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POSITIVE IMPLICATIVE SOJU IDEALS IN BCK-ALGEBRAS

Abstract

The notion of positive implicative soju ideal in BCK-algebra is introduced, and several properties are investigated. Relations between soju ideal and positive implicative soju ideal are considered, and characterizations of positive implicative soju ideal are established. Finally, extension property for positive implicative soju ideal is constructed.

Keywords: soju subalgebra, soju ideal, positive implicative soju ideal.

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1. Introduction

Atanassov [4, 5] introduced the notion of "intuitionistic fuzzy set" as a generalization of the notion of fuzzy set. Using the Atanassov's idea, Jun et al. [15] established the intuitionistic fuzzification of the concept of subalgebras and ideals in BCK-algebras, and investigated some of their properties. They introduced the notion of equivalence relations on the family of all intuitionistic fuzzy ideals of a BCK-algebra and investigated some related properties. In 1999, the notion of soft set was introduced by Molodtsov [21] as a new mathematical tool for dealing with uncertainties. Intuitionistic

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fuzzy set and soft set theory were applied to several aspects (see [1], [2], [3], [6], [7], [8], [12], [13], [14], [16], [17], [19], [22], [23], [24]). Jun et al. [18] introduced a new structure, so called a soju structure, by combining intuitionistic fuzzy set and soft set. They applied it to BCK/BCI-algebras, and introduced the notion of soju subalgebra and soju ideal in BCK/BCI-algebras. They considered the relation between soju subalgebra and soju ideal, and provided conditions for a soju structure to be a soju ideal in a BCK-algebra. They discussed characterizations of soju subalgebra and soju ideal, and considered homomorphic image and preimage of soju subalgebra.

In this paper, we introduce the notion of positive implicative soju ideal in BCK-algebra, and investigate related properties. We discuss relations between soju ideal and positive implicative soju ideal, and establish characterizations of positive implicative soju ideal. We construct extension property for positive implicative soju ideal.

2. Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki (see [10] and [11]).

By a *BCI-algebra*, we mean a set X with a special element 0 and a binary operation $*$ that satisfies the following conditions:

(I) $(\forall x, y, z \in X)$ $(((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)$ $((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)$ $(x * x = 0)$,

(IV) $(\forall x, y \in X)$ $(x * y = 0,\ y * x = 0 \ \Rightarrow\ x = y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x \in X)$ $(0 * x = 0)$,

then X is called a *BCK-algebra*.

Any BCK/BCI-algebra X satisfies the following conditions:

$$(\forall x \in X)\,(x * 0 = x)\,, \tag{2.1}$$

$$(\forall x, y, z \in X)\,(x \leq y \;\Rightarrow\; x * z \leq y * z,\; z * y \leq z * x)\,, \tag{2.2}$$

$$(\forall x, y, z \in X)\,((x * y) * z = (x * z) * y)\,, \tag{2.3}$$

$$(\forall x, y, z \in X)\,((x * z) * (y * z) \leq x * y) \tag{2.4}$$

where $x \leq y$ if and only if $x * y = 0$.

A nonempty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset I of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies:

$$0 \in I, \tag{2.5}$$

$$(\forall x \in X)\,(\forall y \in I)\,(x * y \in I \;\Rightarrow\; x \in I)\,. \tag{2.6}$$

A subset I of a BCK-algebra X is called a *positive implicative ideal* of X if it satisfies (2.5) and

$$(\forall x, y, z \in X)\,((x * y) * z \in I,\; y * z \in I \;\Rightarrow\; x * z \in I)\,. \tag{2.7}$$

We refer the reader to the books [9, 20] for further information regarding BCK/BCI-algebras.

3. Positive implicative soju ideals

In what follows, let U be an initial universe set unless otherwise specified.

Definition 3.1 ([18]). Let X be a set of parameters. For any subset A of X, let $\sigma := (\mu_\sigma,\, \gamma_\sigma)$ be an intuitionistic fuzzy set in A and $(\tilde{F}, A)$ be a soft set over U. Then a pair $(A,\, \langle \sigma; \tilde{F} \rangle)$ is called a *soju structure* over $([0, 1], U)$.

Given a soju structure $(A,\, \langle \sigma; \tilde{F} \rangle)$ over $([0, 1], U)$, $\alpha \in 2^U$ and $(t, s) \in [0, 1] \times [0, 1]$ with $t + s \leq 1$, consider the following sets:

$$(\mu_\sigma; t)^* := \{x \in X \mid \mu_\sigma(x) \geq t\},$$

$$(\gamma_\sigma; s)_* := \{x \in X \mid \gamma_\sigma(x) \leq s\},$$

$$i(\tilde{F}; \alpha) := \{x \in X \mid \tilde{F}(x) \supseteq \alpha\}.$$

Definition 3.2 ([18]). Let A be a subset of a BCK/BCI-algebra X. A soju structure $(A,\, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ is called a *soju subalgebra* based on A (briefly, *soju A-subalgebra*) of X if the following condition is valid.

$$(\forall x, y \in A)\left(x * y \in A \;\Rightarrow \left\{ \begin{array}{l} \mu_\sigma(x*y) \geq \min\{\mu_\sigma(x), \mu_\sigma(y)\} \\ \gamma_\sigma(x*y) \leq \max\{\gamma_\sigma(x), \gamma_\sigma(y)\} \\ \tilde{F}(x*y) \supseteq \tilde{F}(x) \cap \tilde{F}(y) \end{array} \right. \right). \tag{3.1}$$

Definition 3.3 ([18]). Let A be a subalgebra of a BCK/BCI-algebra X. A soju structure $(A,\, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ is called a *soju ideal* based on A (briefly, *soju A-ideal*) of X if the following conditions are valid.

$$(\forall x \in A)(\mu_\sigma(0) \geq \mu_\sigma(x),\; \gamma_\sigma(0) \leq \gamma_\sigma(x),\; \tilde{F}(0) \supseteq \tilde{F}(x)), \tag{3.2}$$

$$(\forall x, y \in A)\left(\begin{array}{l} \mu_\sigma(x) \geq \min\{\mu_\sigma(x*y), \mu_\sigma(y)\} \\ \gamma_\sigma(x) \leq \max\{\gamma_\sigma(x*y), \gamma_\sigma(y)\} \\ \tilde{F}(x) \supseteq \tilde{F}(x*y) \cap \tilde{F}(y) \end{array} \right). \tag{3.3}$$

Definition 3.4. Let A be a subalgebra of a BCK-algebra X. A soju structure $(A,\, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ is called a *positive implicative soju ideal* based on A (briefly, *positive implicative soju A-ideal*) of X if it satisfies the condition (3.2) and

$$(\forall x, y, z \in A)\left(\begin{array}{l} \mu_\sigma(x*z) \geq \min\{\mu_\sigma((x*y)*z), \mu_\sigma(y*z)\} \\ \gamma_\sigma(x*z) \leq \max\{\gamma_\sigma((x*y)*z), \gamma_\sigma(y*z)\} \\ \tilde{F}(x*z) \supseteq \tilde{F}((x*y)*z) \cap \tilde{F}(y*z) \end{array} \right). \tag{3.4}$$

Example 3.5. Let $X = \{0, 1, 2, 3, 4\}$ be a set with the binary operation $*$ which is given in Table 1. Then X is a BCK-algebra (see [20]). For $A = X$, consider a soju structure $(A,\, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ which is defined by Table 2 where $\alpha_1 \supsetneq \alpha_2 \supsetneq \alpha_3 \supsetneq \alpha_4 \neq \emptyset$ in U. It is routine to verify that $(A,\, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X.

We discuss relations between soju ideal and positive implicative soju ideal.

Theorem 3.6. *In a BCK-algebra, every positive implicative soju ideal is a soju ideal.*

Proof: For any subalgebra A of a BCK-algebra X, let $(A,\, \langle\sigma; \tilde{F}\rangle)$ be a positive implicative soju A-ideal of X. If we take $z = 0$ in (3.4) and use (2.1), then we have (3.3). Hence $(A,\, \langle\sigma; \tilde{F}\rangle)$ is a soju A-ideal of X. □

Table 1. Cayley table for the binary operation "$*$"

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	3	2	0

Table 2. Tabular representation of $(A, \langle\sigma; \tilde{F}\rangle)$

A	$\mu_\sigma(x)$	$\gamma_\sigma(x)$	$\tilde{F}(x)$
0	0.7	0.1	α_1
1	0.5	0.2	α_3
2	0.6	0.4	α_2
3	0.3	0.3	α_4
4	0.3	0.4	α_4

The converse of Theorem 3.6 is not true as seen in the following example.

EXAMPLE 3.7. Let $U = \mathbb{Z}$ be the initial universe set and let $X = \{0, 1, 2, 3\}$ be a set with the binary operation $*$ which is given in Table 3.

Table 3. Cayley table for the binary operation "$*$"

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Then $(X, *, 0)$ is a BCK-algebra (see [20]). For $A = X$, let $(A, \langle\sigma; \tilde{F}\rangle)$ be a soju structure over $([0, 1], U)$ which is defined by Table 4. It is routine to check that $(A, \langle\sigma; \tilde{F}\rangle)$ is a soju A-ideal of X. But it is not a positive

Table 4. Tabular representation of $(A,\ \langle\sigma;\tilde{F}\rangle)$

A	$\mu_\sigma(x)$	$\gamma_\sigma(x)$	$\tilde{F}(x)$
0	0.7	0.2	$\mathbb{N}$
1	0.5	0.3	$4\mathbb{N}$
2	0.5	0.3	$4\mathbb{N}$
3	0.4	0.5	$2\mathbb{N}$

implicative soju A-ideal of X since

$$\mu_\sigma(2*1) = \mu_\sigma(1) = 0.5 \not\geq 0.7 = \min\{\mu_\sigma((2*1)*1), \mu_\sigma(1*1)\},$$

$$\gamma_\sigma(2*1) = \gamma_\sigma(1) = 0.3 \not\leq 0.2 = \max\{\gamma_\sigma((2*1)*1), \gamma_\sigma(1*1)\},$$

or $\tilde{F}(2*1) = \tilde{F}(1) = 4\mathbb{N} \not\supseteq \mathbb{N} = \tilde{F}((2*1)*1) \cap \tilde{F}(1*1)$.

Lemma 3.8 ([18]). *Given a subalgebra A of a BCK/BCI-algebra X, let $(A,\ \langle\sigma;\tilde{F}\rangle)$ be a soju A-ideal of X. If the inequality $x \leq y$ holds in A, then $\mu_\sigma(x) \geq \mu_\sigma(y)$, $\gamma_\sigma(x) \leq \gamma_\sigma(y)$ and $\tilde{F}(x) \supseteq \tilde{F}(y)$.*

We provide conditions for a soju ideal to be a positive implicative soju ideal.

Theorem 3.9. *Let $A = X$ be a BCK-algebra. Given a soju structure $(A,\ \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$, the following are equivalent.*

(1) *$(A,\ \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X.*

(2) *$(A,\ \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X satisfying the condition*

$$(\forall x, y \in A) \begin{pmatrix} \mu_\sigma(x*y) \geq \mu_\sigma((x*y)*y) \\ \gamma_\sigma(x*y) \leq \gamma_\sigma((x*y)*y) \\ \tilde{F}(x*y) \supseteq \tilde{F}((x*y)*y) \end{pmatrix}. \tag{3.5}$$

(3) *$(A,\ \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X satisfying the condition*

$$(\forall x, y, z \in A) \begin{pmatrix} \mu_\sigma((x*z)*(y*z)) \geq \mu_\sigma((x*y)*z) \\ \gamma_\sigma((x*z)*(y*z)) \leq \gamma_\sigma((x*y)*z) \\ \tilde{F}((x*z)*(y*z)) \supseteq \tilde{F}((x*y)*z) \end{pmatrix}. \tag{3.6}$$

Proof: Assume that $(A,\ \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Then $(A,\ \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X by Theorem 3.6. Taking $z = y$ in (3.4) implies that

$$\begin{aligned}\mu_\sigma(x*y) &\geq \min\{\mu_\sigma((x*y)*y), \mu_\sigma(y*y)\}\\ &= \min\{\mu_\sigma((x*y)*y), \mu_\sigma(0)\}\\ &= \mu_\sigma((x*y)*y),\end{aligned}$$

$$\begin{aligned}\gamma_\sigma(x*y) &\leq \max\{\gamma_\sigma((x*y)*y), \gamma_\sigma(y*y)\}\\ &= \max\{\gamma_\sigma((x*y)*y), \gamma_\sigma(0)\}\\ &= \gamma_\sigma((x*y)*y),\end{aligned}$$

and

$$\begin{aligned}\tilde{F}(x*y) &\supseteq \tilde{F}((x*y)*y) \cap \tilde{F}(y*y)\\ &= \tilde{F}((x*y)*y) \cap \tilde{F}(y*y)\\ &= \tilde{F}((x*y)*y).\end{aligned}$$

Hence (2) is valid. Let $(A, \langle\sigma; \tilde{F}\rangle)$ be a soju A-ideal of X that satisfies the condition (3.5). Note that

$$((x*(y*z))*z)*z = ((x*z)*(y*z))*z \leq (x*y)*z \tag{3.7}$$

for all $x, y, z \in A$. Hence

$$\mu_\sigma(((x*(y*z))*z)*z) \geq \mu_\sigma((x*y)*z),$$

$$\gamma_\sigma(((x*(y*z))*z)*z) \leq \gamma_\sigma((x*y)*z),$$

and

$$\tilde{F}(((x*(y*z))*z)*z) \supseteq \tilde{F}((x*y)*z)$$

by Lemma 3.8. It follows from (2.3) and (3.5) that

$$\begin{aligned}\mu_\sigma((x*z)*(y*z)) &= \mu_\sigma((x*(y*z))*z)\\ &\geq \mu_\sigma(((x*(y*z))*z)*z)\\ &\geq \mu_\sigma((x*y)*z),\end{aligned}$$

$$\begin{aligned}\gamma_\sigma((x*z)*(y*z)) &= \gamma_\sigma((x*(y*z))*z)\\ &\leq \gamma_\sigma(((x*(y*z))*z)*z)\\ &\leq \gamma_\sigma((x*y)*z),\end{aligned}$$

and

$$\begin{aligned}\tilde{F}((x*z)*(y*z)) &= \tilde{F}((x*(y*z))*z)\\ &\supseteq \tilde{F}(((x*(y*z))*z)*z)\\ &\supseteq \tilde{F}((x*y)*z).\end{aligned}$$

Therefore (3) holds. Finally, let $(A, \langle\sigma;\tilde{F}\rangle)$ be a soju A-ideal of X that satisfies the condition (3.6). Then

$$\begin{aligned}\mu_\sigma(x*z) &\geq \min\{\mu_\sigma((x*z)*(y*z)), \mu_\sigma(y*z)\}\\ &\geq \min\{\mu_\sigma((x*y)*z), \mu_\sigma(y*z)\},\end{aligned}$$

$$\begin{aligned}\gamma_\sigma(x*z) &\leq \max\{\gamma_\sigma((x*z)*(y*z)), \gamma_\sigma(y*z)\}\\ &\leq \max\{\gamma_\sigma((x*y)*z), \gamma_\sigma(y*z)\},\end{aligned}$$

and

$$\begin{aligned}\tilde{F}(x*z) &\supseteq \tilde{F}((x*z)*(y*z)) \cap \tilde{F}(y*z)\\ &\supseteq \tilde{F}((x*y)*z) \cap \tilde{F}(y*z)\end{aligned}$$

for all $x, y, z \in A$ by using (3.3) and (3.6). Therefore $(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. □

THEOREM 3.10. *Let $A = X$ be a BCK-algebra. Then a soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$ is a positive implicative soju A-ideal of X if and only if $(A, \langle\sigma;\tilde{F}\rangle)$ satisfies the condition (3.2) and*

$$(\forall x, y, z \in A)\left(\begin{array}{l}\mu_\sigma(x*y) \geq \min\{\mu_\sigma(((x*y)*y)*z), \mu_\sigma(z)\}\\ \gamma_\sigma(x*y) \leq \max\{\gamma_\sigma(((x*y)*y)*z), \gamma_\sigma(z)\}\\ \tilde{F}(x*y) \supseteq \tilde{F}(((x*y)*y)*z) \cap \tilde{F}(z).\end{array}\right). \tag{3.8}$$

PROOF: Assume that $(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Then $(A, \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X by Theorem 3.6. Hence the condition (3.2) holds. Using (3.3), (2.1), (2.3), (III) and (3.6), we have

$$\begin{aligned}\mu_\sigma(x*y) &\geq \min\{\mu_\sigma((x*y)*z), \mu_\sigma(z)\}\\ &= \min\{\mu_\sigma(((x*z)*y)*(y*y)), \mu_\sigma(z)\}\\ &\geq \min\{\mu_\sigma(((x*z)*y)*y), \mu_\sigma(z)\}\\ &= \min\{\mu_\sigma(((x*y)*y)*z), \mu_\sigma(z)\},\end{aligned}$$

$$\begin{aligned}\gamma_\sigma(x*y) &\le \max\{\gamma_\sigma((x*y)*z), \gamma_\sigma(z)\}\\ &= \max\{\gamma_\sigma(((x*z)*y)*(y*y)), \gamma_\sigma(z)\}\\ &\le \max\{\gamma_\sigma(((x*z)*y)*y), \gamma_\sigma(z)\}\\ &= \max\{\gamma_\sigma(((x*y)*y)*z), \gamma_\sigma(z)\},\end{aligned}$$

and

$$\begin{aligned}\tilde{F}(x*y) &\supseteq \tilde{F}((x*y)*z)\cap\tilde{F}(z)\\ &= \tilde{F}(((x*z)*y)*(y*y))\cap\tilde{F}(z)\\ &\supseteq \tilde{F}(((x*z)*y)*y)\cap\tilde{F}(z)\\ &= \tilde{F}(((x*y)*y)*z)\cap\tilde{F}(z)\end{aligned}$$

for all $x, y, z \in A$, which proves (3.8).

Conversely, suppose that a soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$ satisfies two conditions (3.2) and (3.8). Then

$$\begin{aligned}\mu_\sigma(x) = \mu_\sigma(x*0) &\ge \min\{\mu_\sigma(((x*0)*0)*z), \mu_\sigma(z)\}\\ &= \min\{\mu_\sigma(x*z), \mu_\sigma(z)\},\end{aligned}$$

$$\begin{aligned}\gamma_\sigma(x) = \gamma_\sigma(x*0) &\le \max\{\gamma_\sigma(((x*0)*0)*z), \gamma_\sigma(z)\}\\ &= \max\{\gamma_\sigma(x*z), \gamma_\sigma(z)\},\end{aligned}$$

and $\tilde{F}(x) = \tilde{F}(x*0) \supseteq \tilde{F}(((x*0)*0)*z)\cap\tilde{F}(z) = \tilde{F}(x*z)\cap\tilde{F}(z)$ for all $x, z \in A$. Hence $(A, \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X. If we take $z = 0$ in (3.8), then

$$\begin{aligned}\mu_\sigma(x*y) &\ge \min\{\mu_\sigma(((x*y)*y)*0), \mu_\sigma(0)\}\\ &= \min\{\mu_\sigma((x*y)*y), \mu_\sigma(0)\} = \mu_\sigma((x*y)*y),\end{aligned}$$

$$\begin{aligned}\gamma_\sigma(x*y) &\le \max\{\gamma_\sigma(((x*y)*y)*0), \gamma_\sigma(0)\}\\ &= \max\{\gamma_\sigma((x*y)*y), \gamma_\sigma(0)\} = \gamma_\sigma((x*y)*y),\end{aligned}$$

and

$$\begin{aligned}\tilde{F}(x*y) &\supseteq \tilde{F}(((x*y)*y)*0)\cap\tilde{F}(0)\\ &= \tilde{F}((x*y)*y)\cap\tilde{F}(0) = \tilde{F}((x*y)*y)\end{aligned}$$

for all $x, y \in A$. It follows from Theorem 3.9 that $(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. □

LEMMA 3.11 ([18]). *Given a subalgebra A of a BCK/BCI-algebra X, every soju A-ideal $(A, \langle\sigma;\tilde{F}\rangle)$ of X satisfies the following assertion.*

$$(\forall x, y, z \in A)\left(x * y \leq z \;\Rightarrow\; \left\{\begin{array}{l} \mu_\sigma(x) \geq \min\{\mu_\sigma(y), \mu_\sigma(z)\} \\ \gamma_\sigma(x) \leq \max\{\gamma_\sigma(y), \gamma_\sigma(z)\} \\ \tilde{F}(x) \supseteq \tilde{F}(y) \cap \tilde{F}(z) \end{array}\right.\right). \tag{3.9}$$

THEOREM 3.12. *Given a subalgebra A of a BCK-algebra X, if every soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$ satisfies the condition (3.9), then it is a soju A-ideal of X.*

PROOF: Assume that $(A, \langle\sigma;\tilde{F}\rangle)$ satisfies the condition (3.9). Since $0*x \leq x$ for all $x \in A$, we have $\mu_\sigma(0) \geq \mu_\sigma(x)$, $\gamma_\sigma(0) \leq \gamma_\sigma(x)$ and $\tilde{F}(0) \supseteq \tilde{F}(x)$ for all $x \in A$ by (3.9). Note that $(x*(x*y))*y = 0$, that is, $x*(x*y) \leq y$ for all $x, y \in A$. It follows from (3.9) that $\mu_\sigma(x) \geq \min\{\mu_\sigma(x*y), \mu_\sigma(y)\}$, $\gamma_\sigma(x) \leq \max\{\gamma_\sigma(x*y), \gamma_\sigma(y)\}$, and $\tilde{F}(x) \supseteq \tilde{F}(x*y) \cap \tilde{F}(y)$ for all $x, y \in A$. Therefore $(A, \langle\sigma;\tilde{F}\rangle)$ is a soju A-ideal of X. □

The following corollary can be easily proved by induction.

COROLLARY 3.13. *Given a subalgebra A of a BCK-algebra X, every soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$ is a soju A-ideal of X if and only if the following assertion is valid.*

$$x * \prod_{i=1}^{n} a_i = 0 \;\Rightarrow\; \left\{\begin{array}{l} \mu_\sigma(x) \geq \min\{\mu_\sigma(a_i) \mid i = 1, 2, \cdots, n\} \\ \gamma_\sigma(x) \leq \max\{\gamma_\sigma(a_i) \mid i = 1, 2, \cdots, n\} \\ \tilde{F}(x) \supseteq \bigcap\limits_{i=1,2,\cdots,n} \tilde{F}(a_i) \end{array}\right. \tag{3.10}$$

for all $x, a_1, a_2, \cdots, a_n \in A$, where $x * \prod_{i=1}^{n} a_i = (\cdots(x*a_1)*\cdots)*a_n$.

THEOREM 3.14. *Let $A = X$ be a BCK-algebra. Then a soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0,1], U)$ is a positive implicative soju A-ideal of X if and only if $(A, \langle\sigma;\tilde{F}\rangle)$ satisfies the following assertion.*

$$((x*y)*y)*a \leq b \;\Rightarrow\; \left\{\begin{array}{l} \mu_\sigma(x*y) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\} \\ \gamma_\sigma(x*y) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\} \\ \tilde{F}(x*y) \supseteq \tilde{F}(a) \cap \tilde{F}(b) \end{array}\right. \tag{3.11}$$

for all $x, y, a, b \in A$.

PROOF: Suppose that $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Then $(A, \langle\sigma; \tilde{F}\rangle)$ is a soju A-ideal of X by Theorem 3.6. Suppose that $((x*y)*y)*a \leq b$ for all $x, y, a, b \in A$. Then

$$\mu_\sigma(x*y) \geq \mu_\sigma((x*y)*y) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\},$$

$$\gamma_\sigma(x*y) \leq \gamma_\sigma((x*y)*y) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\},$$

and $\tilde{F}(x*y) \supseteq \tilde{F}((x*y)*y) \supseteq \tilde{F}(a) \cap \tilde{F}(b)$ by (3.5) and Lemma 3.11.

Conversely, assume that $(A, \langle\sigma; \tilde{F}\rangle)$ satisfies the condition (3.11). Let $x, a, b \in A$ be such that $x*a \leq b$, that is, $(x*a)*b = 0$. Then $(((x*0)*0)*a)*b = 0$, that is, $((x*0)*0)*a \leq b$, which implies from (2.1) and (3.11) that $\mu_\sigma(x) = \mu_\sigma(x*0) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\}$, $\gamma_\sigma(x) = \gamma_\sigma(x*0) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\}$ and $\tilde{F}(x) = \tilde{F}(x*0) \supseteq \tilde{F}(a) \cap \tilde{F}(b)$. Hence $(A, \langle\sigma; \tilde{F}\rangle)$ is a soju A-ideal of X by Theorem 3.12. Since $(((x*y)*y)*((x*y)*y))*0 = 0$ for all $x, y \in A$, we get

$$\mu_\sigma(x*y) \geq \min\{\mu_\sigma((x*y)*y), \mu_\sigma(0)\} = \mu_\sigma((x*y)*y),$$

$$\gamma_\sigma(x*y) \leq \max\{\gamma_\sigma((x*y)*y), \gamma_\sigma(0)\} = \gamma_\sigma((x*y)*y),$$

and $\tilde{F}(x*y) \supseteq \tilde{F}((x*y)*y) \cap \tilde{F}(0) = \tilde{F}((x*y)*y)$ by (3.11) and (3.2). Therefore $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X by Theorem 3.9. □

THEOREM 3.15. *Let $A = X$ be a BCK-algebra. Then a soju structure $(A, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ is a positive implicative soju A-ideal of X if and only if $(A, \langle\sigma; \tilde{F}\rangle)$ satisfies the following assertion.*

$$((x*y)*z)*a \leq b \;\Rightarrow\; \begin{cases} \mu_\sigma((x*z)*(y*z)) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\} \\ \gamma_\sigma((x*z)*(y*z)) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\} \\ \tilde{F}((x*z)*(y*z)) \supseteq \tilde{F}(a) \cap \tilde{F}(b) \end{cases} \tag{3.12}$$

for all $x, y, z, a, b \in A$.

PROOF: Assume that $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Then $(A, \langle\sigma; \tilde{F}\rangle)$ is a soju A-ideal of X by Theorem 3.6. Suppose that $((x*y)*z)*a \leq b$ for all $x, y, z, a, b \in A$. Then

$$\mu_\sigma((x*z)*(y*z)) \geq \mu_\sigma((x*y)*z) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\},$$

$$\gamma_\sigma((x*z)*(y*z)) \leq \gamma_\sigma((x*y)*z) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\},$$

and $\tilde{F}((x*z)*(y*z)) \supseteq \tilde{F}((x*y)*z) \supseteq \tilde{F}(a) \cap \tilde{F}(b)$ by (3.6) and Lemma 3.11, which proves (3.12).

Conversely, suppose that $(A, \langle\sigma; \tilde{F}\rangle)$ satisfies the condition (3.12). Let $x, y, a, b \in A$ be such that $((x*y)*y)*a \leq b$. Then

$$\mu_\sigma(x*y) = \mu_\sigma((x*y)*(y*y)) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\},$$

$$\gamma_\sigma(x*y) = \gamma_\sigma((x*y)*(y*y)) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\},$$

and $\tilde{F}(x*y) = \tilde{F}((x*y)*(y*y)) \supseteq \tilde{F}(a) \cap \tilde{F}(b)$ by (III), (2.1) and (3.12). Therefore $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X by Theorem 3.14. □

The above two theorems have more general form.

THEOREM 3.16. *Let $A = X$ be a BCK-algebra. Then a soju structure $(A, \langle\sigma; \tilde{F}\rangle)$ over $([0,1], U)$ is a positive implicative soju A-ideal of X if and only if $(A, \langle\sigma; \tilde{F}\rangle)$ satisfies the following assertion.*

$$((x*y)*y)*\prod_{i=1}^{n} a_i = 0 \;\Rightarrow\; \begin{cases} \mu_\sigma(x*y) \geq \min\{\mu_\sigma(a_i) \mid i = 1, 2, \cdots, n\} \\ \gamma_\sigma(x*y) \leq \max\{\gamma_\sigma(a_i) \mid i = 1, 2, \cdots, n\} \\ \tilde{F}(x*y) \supseteq \bigcap\limits_{i=1,2,\cdots,n} \tilde{F}(a_i) \end{cases} \tag{3.13}$$

for all $x, y, a_1, a_2, \cdots, a_n \in A$.

PROOF: Let $(A, \langle\sigma; \tilde{F}\rangle)$ be a positive implicative soju A-ideal of X. Then $(A, \langle\sigma; \tilde{F}\rangle)$ is a soju ideal of X by Theorem 3.6. Assume that $((x*y)*y)*\prod_{i=1}^{n} a_i = 0$ for all $x, y, a_1, a_2, \cdots, a_n \in A$. Using (3.5) and Corollary 3.13, we get

$$\mu_\sigma(x*y) \geq \mu_\sigma((x*y)*y) \geq \min\{\mu_\sigma(a_i) \mid i = 1, 2, \cdots, n\},$$

$$\gamma_\sigma(x*y) \leq \gamma_\sigma((x*y)*y) \leq \max\{\gamma_\sigma(a_i) \mid i = 1, 2, \cdots, n\},$$

and $\tilde{F}(x*y) \supseteq \tilde{F}((x*y)*y) \supseteq \bigcap\limits_{i=1,2,\cdots,n} \tilde{F}(a_i)$, which proves (3.13).

Conversely, suppose that $(A, \langle\sigma;\tilde{F}\rangle)$ satisfies the condition (3.13). Let $x, y, a, b \in A$ be such that $((x * y) * y) * a \leq b$. Then $\mu_\sigma(x * y) \geq \min\{\mu_\sigma(a), \mu_\sigma(b)\}$, $\gamma_\sigma(x * y) \leq \max\{\gamma_\sigma(a), \gamma_\sigma(b)\}$, and $\tilde{F}(x * y) \supseteq \tilde{F}(a) \cap \tilde{F}(b)$ by (3.13). It follows from Theorem 3.14 that $(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. □

THEOREM 3.17. *Let $A = X$ be a BCK-algebra. Then a soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0, 1], U)$ is a positive implicative soju A-ideal of X if and only if $(A, \langle\sigma;\tilde{F}\rangle)$ satisfies the following assertion.*

$$\begin{aligned} &\mu_\sigma((x * z) * (y * z)) \geq \min\{\mu_\sigma(a_i) \mid i = 1, 2, \cdots, n\}, \\ &\gamma_\sigma((x * z) * (y * z)) \leq \max\{\gamma_\sigma(a_i) \mid i = 1, 2, \cdots, n\}, \\ &\tilde{F}((x * z) * (y * z)) \supseteq \bigcap_{i=1,2,\cdots,n} \tilde{F}(a_i) \end{aligned} \tag{3.14}$$

for all $x, y, z, a_1, a_2, \cdots, a_n \in A$ with $((x * y) * z) * \prod_{i=1}^{n} a_i = 0$.

PROOF: It is similar to the proof of Theorem 3.16. □

LEMMA 3.18 ([18]). *In a BCK-algebra X, every soju ideal is a soju subalgebra.*

THEOREM 3.19. *Let X be a BCK-algebra and let $\alpha \in 2^U$ and $(t, s) \in [0, 1]\times[0, 1]$ with $t+s \leq 1$. Given a soju structure $(A, \langle\sigma;\tilde{F}\rangle)$ over $([0, 1], U)$ where $A = X$, the following are equivalent.*

(1) *$(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X.*

(2) *The sets $(\mu_\sigma; t)^*$, $(\gamma_\sigma; s)_*$ and $i(\tilde{F}; \alpha)$ are positive implicative ideals of X when they are nonempty.*

PROOF: Assume that $(A, \langle\sigma;\tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Then $(A, \langle\sigma;\tilde{F}\rangle)$ is a soju A-subalgebra of X by Theorem 3.6 and Lemma 3.18. Let $\alpha \in 2^U$ and $(t, s) \in [0, 1] \times [0, 1]$ be such that $t + s \leq 1$ and $(\mu_\sigma; t)^*$, $(\gamma_\sigma; s)_*$ and $i(\tilde{F}; \alpha)$ are nonempty. Then there exist $a, b, c \in A$ such that $a \in (\mu_\sigma; t)^*$, $b \in (\gamma_\sigma; s)_*$ and $c \in i(\tilde{F}; \alpha)$. It follows from (III) and (3.1) that

$$\mu_\sigma(0) = \mu_\sigma(a * a) \geq \min\{\mu_\sigma(a), \mu_\sigma(a)\} = \mu_\sigma(a) \geq t,$$

$$\gamma_\sigma(0) = \gamma_\sigma(b * b) \leq \max\{\gamma_\sigma(b), \gamma_\sigma(b)\} = \gamma_\sigma(b) \leq s,$$

and $\tilde{F}(0) = \tilde{F}(c*c) \supseteq \tilde{F}(c)\cap\tilde{F}(c) = \tilde{F}(c) \supseteq \alpha$. This shows that $0 \in (\mu_\sigma; t)^*$, $0 \in (\gamma_\sigma; s)_*$ and $0 \in i(\tilde{F}; \alpha)$. Let $x, y, z \in A$ be such that $(x*y)*z \in (\mu_\sigma; t)^*$

and $y * z \in (\mu_\sigma; t)^*$. Then $\mu_\sigma((x * y) * z) \geq t$ and $\mu_\sigma(y * z) \geq t$. It follows from (3.4) that

$$\mu_\sigma(x * z) \geq \min\{\mu_\sigma((x * y) * z), \mu_\sigma(y * z)\} \geq t$$

and so that $x * z \in (\mu_\sigma; t)^*$. Now, suppose that $(a * b) * c \in (\gamma_\sigma; s)_*$ and $b * c \in (\gamma_\sigma; s)_*$ for all $a, b, c \in A$. Then $\gamma_\sigma((a * b) * c) \leq s$ and $\gamma_\sigma(b * c) \leq s$. Hence

$$\gamma_\sigma(a * c) \leq \max\{\gamma_\sigma((a * b) * c), \gamma_\sigma(b * c)\} \leq s,$$

and so $a * c \in (\gamma_\sigma; s)_*$. Let $x, y, z \in A$ be such that $(x * y) * z \in i(\tilde{F}; \alpha)$ and $y * z \in i(\tilde{F}; \alpha)$. Then $\tilde{F}((x * y) * z) \supseteq \alpha$ and $\tilde{F}(y * z) \supseteq \alpha$, which implies from (3.4) that

$$\tilde{F}(x * z) \supseteq \tilde{F}((x * y) * z) \cap \tilde{F}(y * z) \supseteq \alpha.$$

Hence $x * z \in i(\tilde{F}; \alpha)$. Therefore $(\mu_\sigma; t)^*$, $(\gamma_\sigma; s)_*$ and $i(\tilde{F}; \alpha)$ are positive implicative ideals of X.

Conversely, suppose that $(\mu_\sigma; t)^*$, $(\gamma_\sigma; s)_*$ and $i(\tilde{F}; \alpha)$ are positive implicative ideals of X for all $\alpha \in 2^U$ and $(t, s) \in [0, 1] \times [0, 1]$ with $t + s \leq 1$, $(\mu_\sigma; t)^* \neq \emptyset$, $(\gamma_\sigma; s)_* \neq \emptyset$ and $i(\tilde{F}; \alpha) \neq \emptyset$. Then $(\mu_\sigma; t)^*$, $(\gamma_\sigma; s)_*$ and $i(\tilde{F}; \alpha)$ are subalgebras of X. Let $x, y \in A$ be such that $\mu_\sigma(x) = t_1$ and $\mu_\sigma(y) = t_2$. If we take $t = \min\{t_1, t_2\}$, then $x, y \in (\mu_\sigma; t)^*$ and thus $x * y \in (\mu_\sigma; t)^*$. Hence

$$\mu_\sigma(x * y) \geq t = \min\{t_1, t_2\} = \min\{\mu_\sigma(x), \mu_\sigma(y)\}. \tag{3.15}$$

If we put $x = y$ in (3.15) and use (III), then $\mu_\sigma(0) \geq \mu_\sigma(x)$ for all $x \in A$. Similarly, we know that $\gamma_\sigma(0) \leq \gamma_\sigma(x)$ for all $x \in A$. For any $a, b \in A$, if we take $\alpha = \tilde{F}(a) \cap \tilde{F}(b)$, then $a, b \in i(\tilde{F}; \alpha)$. Thus $a * b \in i(\tilde{F}; \alpha)$, and thus

$$\tilde{F}(a * b) \supseteq \alpha = \tilde{F}(a) \cap \tilde{F}(b).$$

Taking $a = b$ implies $\tilde{F}(0) \supseteq \tilde{F}(a)$ for all $a \in A$. Let $x, y, z \in A$ be such that $\mu_\sigma((x * y) * z) = t_1$ and $\mu_\sigma(y * z) = t_2$. Taking $t = \min\{t_1, t_2\}$ implies that $(x * y) * z \in (\mu_\sigma; t)^*$ and $y * z \in (\mu_\sigma; t)^*$, and so $x * z \in (\mu_\sigma; t)^*$. Hence

$$\mu_\sigma(x * z) \geq t = \min\{t_1, t_2\} = \min\{\mu_\sigma((x * y) * z), \mu_\sigma(y * z)\}.$$

If we put $s = \max\{\gamma_\sigma((x*y)*z), \gamma_\sigma(y*z)\}$ for all $x, y, z \in A$, then $(x*y)*z \in (\gamma_\sigma; s)_*$ and $y*z \in (\gamma_\sigma; s)_*$. Hence $x*z \in (\gamma_\sigma; s)_*$, which implies that

$$\gamma_\sigma(x*z) \leq s = \max\{\gamma_\sigma((x*y)*z), \gamma_\sigma(y*z)\}.$$

For any $a, b, c \in A$, let $\alpha = \alpha_1 \cap \alpha_2$ where $\alpha_1 = \tilde{F}((a*b)*c)$ and $\alpha_2 = \tilde{F}(b*c)$. Then $(a*b)*c \in i(\tilde{F}; \alpha)$ and $b*c \in i(\tilde{F}; \alpha)$, and thus $a*c \in i(\tilde{F}; \alpha)$. It follows that

$$\tilde{F}(a*c) \supseteq \alpha = \alpha_1 \cap \alpha_2 = \tilde{F}((a*b)*c) \cap \tilde{F}(b*c). \tag{3.16}$$

Therefore $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. □

Note that a soju ideal might not be a positive implicative soju ideal (see Example 3.7). But we have the following extension property for positive implicative soju ideal.

THEOREM 3.20. *Given a BCK-algebra X and $A = X$, let $(A, \langle\sigma; \tilde{F}\rangle)$ and $(B, \langle\delta; \tilde{G}\rangle)$ be soju A-ideals of X such that*

(1) $\mu_\sigma(0) = \mu_\delta(0)$, $\gamma_\sigma(0) = \gamma_\delta(0)$, $\tilde{F}(0) = \tilde{G}(0)$.

(2) $(\forall x \in A)$ $(\mu_\sigma(x) \leq \mu_\delta(x), \gamma_\sigma(x) \geq \gamma_\delta(x), \tilde{F}(x) \subseteq \tilde{G}(x))$.

If $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X, then so is $(B, \langle\delta; \tilde{G}\rangle)$.

PROOF: Assume that $(A, \langle\sigma; \tilde{F}\rangle)$ is a positive implicative soju A-ideal of X. Let $x, y, z \in A$. Using (2.3), (3.6) and (III), we have

$$\begin{aligned}
&\mu_\delta(((x*z)*(y*z))*((x*y)*z)) \\
&= \mu_\delta(((x*z)*((x*y)*z))*(y*z)) \\
&= \mu_\delta(((x*((x*y)*z))*z)*(y*z)) \\
&\geq \mu_\sigma(((x*((x*y)*z))*z)*(y*z)) \\
&\geq \mu_\sigma(((x*((x*y)*z))*y)*z) \\
&= \mu_\sigma(((x*y)*((x*y)*z))*z) \\
&= \mu_\sigma(((x*y)*z)*((x*y)*z)) \\
&= \mu_\sigma(0) = \mu_\delta(0),
\end{aligned}$$

$$\begin{aligned}
&\gamma_\delta(((x*z)*(y*z))*((x*y)*z))\\
&=\gamma_\delta(((x*z)*((x*y)*z))*(y*z))\\
&=\gamma_\delta(((x*((x*y)*z))*z)*(y*z))\\
&\leq\gamma_\sigma(((x*((x*y)*z))*z)*(y*z))\\
&\leq\gamma_\sigma(((x*((x*y)*z))*y)*z)\\
&=\gamma_\sigma(((x*y)*((x*y)*z))*z)\\
&=\gamma_\sigma(((x*y)*z)*((x*y)*z))\\
&=\gamma_\sigma(0)=\gamma_\delta(0),
\end{aligned}$$

and

$$\begin{aligned}
&\tilde{G}(((x*z)*(y*z))*((x*y)*z))\\
&=\tilde{G}(((x*z)*((x*y)*z))*(y*z))\\
&=\tilde{G}(((x*((x*y)*z))*z)*(y*z))\\
&\supseteq\tilde{F}(((x*((x*y)*z))*z)*(y*z))\\
&\supseteq\tilde{F}(((x*((x*y)*z))*y)*z)\\
&=\tilde{F}(((x*y)*((x*y)*z))*z)\\
&=\tilde{F}(((x*y)*z)*((x*y)*z))\\
&=\tilde{F}(0)=\tilde{G}(0).
\end{aligned}$$

It follows from (3.2) and (3.3) that

$$\begin{aligned}
\mu_\delta((x*z)*(y*z))&=\min\{\mu_\delta(((x*z)*(y*z))*((x*y)*z)),\mu_\delta((x*y)*z)\}\\
&\geq\min\{\mu_\delta(0),\mu_\delta((x*y)*z)\}=\mu_\delta((x*y)*z),
\end{aligned}$$

$$\begin{aligned}
\gamma_\delta((x*z)*(y*z))&=\max\{\gamma_\delta(((x*z)*(y*z))*((x*y)*z)),\gamma_\delta((x*y)*z)\}\\
&\leq\max\{\gamma_\delta(0),\gamma_\delta((x*y)*z)\}=\gamma_\delta((x*y)*z),
\end{aligned}$$

and

$$\begin{aligned}
\tilde{G}((x*z)*(y*z))&=\tilde{G}(((x*z)*(y*z))*((x*y)*z))\cap\tilde{G}((x*y)*z)\\
&\supseteq\tilde{G}(0)\cap\tilde{G}((x*y)*z)=\tilde{G}((x*y)*z)
\end{aligned}$$

for all $x,y,z\in A$. Therefore $(B,\ \langle\delta;\tilde{G}\rangle)$ is a positive implicative soju A-ideal of X. □

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