop a new perspective on the possibility of applying positional logic to social sciences issues. This paper can also be perceived as an attempt to build a bridge between philosophical and logical concepts and the specific needs of sociology. However, this analysis does not only refer to classic philosophical theories (as sometimes sociologist did in the past), but also presents the proposal of extension and usage of Minimal Realisation (**MR**) logic for solving an important methodological problem: How to combine qualitative and quantitative perspectives in sociology. The

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problem discussed in this paper is very similar to the issue that could recently be found in [6], concerning: how to build a bridge between big data and thick data in the sociology of the Internet. However, our answer is completely different. This work treats tradition (in this case Jerzy Łoś concepts, which are the foundation for **MR** logic) not only as an important point of reference, but also as a practical 'tool' for contemporary research and vital methodological issues of sociology.

In contemporary sociology, there is a clear division between quantitative and qualitative researchers. Quantitative researchers seek to explain social phenomena in the manner of natural science. The emphasis is therefore on the formalisation, validity, reliability, and looking for cause–effect relationships. The qualitatively oriented researchers focus on meanings, understanding (Verstehen), local descriptions, interpretations and reconstructions of collective ways of perceiving the world.

Our proposal is a continuation of the attempt to build a bridge between the two kinds of research orientations. It seems that the grammatical constructions typical for positional logic, especially Minimal Realisation, allow combining the quantitative formalisation with the humanistic coefficient. The humanistic coefficient concept was developed one hundred years ago by Florian Znaniecki [10], who postulated the need not to limit researchers' observation only to their own direct experience of the data, but to reconstruct the experience of the people who are the subject of the research. Thus, it is a kind of qualitative perspective.

This paper is inspired by [7]. In our article, we develop the programme described there. We extend the **MR** logic with new means of expressions. While in [4] the **MR** logic was extended with multiple positions in the range of operator $\mathcal{R}$ and expressions with predicates, here we take another step forward. We add the expressions with nested predicate expressions in the range of operator $\mathcal{R}$ to the language. It allows us to talk about relations and properties 'from some point of view' which is typical for a qualitative description of social phenomena. Although this is not the final level of extension of **MR**, the logic we propose already permits the description of quite complex social situations. Some examples are provided in section 6.

2. Language and semantics

The purpose of the logical part of this paper is to develop a formal framework for social sciences. Our approach to achieving this goal is to follow the programme article [7]. The cited paper established a strategy of extending the system of Minimal Realisation, in a way suitable for our purpose. We attempt to partially execute this task in the following sections. We start by providing the syntactic and semantic base.

We will use the minimal system for $\mathcal{R}$–operator as a basis for further extensions. This logic, abbreviated as **MR**, was presented for the first time in a paper by [5] as the general system of positional logic. Its minimalism is a result of both, semantic and syntactic weaknesses. Indeed, in the context of the systems preceding it—systems constructed by Łoś, Prior and Rescher—**MR** is characterized by the minimal number of assumptions and the poorest language to express them.

The mentioned weaknesses of the system lead to some unfavorable consequences. Among other things, the poor language reduces the expressive power of the theory built upon it. Such theory may not be sufficient to express facts regarding complex phenomena. On the other hand, the minimalism of the system makes it easy to extend.

In our investigations, we follow the design of extension partially outlined in ([7], pp. 13–16). It requires addition of predicate symbols to the alphabet. By doing so, the language of our logic will consists of: logical connectives $\mathsf{Con} = \{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$, variables $\mathsf{Var} = \{p_i : i \in \mathbb{N}\}$, positional letters $\mathsf{PL} = \{a_i : i \in \mathbb{N}\}$, predicates $\mathsf{PS} = \{P^i_n : i, n \in \mathbb{N}\}$, realisation operator $\mathcal{R}$ and brackets:), (, where $\mathbb{N}$ denotes the set of natural numbers. For the definitions and theorems ahead, let us denote the set of predictate expressions: $\mathsf{PE} = \{P^{\mathsf{i}}_n(\alpha_1, ..., \alpha_i) : P^{\mathsf{i}}_n \in \mathsf{PS},\ \alpha_1, ..., \alpha_i \in \mathsf{PL}$, for some $i, n \in \mathbb{N}\}$. Additional changes are carried out on the level of the grammatical rules. We prefer to extend the class of expressions in a way that allows speaking about the context in the manner of a more complex structure. Therefore, expressions consisting of the $\mathcal{R}$–operator will not contain one positional letter, but a sequence of positional letters of any length. From the semantic point of view, it will allow accounting for more than one context factor, considering the truth value of a given expression (notice that both changes were examined in [4]).

However, the crucial new modification is also on the level of grammatical rules. What is new is that we add to the language the expressions with nested predicate expressions in the range of operator $\mathcal{R}$. The new expressions allow us to talk about relations and properties 'from some point of view' which is characteristic of social phenomena.

Let us start with the introduction of basic syntactic notions.

Definition 2.1 (Auxiliary Expressions). The set of auxiliary expressions AE is the smallest set satisfying the conditions stated as follows:

1. $\mathsf{Var} \subseteq \mathsf{AE}$,
2. $\mathsf{PE} \subseteq \mathsf{AE}$,
3. $\neg A \in \mathsf{AE}$, where $A \in \mathsf{AE}$,
4. $A * B \in \mathsf{AE}$, where $A, B \in \mathsf{AE}$, and $* \in \mathsf{Con} \setminus \{\neg\}$.

Those expressions are in relation to the expressions constructed using the $\mathcal{R}$–operator. That is, the elements of AE are the only expressions that can be in a range of the $\mathcal{R}$–operator. This fact is outlined in the next definition.

Definition 2.2 (Formulas). The set of formulas For is the smallest set satisfying conditions stated as follows:

1. $\mathcal{R}_{\alpha_1,\ldots,\alpha_i}(A) \in \mathsf{For}$, where $A \in \mathsf{AE}$ and $\alpha_1, \ldots, \alpha_i \in \mathsf{PL}$ for some $i \in \mathbb{N}$,
2. $\mathsf{PE} \subseteq \mathsf{For}$,
3. $\neg\phi \in \mathsf{For}$, where $\phi \in \mathsf{For}$,
4. $\phi * \psi \in \mathsf{For}$, where $\phi, \psi \in \mathsf{For}$ and $* \in \mathsf{Con} \setminus \{\neg\}$.

From the set of all formulas, the subset of all formulas that do not contain standard logical connectives outside the range of the $\mathcal{R}$–operator or belong to PE, can be distinguished. We will denote it by $\mathsf{For}_{\mathsf{AT}}$.

To simplify the notation, let us abbreviate $\Gamma, \Gamma_1, \Gamma_2, \ldots$ for any sequences of positional letters $\alpha_1, \ldots \alpha_n$, for some $n \in \mathbb{N}$. The set of all finite sequences of positional letters will be denoted by SE. We can formally construct this set as follows:

$$\mathsf{SE} = \{\alpha_1, \ldots, \alpha_i : \exists_{i \in \mathbb{N}} \forall_{n \in \{1,\ldots,i\}} \alpha_n \in \mathsf{PL}\}.$$

To express the information of the length of a sequence, we will use an upper index. Therefore a sequence of positional letters of a length $i \in \mathbb{N}$

will be denoted by Γ^{i}. Similarly, we will denote the set of all sequences of a given length $i \in \mathbb{N}$ by adding an upper index to the name of this set. For example, the symbol for the set of all sequences of positional letters of the length $i \in \mathbb{N}$, would be SE^{i}. Futher conventions are that any non-empty set of objects will be symbolized by W. Further, $w, w_1, w_2, ...$ will denote its elements and by $\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2, ...$ we will denote sequences of elements from W of any length. By $\mathbf{W}$ will denote the class of those sequences. Of course, the previous conventions are applicable.

Based on the notions defined above, we present semantics. First, we define the notion of a model for our language. Its definition will be an extension of a corresponding definition provided for Minimal Realisation given in ([5], p. 9).

Definition 2.3 (Model). A model $\mathfrak{M}$ for the set For is any quintuple $\langle W, \mathsf{d}, \delta, \{\delta_{\mathbf{w}}\}_{\mathbf{w}\in\mathbf{W}}, \mathsf{v}\rangle$, where:

- W is a non–empty set of objects,
- $\mathsf{d}\colon \mathsf{SE} \longrightarrow \mathbf{W}$ is such a function that $\forall_{i\in\mathbb{N}}\, \mathsf{d}(\Gamma^{\mathsf{i}}) \in \mathbf{W}^{\mathsf{i}}$,
- $\delta : \mathsf{PS} \longrightarrow \mathcal{P}(\mathbf{W})$ is such a function that $\forall_{i,n\in\mathbb{N}}\, \delta(P^{\mathsf{i}}_n) \subseteq \mathbf{W}^{\mathsf{i}}$,
- $\{\delta_{\mathbf{w}}\}_{\mathbf{w}\in\mathbf{W}}$ is a family of functions $\delta_{\mathbf{w}}$ that fulfil the condition given for δ,
- $\mathsf{v}\colon \mathbf{W} \times \mathsf{AE} \longrightarrow \{0,1\}$ is a function that for any $\mathbf{w} \in \mathbf{W}$, any $i, n \in \mathbb{N}$, $P^{\mathsf{i}}_n \in \mathsf{PS}$, and $A, B \in \mathsf{AE}$ satisfies the following conditions:
  1. $\mathsf{v}(\mathbf{w}, P^{\mathsf{i}}_n(\Gamma^{\mathsf{i}})) = 1$ iff $\mathsf{d}(\Gamma^{\mathsf{i}}) \in \delta_{\mathbf{w}}(P^{\mathsf{i}}_n)$,
  2. $\mathsf{v}(\mathbf{w}, \neg A) = 1$ iff $\mathsf{v}(\mathbf{w}, A) = 0$,
  3. $\mathsf{v}(\mathbf{w}, A \wedge B) = 1$ iff $\mathsf{v}(\mathbf{w}, A) = 1$ and $\mathsf{v}(\mathbf{w}, B) = 1$,
  4. $\mathsf{v}(\mathbf{w}, A \vee B) = 1$ iff $\mathsf{v}(\mathbf{w}, A) = 1$ or $\mathsf{v}(\mathbf{w}, B) = 1$,
  5. $\mathsf{v}(\mathbf{w}, A \to B) = 1$ iff $\mathsf{v}(\mathbf{w}, A) = 0$ or $\mathsf{v}(\mathbf{w}, B) = 1$,
  6. $\mathsf{v}(\mathbf{w}, A \leftrightarrow B) = 1$ iff $\mathsf{v}(\mathbf{w}, A) = \mathsf{v}(\mathbf{w}, B)$.

It is worth pointing out three facts. First, in our definition, the arguments of the function d, are sequences of positional letters, not the letters themselves. Therefore, the consistency requires that the function returns a value from the set $\mathbf{W}$ of the length corresponding to the length of the interpreted positional sequence. Second, the same restriction must be imposed on the function δ which ranges across the set of all predicates. Finally, the interpretation of predicates must enable us to treat predicate expressions differently, depending on whether one is in the range of the $\mathcal{R}$–operator or not. In the case of the latter, such expressions could be interpreted by a standard δ function. However, in the case of the former, the interpretation of the expression should be related to the interpretation of the positional sequence bounded by the $\mathcal{R}$–operator. This condition is satisfied by creating a family of $\delta_{\mathbf{w}}$ functions which are defined in the same manner as δ but depending on the $\mathbf{w} \in \mathbf{W}$.

The class of all models satisfying the conditions stated above, will be denoted by $\mathbf{M}$. Considering any model of this class, we would like to evaluate the truth value for any formula in this model. The relation constructed in the next definition, enables us to do so.

Definition 2.4 (Truth in a Model). Let $\mathfrak{M} = \langle W, \mathsf{d}, \delta, \{\delta_{\mathbf{w}}\}_{\mathbf{w}\in\mathbf{W}}, \mathsf{v}\rangle$ and $\mathfrak{M} \in \mathbf{M}$, $\phi \in \mathsf{For}$. A formula ϕ is *true in* $\mathfrak{M}$ (in short: $\mathfrak{M} \models \phi$) iff it satisfies the following conditions:

1. if $\phi = \mathcal{R}_\Gamma(A)$ for some $\Gamma \in \mathsf{SE}$ and $A \in \mathsf{AE}$, then $\mathsf{v}(\mathsf{d}(\Gamma), A) = 1$,
2. if $\phi = P^{\mathsf{i}}_n(\Gamma^{\mathsf{i}})$ for some $i, n \in \mathbb{N}$, $\Gamma^{\mathsf{i}}_n \in \mathsf{SE}$ and $P^{\mathsf{i}}_n \in \mathsf{PS}$ then $\mathsf{d}(\Gamma^{\mathsf{i}}) \in \delta(P^{\mathsf{i}}_n)$,
3. if $\phi = \neg\psi$ for some ψ, then it is not that $\mathfrak{M} \models \phi$ (in short: $\mathfrak{M} \not\models \psi$),
4. if $\phi = \psi \wedge \chi$ for some ψ, χ, then $\mathfrak{M} \models \psi$ and $\mathfrak{M} \models \chi$,
5. if $\phi = \psi \vee \chi$ for some ψ, χ, then $\mathfrak{M} \models \psi$ or $\mathfrak{M} \models \chi$,
6. if $\phi = \psi \to \chi$ for some ψ, χ, then $\mathfrak{M} \not\models \psi$ or $\mathfrak{M} \models \chi$,
7. if $\phi = \psi \leftrightarrow \chi$ for some ψ, χ, then $\mathfrak{M} \models \psi$ and $\mathfrak{M} \models \chi$ or $\mathfrak{M} \not\models \psi$ and $\mathfrak{M} \not\models \chi$.

Definition 2.5 (Semantic Consequence Relation). Let $\Lambda \cup \{\phi\} \subseteq \mathsf{For}$. The formula ϕ *follows from the set* Λ *with respect to the set of models* $\mathbf{M}$ (in short: $\Lambda \vDash_{\mathbf{M}} \phi$) iff for any $\mathfrak{M} \in \mathbf{M}$, if for all $\psi \in \Lambda$, $\mathfrak{M} \vDash \psi$ (in short: $\mathfrak{M} \vDash \Lambda$), then $\mathfrak{M} \vDash \phi$. When $\emptyset \vDash_{\mathbf{M}} \phi$, the formula ϕ is called a *tautology* of **M**.

However, as the set of models **M** will be the only one considered here, we will omit its symbol in such contexts. The logic that is determined by the class of models **M**, will be denoted by $\mathbf{MR_{np}}$ as an abbreviation for Minimal Realisation with Nested Predicates.

3. Axiomatic system

In this section, the relation of the syntactic consequence for $\mathbf{MR_{np}}$ is defined. This will be achieved by providing a set of axioms and syntactic rules for the logic. Since **MR** was already presented as an axiomatic system, we take advantage of this fact as it is possible to reuse some of the results concerning the original version of our system.

For this purpose, four axiom schemes are used. To introduce the first one, let us denote the set of all formulas of Classical Propositional Logic (CPL) by $\mathsf{For}_{\mathsf{CPL}}$ and the set of all its tautologies by $\mathbf{Taut}_{\mathsf{CPL}}$. Additionally, we will define the notion of substitution function.

Definition 3.1 (Substitution Function). Substitution function for CPL formulas is any function $\mathfrak{s}\colon \mathsf{For}_{\mathsf{CPL}} \longrightarrow \mathsf{For}$ that for any $\phi, \psi \in \mathsf{For}_{\mathsf{CPL}}$ and $* \in \mathsf{Con} \setminus \{\neg\}$ satisfies following conditions:

1. $\mathfrak{s}(\neg\phi) = \neg\mathfrak{s}(\phi)$,
2. $\mathfrak{s}(\phi * \psi) = \mathfrak{s}(\phi) * \mathfrak{s}(\psi)$.

The first axiom scheme is restricted to CPL tautologies in our language – namely, each substitution of a CPL tautology is an axiom of our logic. The formulation of this scheme in the formal languages looks identical to its formulation in the original version.

Axiom 3.1. $\mathfrak{s}(\phi)$, if $\phi \in \mathbf{Taut}_{\mathsf{CPL}}$ and $\mathfrak{s}$ is a substitution function.

The next two axiom schemes differ in formulation from the corresponding axiom schemes formulations in the original system. Specifically, the $\mathcal{R}$–operator does not bind a positional constant. In our version, it binds a finite sequence of any given length of positional letters. For any expressions $A, B \in \mathsf{AE}$ and any sequence of positional letters $\Gamma \in \mathsf{SE}$ those schemes appear as follows:

AXIOM 3.2. $\neg\mathcal{R}_\Gamma A \leftrightarrow \mathcal{R}_\Gamma\neg A$.

AXIOM 3.3. $(\mathcal{R}_\Gamma A \wedge \mathcal{R}_\Gamma B) \to \mathcal{R}_\Gamma(A \wedge B)$.

The last axiom scheme presents the idea that a CPL tautology is true in any given context.

AXIOM 3.4. $\mathcal{R}_\Gamma A$, if $A \in \mathbf{Taut}_{\mathsf{CPL}}$.

Besides the aforementioned schemes, we assume the Modus Ponens rule (in short: MP).

$$\frac{\phi, \phi \to \psi}{\psi}$$

The set of axioms we will denote by $\mathbf{MR^{ax}}$. Having $\mathbf{MR^{ax}}$, we accept the standard notion of syntactic consequence relation.

DEFINITION 3.2 (Syntactic Consequence Relation). Let $\Lambda \cup \{\phi\} \subseteq \mathsf{For}$. The formula ϕ *is provable based on the set* Λ *with respect to* $\mathbf{MR^{ax}}$ (in short: $\Lambda \vdash_{\mathbf{MR^{ax}}} \phi$) iff there is such a sequence of formulas: $\psi_1, \ldots, \psi_n$ that $\psi_n = \phi$ and for all $1 \leq i \leq n$ if at least one of the below conditions is fulfilled:

1. $\psi_i \in \Lambda$
2. $\psi_i \in \mathbf{MR^{ax}}$
3. for some $j, k < i$ there exist such ψ_j, ψ_k that $\psi_k = \psi_j \to \psi_i$.

When $\emptyset \vdash_{\mathbf{MR^{ax}}} \phi$, the formula ϕ is called a *thesis.*

Since we consider only one axiomatic system, we will write $\vdash$ rather than $\vdash_{\mathbf{MR^{ax}}}$ to simplify the notation. Using those concepts, we will introduce the notion of a maximal consistent set.

DEFINITION 3.3 ($\mathbf{MR^{ax}}$–consistent | $\mathbf{MR^{ax}}$–inconsistent Set of Formulas). Let $\Delta \subseteq \mathsf{For}$. Then:

- Δ is called an $\mathbf{MR^{ax}}$–consistent set of formulas iff $\Delta \nvdash \phi$, for some $\phi \in \mathsf{For}$,

- Δ is called an $\mathbf{MR^{ax}}$–inconsistent set of formulas iff it is not $\mathbf{MR^{ax}}$–consistent.

For any $\mathbf{MR^{ax}}$–consistent set the standard facts about consistent sets hold. This is a consequence of the fact that our logic is founded on CPL as it contains axiom 3.1 and MP. Basing on the previous definition, we can construct the notion of a maximal $\mathbf{MR^{ax}}$–consistent set of formulas.

Definition 3.4 (Maximal $\mathbf{MR^{ax}}$–consistent Set). Let $\Delta \subseteq \mathsf{For}$. We call Δ a maximal $\mathbf{MR^{ax}}$–consistent iff both:

1. Δ is $\mathbf{MR^{ax}}$–consistent,
2. for any $\Lambda \subseteq \mathsf{For}$ if $\Delta \subset \Lambda$, then Λ is $\mathbf{MR^{ax}}$–inconsistent.

Using the symbol $\mathbf{Max_{MR^{ax}}}$, we will denote the class of all maximal $\mathbf{MR^{ax}}$–consistent sets. Some intuitions about the properties of those sets are expressed by the next three facts.

The first fact says that such sets are closed under the syntactic consequence relation. Therefore any formula $\phi \in \mathsf{For}$, for which there exists proof based on the maximal $\mathbf{MR^{ax}}$–consistent set, has to be a element of such set. And conversely, if a formula is an element of a maximal $\mathbf{MR^{ax}}$–consistent set, there is a proof of the formula on the ground of this set.

Fact 3.5. Let $\Delta \in \mathbf{Max_{MR^{ax}}}$ and $\phi \in \mathsf{For}$. Then $\Delta \vdash \phi$ iff $\phi \in \Delta$.

The next fact expresses the relation between the set $\mathbf{MR^{ax}}$ and a maximal $\mathbf{MR^{ax}}$–consistent set. More specifically it states that all formulas from $\mathbf{MR^{ax}}$ are contained in such a set.

Fact 3.6. Let $\Delta \in \mathbf{Max_{MR^{ax}}}$. Then $\mathbf{MR^{ax}} \subseteq \Delta$.

The last of the aforementioned facts states that any maximal $\mathbf{MR^{ax}}$ –consistent set is closed under the listed conditions.

Fact 3.7. Let $\Delta \in \mathbf{Max_{MR^{ax}}}$. Then for any $\phi, \psi \in \mathsf{For}$ it is true that:

- $\neg\phi \in \Delta$ iff $\phi \notin \Delta$,
- $\phi \wedge \psi \in \Delta$ iff $\phi \in \Delta$ and $\psi \in \Delta$,
- $\phi \vee \psi \in \Delta$ iff $\phi \in \Delta$ or $\psi \in \Delta$,

- $\phi \to \psi \in \Delta$ iff $\phi \notin \Delta$ or $\psi \in \Delta$,
- $\phi \leftrightarrow \psi \in \Delta$ iff $\phi \in \Delta$ and $\psi \in \Delta$ or $\phi \notin \Delta$ and $\psi \notin \Delta$.

The most important theorem concerning the maximal $\mathbf{MR^{ax}}$–consistent sets is the so–called Lindenbaum's Lemma. It states that any $\mathbf{MR^{ax}}$–consistent set is a subset of some maximal $\mathbf{MR^{ax}}$–consistent set.

Lemma 3.8. *Let $\Lambda \subseteq \mathsf{For}$. Then if Λ is $\mathbf{MR^{ax}}$–consistent, there is such $\Delta \subseteq \mathsf{For}$ that $\Lambda \subseteq \Delta$ and $\Delta \in \mathbf{Max}_{\mathbf{MR^{ax}}}$.*

4. Soudness and completeness

In the previous sections, we established the relations of semantic and syntactic consequences. With that in mind, in this section, we investigate a relationship between those two relations and provide a list of theorems and facts regarding this relationship. Two of the main results that we want to present in this section are soundness and completeness of our logic.

To obtain the former result, we will need to first prove the following lemma.

Lemma 4.1. *For any formula $\phi \in \mathbf{MR^{ax}}$, it is also a tautology.*

Proof: Of course, the substitution of any tautology of CPL is a tautology of our logic by the notion of the substitution function defined in 3.1 and the truth conditions 2.4. Therefore, any formula that is an instance of an axiom 3.1 is a tautology of our system.

Now let us assume that for any $\mathfrak{M} \in \mathbf{M}$, $\mathfrak{M} \models \neg\mathcal{R}_\Gamma(A)$, for some $\Gamma \in \mathsf{SE}$ and $A \in \mathsf{AE}$. Then according to definition 2.4, it is the case iff $\mathfrak{M} \not\models \mathcal{R}_\Gamma(A)$ and thus $\mathsf{v}(\mathsf{d}(\Gamma), A) = 0$. By definition 2.3, it is equivalent to the $\mathsf{v}(\mathsf{d}(\Gamma), \neg A) = 1$ and thus $\mathfrak{M} \models \mathcal{R}_\Gamma(\neg A)$.

To prove that the axiom scheme 3.3 is tautological, let us assume that for a $\mathfrak{M} \in \mathbf{M}$, $\mathfrak{M} \models (\mathcal{R}_\Gamma A \wedge \mathcal{R}_\Gamma B)$. Then, based on the definition 2.4, it is the case iff $\mathfrak{M} \models \mathcal{R}_\Gamma A$ and $\mathfrak{M} \models \mathcal{R}_\Gamma B$. According to the same definition, by equivalence we obtain $\mathsf{v}(\mathsf{d}(\Gamma), A) = 1$ and $\mathsf{v}(\mathsf{d}(\Gamma), B) = 1$ and using definition 2.3, it is the case iff $\mathsf{v}(\mathsf{d}(\Gamma), A \wedge B) = 1$. And thus, equivalently, $\mathfrak{M} \models \mathcal{R}_\Gamma(A \wedge B)$.

Further, let us assume that $A \in \mathbf{Taut}_{\mathsf{CPL}}$ and $\mathfrak{M} \not\models \mathcal{R}_\Gamma A$, for some $\mathfrak{M} \in \mathbf{M}$. According to the definition 2.4, $\mathfrak{M} \not\models \mathcal{R}_\Gamma A$ iff $\mathsf{v}(\mathsf{d}(\Gamma), A) = 0$ for some valuation function v. Then by definition 2.3, valuation v falsifies formula A. However, as A is a tautology, it leads to an immediate contradiction. $\square$

THEOREM 4.2 (Soundness). *Let* $\Lambda \cup \{\phi\} \subseteq \mathsf{For}$. *If* $\Lambda \vdash \phi$, *then* $\Lambda \vDash \phi$.

PROOF: Let us assume that $\Lambda \vdash \phi$, $\mathfrak{M} \in \mathbf{M}$, and that all elements of Λ are true in the model $\mathfrak{M} \in \mathbf{M}$. Thus, according to definition 3.2 there exists such a sequence of formulas $\psi_1, ..., \psi_n$ that $\psi_n = \phi$, for some $n \in \mathbb{N}$. We prove that $\mathfrak{M} \vDash \psi_i$, for $1 \leq i \leq n$, and thus $\mathfrak{M} \vDash \phi$, since $\psi_n = \phi$.

If we assume that $n = 1$, there are two possible cases — $\psi_1 \in \Lambda$ or $\psi_1 \in \mathbf{MR^{ax}}$. Consider the first one according to the assumption $\mathfrak{M} \vDash \psi_1$. In the second case, due to lemma 4.1, $\mathfrak{M} \vDash \psi_1$. Since the sequence is of the length one, we obtain $\mathfrak{M} \vDash \phi$.

Let us assume that $n > 1$. We make an induction, based on the length of the assumed sequence. The initial step is similar to the case when $n = 1$. So, for the inductive step we assume that for some $1 \leq k < n$, if $j \leq k$, then $\mathfrak{M} \vDash \psi_j$. Now, let us consider the formula ψ_{k+1}. There are the following three possibilities:

1. $\psi_{k+1} \in \mathbf{MR^{ax}}$,
2. $\psi_{k+1} \in \Lambda$,
3. there exists ψ_l, ψ_m, such that $\psi_m = \psi_l \to \psi_{k+1}$, for some $l, m \leq k$.

The first two cases are similar to the case when $n = 1$. Now consider the third one. As $l, m \leq k$, so by the inductive hypothesis, $\mathfrak{M} \vDash \psi_l$ and $\mathfrak{M} \vDash \psi_l \to \psi_{k+1}$. Now, according to definition 2.4, we get that $\mathfrak{M} \nvDash \psi_l$ or $\mathfrak{M} \vDash \psi_{k+1}$. Thus, $\mathfrak{M} \vDash \psi_{k+1}$. $\square$

Having thus proved the soundness of $\mathbf{MR_{np}}$, we will prove the converse implication—that is the completeness theorem. It requires us to define the notion of a canonical model. It is a special structure that is also a model for our logic interpreted within the set of formulas. However, first we define the notion of canonical quasi-model.

DEFINITION 4.3 (Canonical Quasi-Model). Let $\Delta \in \mathbf{Max_{MR^{ax}}}$. *A canonical quasi-model* is a quintuple $\langle W_\Delta, \mathsf{d}_\Delta, \delta_\Delta, \{\delta_{\Delta_\Gamma}\}_{\Gamma \in W_\Delta}, \mathsf{v}_\Delta \rangle$ such that:

- $W_\Delta = \mathsf{SE}$,
- $\mathsf{d}_\Delta \colon \mathsf{SE} \longrightarrow W_\Delta$ such that $\forall_{i \in \mathbb{N}}\, \mathsf{d}_\Delta(\Gamma^\mathsf{i}) = \Gamma^\mathsf{i}$,

- $\delta_\Delta \colon \mathsf{PS} \longrightarrow \mathcal{P}(W_\Delta)$ such that $\forall_{i,n\in\mathbb{N}}\, \delta_\Delta(P^{\mathsf{i}}_n) = \{\Gamma^{\mathsf{i}} \colon P^{\mathsf{i}}_n(\Gamma^{\mathsf{i}}) \in \Delta\}$,
- $\{\delta_{\Delta_\Gamma}\}_{\Gamma\in W_\Delta}$ is the family of functions δ_{Δ_Γ} such that $\forall_{i,n\in\mathbb{N}}\, \delta_{\Delta_\Gamma}(P^{\mathsf{i}}_n) = \{\Gamma^{\mathsf{i}} \colon \mathcal{R}_\Gamma(P^{\mathsf{i}}_n(\Gamma^{\mathsf{i}})) \in \Delta\}$,
- $v_\Delta \colon W_\Delta \times (\mathsf{Var} \cup \mathsf{PS}) \longrightarrow \{0, 1\}$ such that:
  1. for any $A \in \mathsf{Var}$, $v_\Delta(\Gamma, A) = 1$ iff $\mathcal{R}_\Gamma(A) \in \Delta$,
  2. for any $P^{\mathsf{i}}_n \in \mathsf{PS}$, $i, n \in \mathbb{N}$ and $\Gamma_1 \in \mathsf{SE}$,
     $v_\Delta(\Gamma, P^{\mathsf{i}}_n(\Gamma_1)) = 1$ iff $\mathsf{d}_\Delta(\Gamma_1) \in \delta_{\Delta_\Gamma}(P^{\mathsf{i}}_n)$.

The above definition presents a structure that does not fully correspond to the definition of a model for $\mathbf{MR_{np}}$. The conditions for the function of valuation in definition 2.3 contain cases of complex expressions formed with logical connectives. In the latter definition, only the primitive cases are considered explicitly. The following lemma will prove that a structure satisfying conditions given in definition 4.3 satisfies the conditions from definition 2.3—for complex expressions within the $\mathcal{R}$–operator.

Fact 4.4. Let $\Delta \in \mathbf{Max}_{\mathbf{MR}^{\mathbf{ax}}}$ and $\mathfrak{M} = \langle W_\Delta, \mathsf{d}_\Delta, \delta_\Delta, \{\delta_{\Delta_\Gamma}\}_{\Gamma\in W_\Delta}, \mathsf{v}_\Delta\rangle$ be a canonical quasi-model. Then it can be extended to a canonical model (in short: Δ–model).

Proof: Assume all the hypotheses. The fact that $\mathfrak{M}$ can be extended to a Δ–model is equivalent to the fact that the function v_Δ can be extended to range over $W_\Delta \times \mathsf{AE}$. Thus, that it satisfies the following conditions:

1. for any $A \in \mathsf{Var}$, $v_\Delta(\Gamma, A) = 1$ iff $\mathcal{R}_\Gamma(A) \in \Delta$,
2. for any $P^{\mathsf{i}}_n \in \mathsf{PS}$, $i, n \in \mathbb{N}$ and $\Gamma_1 \in \mathsf{SE}$,
   $v_\Delta(\Gamma, P^{\mathsf{i}}_n(\Gamma_1)) = 1$ iff $\mathsf{d}_\Delta(\Gamma_1) \in \delta_{\Delta_\Gamma}(P^{\mathsf{i}}_n)$,
3. $\mathsf{v}_\Delta(\Gamma, \neg A) = 1$ iff $\mathsf{v}_\Delta(\Gamma, A) = 0$,
4. $\mathsf{v}_\Delta(\Gamma, A \wedge B) = 1$ iff $\mathsf{v}_\Delta(\Gamma, A) = 1$ and $\mathsf{v}_\Delta(\Gamma, B) = 1$,
5. $\mathsf{v}_\Delta(\Gamma, A \vee B) = 1$ iff $\mathsf{v}_\Delta(\Gamma, A) = 1$ or $\mathsf{v}_\Delta(\Gamma, B) = 1$,
6. $\mathsf{v}_\Delta(\Gamma, A \to B) = 1$ iff $\mathsf{v}_\Delta(\Gamma, A) = 0$ or $\mathsf{v}_\Delta(\Gamma, B) = 1$,
7. $\mathsf{v}_\Delta(\Gamma, A \leftrightarrow B) = 1$ iff $\mathsf{v}_\Delta(\Gamma, A) = \mathsf{v}_\Delta(\Gamma, B)$.

Assume that $C \in \mathsf{AE}$. Let us provide a proof for this fact by induction over complexity of the considered expression. Thus if $C \in \mathsf{Var} \cup \mathsf{PS}$ the theorem is fulfilled due to the fact that $\mathfrak{M}$ is a canonical quasi-model.

Suppose that the theorem is true for the expressions of the complexity equal to n. Let us consider an expression C of the complexity equal to $n+1$. Then it is the case that for some $A, B \in \mathsf{AE}$, C is one of the listed forms $\neg A, A \wedge B, A \vee B, A \rightarrow B, A \leftrightarrow B$. Therefore, basing on the definition 4.3, distributivity laws for $\mathcal{R}$ and the classical connectives (see [5], pp. 151–153) and facts 3.7 and 3.5, we obtain the thesis. $\square$

A structure named as the Δ–model should possess a certain property. Namely, any formula that is true in that model should be also an element of the maximal $\mathbf{MR^{ax}}$–consistent set on which the canonical model is based. The fact of possessing this mentioned property is expressed by the next lemma.

Lemma 4.5. *Let $\Delta \in \mathbf{Max}_{\mathbf{MR^{ax}}}$, $\mathfrak{M}$ be the Δ–model. Then for any $\phi \in \mathsf{For}$ it is the case that $\mathfrak{M} \models \phi$ iff $\phi \in \Delta$.*

Proof: We will present the proof by induction over the complexity of the formulas. Consider the initial case where $\phi \in \mathsf{For_{AT}}$. Then the formula ϕ is an expression created with the $\mathcal{R}$–operator, a positional sequence and an expression of the set AE or ϕ is a predicate expression.

Let us assume the former case, that is $\phi = \mathcal{R}_\Gamma(A)$ for some $\Gamma \in \mathsf{SE}$ and $A \in \mathsf{AE}$. We will use here fact 4.4. We have $\mathfrak{M} \models \mathcal{R}_\Gamma(A)$ iff $\mathcal{R}_\Gamma(A) \in \Delta$, if $A \in \mathsf{Var}$. If $A \in \mathsf{PE}$, then let us notice that the condition $\mathsf{d}_\Delta(\Gamma_1) \in \delta_{\Delta_\Gamma}(P^{\mathsf{i}}_n)$, for some $P^{\mathsf{i}}_n \in \mathsf{PS}$, $i, n \in \mathbb{N}$ and $\Gamma_1 \in \mathsf{SE}$, is equivalent to the statement $\mathcal{R}_\Gamma(P^{\mathsf{i}}_n(\Gamma_1)) \in \Delta$. Cases for non-atomic expressions follows also from the fact 4.4. Consider the latter case that ϕ is a predicate expression. Then the condition $\mathsf{d}_\Delta(\Gamma_1) \in \delta_\Delta(P^{\mathsf{i}}_n)$, for all $P^{\mathsf{i}}_n \in \mathsf{PS}$, $i, n \in \mathbb{N}$ and $\Gamma_1 \in \mathsf{SE}$, is equivalent to the statement $P^{\mathsf{i}}_n(\Gamma_1) \in \Delta$.

Assume that the hypothesis is satisfied for all formulas ϕ with complexity equal or lesser than n, for some $n \in \mathbb{N}$. We will prove also that it is satisfied for expressions of the complexity equal to $n+1$. In such case, we should consider the following formulas: $\neg\psi$, $\psi \wedge \chi$, $\psi \vee \chi$, $\psi \rightarrow \chi$ or $\psi \leftrightarrow \chi$ for some ψ and χ with the complexities equal or lesser than n. We analyse the cases for $\neg$ and $\wedge$. For the rest of them, the proof could be carried out analogously.

Assume the former case first. Then $\mathfrak{M} \models \neg\psi$ is equivalent to $\mathfrak{M} \not\models \psi$. From the inductive hypothesis, it is equivalent to the fact that $\psi \notin \Delta$. From the assumption that Δ is the maximal $\mathbf{MR^{ax}}$–consistent, we get the next equivalent fact $\neg\psi \in \Delta$.

Assume the second case. Then $\mathfrak{M} \vDash \psi \wedge \chi$ is equivalent to $\mathfrak{M} \vDash \psi$ and $\mathfrak{M} \vDash \chi$. From the fact that complexity of both formulas is lesser or equal to n and the inductive hypothesis, we obtain by equivalence $\psi, \chi \in \Delta$. As Δ is the maximal $\mathbf{MR^{ax}}$–consistent, it is equivalent to the fact that $\psi \wedge \chi \in \Delta$. □

This lemma is crucial for the next theorem. It expresses the fact that for any $\mathbf{MR^{ax}}$–consistent set of formulas, there exists a canonical model in which all the formulas from the set are true.

THEOREM 4.6. *Let $\Lambda \subseteq \mathsf{For}$ be a $\mathbf{MR^{ax}}$–consistent set. Then there exists such $\Delta \in \mathbf{Max}_{\mathbf{MR^{ax}}}$ that $\mathfrak{M}$ is a Δ–model and $\mathfrak{M} \vDash \Lambda$.*

PROOF: Assume the hypothesis. As Λ is a $\mathbf{MR^{ax}}$–consistent set of formulas, from Lindenbaum's Lemma (lemma 3.8), there exists such a set of formulas $\Delta \in \mathbf{Max}_{\mathbf{MR^{ax}}}$ that $\Lambda \subseteq \Delta$. Then according to the definition 4.3 and the fact 4.4 there exists a Δ–model $\mathfrak{M}$. According to the lemma 4.5 – for any formula $\phi \in \Delta$ it is the case that $\mathfrak{M} \vDash \phi$, where $\mathfrak{M}$ is Δ-model. It leads to the conclusion that $\mathfrak{M} \vDash \Lambda$. □

THEOREM 4.7 (Completness Theorem). *Let $\Lambda \cup \{\phi\} \subseteq \mathsf{For}$. If $\Lambda \vDash \phi$, then $\Lambda \vdash \phi$.*

PROOF: Assume all hypotheses. Moreover, suppose that $\Lambda \nvdash \phi$. We know that $\Lambda \cup \{\neg\phi\}$ is $\mathbf{MR^{ax}}$–consistent. According to the lemma 3.8, there exists such a maximal $\mathbf{MR^{ax}}$–consistent set $\Delta \in \mathbf{Max}_{\mathbf{MR^{ax}}}$ that $\Lambda \cup \{\neg\phi\} \subseteq \Delta$. Therefore, according to the theorem 4.6, $\mathfrak{M} \vDash \Lambda \cup \{\neg\phi\}$ where $\mathfrak{M}$ is the Δ–model. As a result, we obtain $\Lambda \nvDash \phi$. □

5. Expressive power of presented logic

As we outlined in the previous sections, our system emerges from the Minimal Realisation logic by expanding its alphabet and grammatical rules. The axioms schemes for $\mathbf{MR_{np}}$ have a similar form to the corresponding axiom schemes of the original system. Therefore, in this paper, we expanded the language of our logic, preserving the theses of the base system.

Knowing this, we show, that there exists a mapping from the set of $\mathbf{MR_{np}}$ formulas (in short: $\mathsf{For}_{\mathbf{MR_{np}}}$) into the set of $\mathbf{MR}$ formulas (in short: $\mathsf{For}_{\mathbf{MR}}$). Using this mapping, we prove that any expression of the former system is a thesis if and only if it can be mapped into a corresponding thesis of the latter. The mentioned proof will be provided in a manner similar as in [4] (see: [4], p. 361).

To construct such a mapping, first, we will focus on the expressions that are in the range of the $\mathcal{R}$–operator, that is expressions from the class AE. As this class was extended by the addition of predicate expressions, in a comparison to the system of Minimal Realisation, the mapping must take into account such cases. To distinguish the mentioned classes of expressions for the two systems, we consider in the paper, we add a system name in the lower index, similar to the classes of formulas.

Definition 5.1. $\mu : \mathsf{AE}_{\mathbf{MR_{np}}} \longrightarrow \mathsf{AE}_{\mathbf{MR}}$:

- $\mu(A) = A$ if $A \in \mathsf{Var}$,
- $\mu(A) = p$ if $A \in \mathsf{PE}$, for some $p \in \mathsf{Var}$,
- $\mu(\neg A) = \neg\mu(A)$,
- $\mu(A * B) = \mu(A) * \mu(B)$ for $* \in \mathsf{Con} \setminus \{\neg\}$.

Of course, we have the continuum of such mappings. Any of them serves as the identity function for all such expressions, that do not contain predicates. Otherwise, it translates them into their version in which every occurrence of a predicate expression is replaced by a given sentential variable. Thus, we will define it as a mapping ranging over the class of formulas.

Definition 5.2. $\sigma\colon \mathsf{For}_{\mathbf{MR_{np}}} \longrightarrow \mathsf{For}_{\mathbf{MR}}$:

- $\sigma(\mathcal{R}_{\alpha_1,\ldots,\alpha_n}(A)) = \mathcal{R}_{\alpha_i}(\mu(A))$, for some $1 \leq i \leq n$, any $n \in \mathbb{N}$, $A \in \mathsf{AE}$, and some μ,
- $\sigma(\phi) = \mathcal{R}_\alpha(\mu(\phi))$, if $\phi \in \mathsf{PE}$, for some $\alpha \in \mathsf{PL}$, and for some μ,
- $\sigma(\neg\phi) = \neg\sigma(\phi)$,
- $\sigma(\phi * \psi) = \sigma(\phi) * \sigma(\psi)$ for $* \in \mathsf{Con} \setminus \{\neg\}$.

As $\mathbf{MR_{np}}$ is extended by allowing the $\mathcal{R}$–operator to bind sequences of positional letters, defined mapping must accordingly translate them into the expressions of **MR**. This is done by mapping the positional sequence

$\alpha_1, \ldots, \alpha_n$ onto one of its elements. Another case that was not present in the original system is the validity of the formula containing a predicate expression outside the $\mathcal{R}$—operator. This case is considered in the second point of the above definition. A predicate expression is translated into an $\mathcal{R}$—operator expression in the context symbolized by some arbitrarily chosen positional letter. The expression within the $\mathcal{R}$—operator is a translation of the predicate by the previously defined mapping μ.

With the notions defined above, we will attempt to set a certain correspondence between both systems. The following theorem will present this relationship.

THEOREM 5.3. *Let $\phi \in \mathsf{For}$. Then $\vDash_{\mathbf{MR_{np}}} \phi$ iff for any σ, $\vDash_{\mathbf{MR}} \sigma(\phi)$.*

PROOF: Assume the hypothesis. Moreover, let us assume that $\vDash_{\mathbf{MR_{np}}} \phi$. Then, according to theorem 4.7 $\vdash_{\mathbf{MR_{np}}} \phi$. Thus, there exists a proof of a formula ϕ within our system. As the class of axioms does not contain any specific axiom for predicate expressions, it suffices that the proof will be repeated for $\sigma(\phi)$ just by mapping all its elements into the class $\mathsf{For}_{\mathbf{MR}}$. This is since any instance of an $\mathbf{MR_{np}}$ axiom scheme χ is an instance of the $\mathbf{MR}$ axiom scheme after translating $\sigma(\chi)$ for any mapping σ. Hence, if $\psi_1, ..., \psi_i$ for some $i \in \mathbb{N}$ is a proof of ϕ in $\mathbf{MR_{np}}$, then $\sigma(\psi_1), ..., \sigma(\psi_i)$ is a proof of $\sigma(\phi)$ in $\mathbf{MR}$. By the correctness theorem for $\mathbf{MR}$ ([5], p. 155), we obtain $\vDash_{\mathbf{MR}} \sigma(\phi)$.

Now let us assume that $\vDash_{\mathbf{MR}} \sigma(\phi)$ for any σ. If it is the case for any function σ, then especially it is the case for all injective functions. Let us consider such a function. After restricting, the function would satisfy conditions for a bijection $\sigma_{\mathbf{bi}}$. Thus, there exists an inverse function $\sigma_{\mathbf{bi}}^{-1}$. According to the completness theorem for $\mathbf{MR}$ ([5], p. 159), we obtain $\vdash_{\mathbf{MR}} \sigma_{\mathbf{bi}}(\phi)$. It is equivalent to the fact that there exists a proof $\psi_1, ..., \psi_n$ for some $n \in \mathbb{N}$ and for $1 \leq i \leq n$, $\psi_i \in \mathsf{For}_{\mathbf{MR}}$, where $\psi_n = \sigma_{\mathbf{bi}}(\phi)$. Due to the fact, that $\sigma_{\mathbf{bi}}^{-1}$ maps axioms of $\mathbf{MR}$ into some form of $\mathbf{MR_{np}}$ axioms and the MP preserves its properties after such mapping, we get that there exists such a proof $\sigma_{\mathbf{bi}}^{-1}(\psi_1), ..., \sigma_{\mathbf{bi}}^{-1}(\psi_n)$ for some $n \in \mathbb{N}$ and for $1 \leq i \leq n$, $\psi_i \in \mathsf{For}_{\mathbf{MR_{np}}}$ where $\psi_n = \sigma_{\mathbf{bi}}^{-1}(\sigma_{\mathbf{bi}}(\phi))$. It is equivalent to the fact that $\vdash_{\mathbf{MR_{np}}} \phi$. According to the theorem 4.7, we obtain $\vDash_{\mathbf{MR_{np}}} \phi$. $\square$

6. Applications in social sciences

This part of the article is devoted to the presentation of an example of the application of $\mathbf{MR_{np}}$ logic in the sociological perspective. As was stated in the introduction (section 1), of all social sciences we identifies sociology as the science with greater methodological challenges.

Sociology functions as a conglomerate of various theoretical approaches, paradigms, and at the same time it operates in a multitude of sub–disciplines that often intersect with other social sciences (e.g. sociology of politics with political science or sociology of knowledge with philosophy). Among the classic distinction between ways of conducting sociological research, there is also a permanent division into macro, meso and micro–sociology. Each of these subdivisions focuses on other objects and dimensions of social life – from long–term global or national processes, through analyzes of the life of an organization, to sociometric analyzes of interpersonal relations created at the crossroad of sociology and psychology. Whether we are talking about macrosociology or the analysis of small social groups, at each level there is an attempt to find repetitive patterns in complex, multi–layered interpersonal relations. At each of these levels, three issues are also present as follows:

(1) The problem of the complexity of the analyzed world (including individual and collective actors, interactions, cultural patterns, power mechanisms, material resources, etc.),

(2) The problem of actors' self–awareness, and

(3) How this awareness influences the course of the analyzed processes over time and space.

The application of extensions of **MR** logic to sociological concepts and theories is a vast task considering how vast and complex is the field of sociological theory. Therefore, we propose that such a task must start with a reference to basic sociological concepts such as behaviour or interaction. The proposal stated below should be perceived as a kind of a 'sample' in the wider project of application of $\mathbf{MR_{np}}$ logic to the sociological perspective. For this, we decided to use the classical, behavioral postulates by George C. Homans. There are two reasons for this choice. First, there have been attempts in sociology to present Homans' concepts using the language of logic [8]. So our proposal fits into historical research. Second, in our opinion since this is specifically the first case of application of $\mathbf{MR_{np}}$ logic to the social sciences – starting with postulates about basic forms of social relations is the right way to do. In his works Homans presented a very reductionist perspective on human relations and famously formulated 5 postulates (laws) of human interactions. In Homans theory, each social process starts with specific human behaviour and each social interaction starts with human contact. The general concept is that there are patterns of human behaviour that influence interactions and therefore have an impact on the shape of the whole society. Homans' theory is an example of sociologists' attempts to formulate theorems that would have a general range, as much as possible. However it is also an example of a theory that lacks a humanistic approach and is blind to the issue of providing an insight into deeper meanings and understandings of social situations [3].

Therefore we have picked two postulates from Homans' 5 laws of interaction (given below as (P1) and (P2)). These postulates are formulated from a rather 'objective' perspective and are already in quite a formal manner. This means that they lack the humanistic coefficient, or a kind of an in–sight into a subjective perception of human behaviour.

(P1) The more often a particular action of a person is rewarded, the more likely the person is to perform that action.

(P2) If in the past the occurrence of a particular stimulus, or set of stimuli, has been the occasion on which a person's action was rewarded, then the more similar the present stimuli are to the past ones, the more likely the person is to perform the action, or some similar action.

To formalize (P1) and (P2) we need a theory built upon $\mathbf{MR_{np}}$. In the language we distinguish three predicate constants that are read as given on the right:

$\mathsf{Fr}(x, y)$ — *x is a smaller frequency than y*

$\mathsf{Prob}(x, y)$ — *x is a less probability than y*

$\mathsf{Sim}(x, y, z)$ — *y is a stimulus less similar to the stimulus x than the stimulus z is.*

We assume that predicates $\mathsf{Fr}(x, y)$ and $\mathsf{Prob}(x, y)$ are among others irreflexive and transitive (so, also asymmetric):

(Irreflexivity Fr) $\neg\mathsf{Fr}(\alpha_1, \alpha_1)$

(Transitivity Fr) $\mathsf{Fr}(\alpha_1, \alpha_2) \wedge \mathsf{Fr}(\alpha_2, \alpha_3) \to \mathsf{Fr}(\alpha_1, \alpha_3)$

(Irreflexivity Prob) $\neg\mathsf{Prob}(\alpha_1, \alpha_1)$

(Transitivity Prob) $\mathsf{Prob}(\alpha_1, \alpha_2) \wedge \mathsf{Prob}(\alpha_2, \alpha_3) \to \mathsf{Prob}(\alpha_1, \alpha_3)$

Instead of single-argument predicates *being a reward* and *perform*, to simplify a description, we introduce positional letters $\mathsf{perform}$ and reward. Further an agent will be denoted by positional letter a. In the end, we should add that with metavariable A, B with the set of values AE we will understand objects of an agents's activity in a specific context. Thus such activities can be quite complex.

(P1 form) $$\mathsf{Fr}(b_1, b_2) \wedge \mathsf{Prob}(c_1, c_2) \wedge \mathcal{R}_{a,b_1,\mathsf{reward}}(A) \wedge \mathcal{R}_{a,b_2,\mathsf{reward}}(B) \to \\ \to \mathcal{R}_{a,c_1,\mathsf{perform}}(A) \wedge \mathcal{R}_{a,c_2,\mathsf{perform}}(B)$$

Homans' (P1) has been written above using the language of our logic, highlighted predicates and positional letters. It must be mentioned here, that our formalisation reveals a hidden comparison that is stated in the original postulate (formulated in the English language). The same happens in the formalisation of (P2). The expressions used there, namely, 'the more often...' and 'the more likely...' express a comparison of the level of the rewarding degree of a subject's action and the probability of taking on this action, with the level of rewarding and the probability of taking up another, different action, from what was presented in our formalisation. We compare actions A and B. The formula $\mathcal{R}_{a,b_1,\mathsf{reward}}(A)$ says, that an agent a is rewarded for activity A with a frequency b_1, and the formula $\mathcal{R}_{a,b_2,\mathsf{reward}}(B)$ states, that an agent a is rewarded for activity B with a frequency b_2. Because the frequency b_1 is smaller than b_2 (the formula $\mathsf{Fr}(b_1, b_2)$), the agent a will take on activity B (formula $\mathcal{R}_{a,c_2,\mathsf{perform}}(B)$) with greater probability (formula $\mathsf{Prob}(c_1, c_2)$) than activity A (formula $\mathcal{R}_{a,c_1,\mathsf{perform}}(A)$).

Let us address the formalisation of Homans' (P2):

(P2 form) $$\mathcal{R}_{a,s_1,\mathsf{perform},\mathsf{reward}}(A) \wedge \mathsf{Sim}(s_1, s_2, s_3) \wedge \mathsf{Prob}(c_1, c_2) \to \\ \to \mathcal{R}_{a,s_2,c_1,\mathsf{perform}}(A) \wedge \mathcal{R}_{a,s_3,c_2,\mathsf{perform}}(A)$$

Formula $\mathcal{R}_{a,s_1,\mathsf{perform},\mathsf{reward}}(A)$ says, that an agent a performed an activity A because of an stimulus s_1, which later has been rewarded. If this formula is true and a stimulus s_2 is less likely than stimulus s_1 or stimulus s_3 (formula $\mathsf{Sim}(s_1, s_2, s_3)$), then it is less likely that an agent a will take on an activity A because of the stimulus s_2 (formula $\mathcal{R}_{a,s_2,c_1,\mathsf{perform}}(A)$) than because of the stimulus s_3 (formula $\mathcal{R}_{a,s_3,c_2,\mathsf{perform}}(A)$), where $\mathsf{Prob}(c_1, c_2)$ says, that c_1 is a smaller probability than c_2.

Homans' postulates are formulated from the point of view of an objective observer, a scientist who studies human interactions and behaviours, for instance a biologist who observes and studies interactions between animals in a laboratory. Therefore to use Homans' postulates to $\mathbf{MR_{np}}$ logic application, we introduce some changes to his original statements. Below we present them with the addition of the aspect of individual beliefs. So, instead of general statements about human behaviour, we provide statements that contain agents' beliefs about some aspects of the nature of social interactions ((P1h) and (P2h)).

(P1h) If an agent **beliefs** that the more often a particular action of a person is rewarded, the more likely the person is to perform that action.

(P2h) If in the past the occurrence of a particular stimulus, or set of stimuli, was the occasion on which a person's action was rewarded and he **beliefs** that the more similar the present stimuli are to the past ones, the more likely the person is to perform the action, or some similar action.

The reformulation of Homans' postulates has been made to add the humanistic coefficient to the statements about human behaviour. Intending to formalize (P1) and (P2), instead of introducing a single-argument predicate *being a belief*, and to simplify a record, we introduce a positional letter belief.

(P1h form)
$$\mathcal{R}_{a,\mathsf{believe}}(\mathsf{Fr}(b_1, b_2)) \wedge \mathsf{Prob}(c_1, c_2) \wedge \mathcal{R}_{a,b_1,\mathsf{reward}}(A) \wedge \\ \wedge \mathcal{R}_{a,b_2,\mathsf{reward}}(B) \rightarrow \mathcal{R}_{a,c_1,\mathsf{perform}}(A) \wedge \mathcal{R}_{a,c_2,\mathsf{perform}}(B)$$

(P2h form)
$$\mathcal{R}_{a,s_1,\mathsf{perform},\mathsf{reward}}(A) \wedge \mathcal{R}_{a,\mathsf{believe}}(\mathsf{Sim}(s_1, s_2, s_3)) \wedge \mathsf{Prob}(c_1, c_2) \rightarrow \\ \rightarrow \mathcal{R}_{a,s_2,c_1,\mathsf{perform}}(A) \wedge \mathcal{R}_{a,s_3,c_2,\mathsf{perform}}(A)$$

In the formulas (P1h form) and (P2h form) the subformulas $\mathcal{R}_{a,\mathsf{believe}}(\mathsf{Fr}(b_1, b_2))$ and $\mathcal{R}_{a,\mathsf{believe}}(\mathsf{Sim}(s_1, s_2, s_3))$ express a subjective point of view of the agent a. Therefore to conclude from two Homans' postulates one needs something more than $\mathsf{Fr}(b_1, b_2)$ or $\mathsf{Sim}(s_1, s_2, s_3)$ as premises.

Our examples and the whole proposal is of course rather 'modest' and simple. However, they show that we need to develop this project with the addition of the nesting of the $\mathcal{R}$ operator and quantifiers.

7. Further developments

One of our hopes for $\mathbf{MR_{np}}$ logic and the Łoś $\mathcal{R}$–operator is that they will contribute to sociology by connecting the qualitative perspective with the quantitative one. Our proposal for applying Homans' postulates is however, just the beginning, as stated before. Sociological theories that seek to describe complex social phenomena need more accurate modelling of contexts, in which many agents participate in a collective action. Computational sociology with the references and usage of agent–based models (ABMs) is trying to achieve this goal as well. ABMs are considered to be especially instrumental:

> "...when the macro patterns of sociological interest are not the simple aggregation of individual attributes but the result of bottom–up processes at a relational level" [1].

However, one of the many conclusions resulting from the studies on ABMs and sociology, is that this type of modelling of social phenomena has many features typical of methodological individualism [2]. Therefore it presents rather a individualistic point of view and still stumbles upon an issue of 'strong commitment to minimal behavioural complexity' [9].

Nevertheless, our goals are not entirely different from those formulated for ABMs, we try to achieve (as for now) less pragmatic, more theoretical results. Our proposal of extension of **MR** logic is a further step for the programme that was laid out in [7]. The need to combine the humanistic coefficient with the formalisation that was expressed there, can be fulfilled with the language of the **MR** system. The application to sociological theorems and postulates has shown that it is possible to grasp not only one's behavior, but also a set of beliefs as separate variables. The idea of changing George Homans' postulates from an 'objective' style to a more 'subjective' one (with visible convictions of the agent), makes us suppose that it will be possible to represent other quantitative, more formalised sociological theses more qualitatively. We need to add quantifiers and nesting of the $\mathcal{R}$-operator expressions to the language of $\mathbf{MR_{np}}$.

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