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DEONTIC PARADOXES AND TABLEAU SYSTEM FOR KALINOWSKI'S DEONTIC LOGIC $K1$

Abstract

In 1953, Jerzy Kalinowski published his paper on the logic of normative sentences. The paper is recognized as one of the first publications on the formal system of deontic logic. The aim of this paper is to present a tableau system for Kalinowski's deontic logic and to discuss some of the topics related to the paradoxes of deontic logic.

Keywords: deontic logic, *K1*, Kalinowski's logic, paradoxes, tableaux

1. Introduction

Roughly speaking, deontic logic is a logic which deals with (or - is concerned with) normative concepts such as obligation, permission or prohibition. Standard Deontic Logic (*SDL*, for short) is probably one of the well-known and most studied systems of deontic logic. The language of *SDL* is like that of classical propositional logic, i.e it includes (the nonempty and denumerable set of) propositional variables and Boolean connectives, except that it is enriched with the deontic operator O. A Hilbert-style axiom system for *SDL* consists of axioms for classical propositional logic, (K) $O(\phi \to \chi) \to (O\phi \to O\chi)$, (D) $\sim (O\phi \wedge O \sim \phi)$ and two rules of inferences: Detachment and Necessitation for O. However, this approach gives rise to some problem which was aptly described by Royakkers in his book: 'Deontic logic has been bothered by a number of paradoxes during its entire development. These paradoxes are logical expressions that have validity in a deontic system, such as *SDL*, but are counter-intuitive in a common sense context, or they are logical expressions that are inconsistent.'[1]

Consider, as a simple example, two sentences:

(1) Grandma should love her grandchildren: Op.

(2) Grandma should love her grandchildren or kill her noisy neighbour: $O(p \vee q)$.

The sentences mean different things, but according to *SDL*, Grandma also has an obligation that can be fulfilled by killing her noisy neighbour. It is because

(a) $p \to (p \vee q)$ Disjunction Introduction Principle
(b) $O(\to (p \vee q))$ by Necessitation Rule
(c) $Op \to O(p \vee q)$ by (K) and Detachment Rule.

The *problem* is known as Ross's paradox.

Kalinowski's approach to deontic operators differs from that of the standard modal approach. There is an important assumption behind Kalinowski's logic that norms, just like logical propositions, are true or false. To say that a norm is true (or false) is to determine the moral value of action. 'Kalinowski believed that (...) logical value of norms depends on moral value of actions (...) every action *in genere* is either good (g), or bad (b) or neutral (n), although actions *in concreto* are always good or bad. Good (bad) actions are such by nature and remain good (bad) in all circumstances. On the other hand neutral actions are those which in some circumstances are good and in other circumstances bad'([7], pp. 54-55). The source of inspiration is clear, at least from the formal point of view. It was Łukasiewicz and his three-valued logic that exerted a strong influence on Kalinowski. As we will see below, moral value of actions was given in terms of 3-valued truth tables. Before going into more details, let us introduce some notation and definitions.

Let $Act_0 = \{\alpha_1, \alpha_2, \alpha_3, ...\}$ be a finite set of *basic action names* and $\bar{\alpha}_i$ denote the complement of α_i (where $i \in N$), then, for a fixed Act_0, the set of *action names* can be defined as follows:

$$Act = Act_0 \cup \{\bar{\alpha_1}, \bar{\alpha_2}, \bar{\alpha_3}, \ldots, \bar{\bar{\alpha}}_1, \bar{\bar{\alpha}}_2, \bar{\bar{\alpha}}_3, \ldots, \bar{\bar{\bar{\alpha}}}_1, \bar{\bar{\bar{\alpha}}}_2, \bar{\bar{\bar{\alpha}}}_3, \ldots\}.$$

The set of formulas of Kalinowski's deontic logic *K1*, *K1*-formulas for short, is inductively defined in the following way:

[1] [8], p. 45.

(1) if $\alpha \in Act$, then $P(\alpha)$ is a *K1*-formula
(2) if ϕ and χ are *K1*-formulas, then $\sim \phi$ and $\phi \rightarrow \chi$ are *K1*-formulas.

where P is a deontic operator meaning: *it is permitted that*, and the symbols: $\sim$, $\rightarrow$ denote classical negation and implication, respectively.

Every *K1*-formula of the form $P(\alpha)$ is called *atomic*. The connectives of conjunction, disjunction and equivalence can be introduced *via* definitions as in classical propositional logic. In standard deontic logics, it is possible to define all the deontic operators in terms of one operator. Notice that it also happens in Kalinowski's deontic logic. The deontic operators O (it is *obligatory* that) and F (it is *forbidden* that) is defined in terms of the operator P:

$O(\alpha) =^{df} \sim P(\bar{\alpha})$
$F(\alpha) =^{df} \sim P(\alpha)$, where $\alpha \in Act$.

Additionally, we can introduce by definition other deontic operators, for example neutrality N:

$N(\alpha) =^{df} P(\alpha) \wedge P(\bar{\alpha})$, where $\alpha \in Act$.

Now let us return for a moment to Ross's paradox. In SDL, we are made to accept $Op \rightarrow O(p \vee q)$ as a tautology. The problem does not exist in *K1*. Neither $O(p \rightarrow (p \vee q))$ nor $Op \rightarrow O(p \vee q)$ can be deduced from $p \rightarrow (p \vee q)$. This is a consequence of the definition of *K1*-formulas. The paradox cannot be formulated in the language of *K1*, and neither can many other paradoxes, such as, the Good Samaritan Paradox, the Paradox of Derived Obligation, Chisholm's Paradox, etc. There is no need to look for any counter-intuitive example of the formula $O(\phi \rightarrow \varphi)$ or $O(\phi \wedge \varphi)$ in everyday language. The reason for that is quite simple: neither of them is a *K1*-formula.

Kalinowski's logic is based on the classical propositional calculus. $K1$ is axiomatized by adding to the usual axioms and rules of classical propositional calculus the following axiom:

$(AP) \sim P(\bar{\alpha}) \rightarrow P(\alpha)$

and double complement elimination rule:

(DCE) $\phi(\bar{\bar{\alpha}})/\phi(\bar{\bar{\alpha}}//\alpha)$.

The notation $\phi(\bar{\bar{\alpha}}//\alpha)$ means that we may replace $\bar{\bar{\alpha}}$ with α (and *vice versa*) in the formula ϕ. A formula ϕ is provable in $K1$ if there is a formal proof of ϕ within $K1$, i.e. there is a finite sequence of formulas, φ_1, φ_2, ... φ_n, each of which is an axiom of $K1$ or follows from the preceding formulas in the sequence by the rules of $K1$. This sequence is a proof for ϕ if $\varphi_n = \phi$.

As we have noticed there is an important assumption behind Kalinowski's logic which posits that norms, just like logical propositions, are true or false, but moral value of actions is given in terms of 3-valued truth tables. In $K1$, the truth value of a compound proposition depends only on the value of its components. To say that a norm is $true$ (or $false$) is to determine what the moral value of action is. But here the question arises: How to determine the moral value of action? Kalinowski's answer is as follows: the action α is good (g, in symbols) if and only if the complement of α is bad (b). The action α is bad (b) iff the complement of α is good (g). And finally, the action α is neutral (n, in symbols) iff the complement of α is also neutral (n). It now becomes possible to set up the truth table for the complement of action:

α	$\bar{\alpha}$
g	b
b	g
n	n

The next question is, how to determine the logical value of norms? We reformulate the question to ask: What kind of action is *permitted*, *obligatory* or *forbidden*? For instance, if the action α is either good (g) or neutral (n), then it is true (t, for short) that the action α is permitted ($P(\alpha)$). If the action α is bad (b), then it is false (f) that the action α is permitted ($P(\alpha)$). All these 'if-thens' are based on some underlying philosophical assumptions about what makes it true (or false) that an action is permitted, obligatory or forbidden. Obviously, one can disagree with Kalinowski and express doubts about the validity of his assumptions.

Here are the truth tables for the deontic operators:

α	$P(\alpha)$	$F(\alpha)$	$O(\alpha)$	$N(\alpha)$
g	t	f	t	f
b	f	t	f	f
n	t	f	f	t

To put it more precisely, $K1$-matrix is a triple ([6], p.186):

$M_{K1} = \langle \{g, b, c\}, \{P, F, O, N\}, v \rangle$

where $\{g, b, c\}$ is a set of deontic values, $\{P, F, O, N\}$ is a set of functions from $\{g, b, c\}$ to $\{t, f\}$ which corresponds to the deontic operators, v is a function from $\{g, b, c\}$ to $\{g, b, c\}$ which attaches a deontic value to complex actions.

Below we define a family of interpretation and valuation functions based on them:

$$Int : Act \longrightarrow \{g, b, c\}$$

For each function Int there exists a $K1$-valuation v^*: $K1$-formulas $\longrightarrow \{t, f\}$. A $K1$-formula ϕ is a $K1$-tautology iff ϕ takes the value t under any $K1$-valuation.

Theorem 1. A formula ϕ is provable in $K1$ iff ϕ a $K1$-tautology.

Proof. (*Ibidem*, pp. 187-189).

2. $K1$-Tableau System

In Section 2, we present a tableau based proof technique that can be used for proving theorems in $K1$. We assume some familiarity with tableau methods.[2] In a nutshell, *tableau lines* are of the form $\sigma : \phi$ or $\tau : \alpha$, where ϕ is a $K1$-formula and σ is a logical value (i.e. $\sigma \in \{t, f\}$); or α is an action name and τ is a moral value (i.e. $\tau \in \{g, b, n\}$). The notation $\sigma : \phi$ intuitively means 'ϕ is *true*' if $\sigma = t$, or 'ϕ is *false*' if $\sigma = f$. The notation $\tau : \alpha$ means one of the following: 'the moral value of α is *good*' if $\tau = g$, 'the moral value of α is *bad*' if $\tau = b$ or 'the moral value of α is *neutral*' if $\tau = n$. We start with a line of the form $f : \phi$. By a *tableau proof* of ϕ (*K1*-tableau proof) is meant a closed tableau with $f : \phi$. All the branches of a tree are obtained by the following rules:

Classical Rules

The rules for negation and implication (plus conjunction, disjunction and equivalence) are identical to the ones used in classical propositional logic.

Negation:

$$(\mathbf{t} \sim) \ \frac{t :\sim \phi}{f : \phi} \qquad\qquad (\mathbf{f} \sim) \ \frac{f :\sim \phi}{t : \phi}$$

Implication:

$$(\mathbf{t} \rightarrow) \ \frac{t : \phi \rightarrow \chi}{f : \phi \mid t : \chi} \qquad\qquad (\mathbf{f} \rightarrow) \ \frac{f : \phi \rightarrow \chi}{\begin{array}{c} t : \phi \\ f : \chi \end{array}}$$

[2] See [1] and [3], for details.

Notice that the rule $(f \rightarrow)$ is linear, but $(t \rightarrow)$ is a branching rule, as expected.

Action Rules

The rules presented here are easily adapted from the truth table for the complement of action. To be more specific:

Complement:

$$(\mathbf{gA})\ \frac{g : \bar{\alpha}}{b : \alpha} \qquad (\mathbf{bA})\ \frac{b : \bar{\alpha}}{g : \alpha} \qquad (\mathbf{nA})\ \frac{n : \bar{\alpha}}{n : \alpha}$$

Deontic Rules

The rules coincide with the lines of the truth tables of deontic operators.

Permission:

$$(\mathbf{tP})\ \frac{t : P(\alpha)}{g : \alpha \mid n : \alpha} \qquad (\mathbf{fP})\ \frac{f : P(\alpha)}{b : \alpha}$$

If it is true that the action α is permitted, then α is either good or bad. And similarly for (**fP**), if $P(\alpha)$ is false, then the action α is bad. Note that the rule (**fP**) is linear while (**tP**) is branching.

Obligation:

$$(\mathbf{tO})\ \frac{t : O(\alpha)}{g : \alpha} \qquad (\mathbf{fO})\ \frac{f : O(\alpha)}{b : \alpha \mid n : \alpha}$$

If it is true that the action α is obligatory, then the action is good. But if $O(\alpha)$ is false, then two possibilities must be taken into account: the action α is bad or the action α is neutral. The rule (**tO**) is linear, whereas (**fO**) is branching.

Forbiddance:

$$(\mathbf{tF})\ \frac{t : F(\alpha)}{b : \alpha} \qquad (\mathbf{fF})\ \frac{f : F(\alpha)}{g : \alpha \mid n : \alpha}$$

If it is true that α is forbidden, then the action is bad. On the other hand, if the formula $F(\alpha)$ is false, then two cases need to be considered: the action α is good or the action α is neutral. Likewise, the rule (**tF**) is linear, but (**fF**) is branching.

Neutrality:

$$(\mathbf{tN})\ \frac{t : N(\alpha)}{n : \alpha} \qquad (\mathbf{fN})\ \frac{f : N(\alpha)}{g : \alpha \mid b : \alpha}$$

If the formula $N(\alpha)$ is true, then the action is neutral. But, if the formula is false, then the action α is either good or bad. In the same way, the rule (**tN**) is linear, but (**fN**) is branching.

A branch of a tableau is *closed* if we can apply the closure rule:

Closure rules

$$(\perp_{\mathbf{A}})\quad \frac{\begin{array}{c} x:\alpha \\ y:\alpha \end{array}}{\text{closed}}$$

where $\alpha \in Act$; $x,\ y \in \{b, g, n\}$ and $x \neq y$.

Otherwise the branch is *open*. A tableau is closed if all of its branches are closed, otherwise the tableau is open. Let ϕ be a formula. By a *tableau proof* of ϕ ($K1$-tableau proof) we mean a closed tableau with $f : \phi$.

Conceptually, *K1*-tableau proof system is an easy to used system for proving the validity of a *K1*-formula.

Theorem 2. A formula ϕ has a $K1$-tableau proof iff ϕ is valid in $K1$.

The examples below illustrate the usage of the tableau based proof technique.

Example 1. A tableau proof of $\sim P(\bar{\alpha}) \rightarrow P(\alpha)$.

(1)	$f : \sim P(\bar{\alpha}) \rightarrow P(\alpha)$	(start)
(2)	$t : \sim P(\bar{\alpha})$	($\mathbf{f} \rightarrow$),(1)
(3)	$f : P(\alpha)$	($\mathbf{f} \rightarrow$),(1)
(4)	$b : \alpha$	(**fP**),(3)
(5)	$f : P(\bar{\alpha})$	($\mathbf{t} \sim$),(2)
(6)	$b : \bar{\alpha}$	(**fP**),(5)
(7)	$g : \alpha$	(**bA**),(6)
	closed	($\perp_{\mathbf{A}}$),(4),(7)

This might seem a rather trivial example, but it clearly demonstrates the usage of the *action* rules.

Example 2. A tableau proof of $F(\alpha) \rightarrow O(\bar{\alpha})$.

(1)	$f : F(\alpha) \rightarrow O(\bar{\alpha})$	(start)
(2)	$t : F(\alpha)$	($\mathbf{f} \rightarrow$),(1)
(3)	$f : O(\bar{\alpha})$	($\mathbf{f} \rightarrow$),(1)
(4)	$b : \alpha$	(**tF**),(2)

(5)	$b : \bar{\alpha}$	(**fO**),(3)	(7)	$n : \bar{\alpha}$	(**fO**),(3)
(6)	$g : \alpha$	(**gA**),(5)	(8)	$n : \alpha$	(**nA**),(7)
	closed ($\perp_{\mathbf{A}}$), (4), (6)			closed ($\perp_{\mathbf{A}}$), (4), (8)	

Example 3. Closed tableau for $\sim F(\alpha) \rightarrow (\sim N(\alpha) \rightarrow O(\alpha))$.

(1)	$f : \sim F(\alpha) \rightarrow (\sim N(\alpha) \rightarrow O(\alpha))$	(start)
(2)	$t : \sim F(\alpha)$	$(\mathbf{f} \rightarrow)$,(1)
(3)	$f : \sim N(\alpha) \rightarrow O(\alpha)$	$(\mathbf{f} \rightarrow)$,(1)
(4)	$f :\ F(\alpha)$	$(\mathbf{t} \sim)$,(2)
(5)	$t : \sim N(\alpha)$	$(\mathbf{f} \rightarrow)$,(3)
(6)	$f :\ O(\alpha)$	$(\mathbf{f} \rightarrow)$,(3)
(7)	$f :\ N(\alpha)$	$(\mathbf{t} \sim)$,(5)

(8) $g : \alpha$ (**fN**),(7)		(9) $b : \alpha$ (**fN**),(7)	
(10) $b : \alpha$ (**fO**),(6)	(11) $n : \alpha$ (**fO**),(6)	(12) $g : \alpha$ (**fF**),(4)	(13) $n : \alpha$ (**fF**),(4)
closed $(\perp_{\mathbf{A}})$, (8), (10)	closed $(\perp_{\mathbf{A}})$, (8), (11)	closed $(\perp_{\mathbf{A}})$, (9), (12)	closed $(\perp_{\mathbf{A}})$, (9), (13)

References

[1] M. D'Agostino, D. Gabbay, R. Haenle and J. Possegga (eds), **Handbook of Tableau Methods**, Kluwer Academic Publishers, Dordrecht-Boston-London, 1999.

[2] M. Fisher, *A three-valued calculus for deontic logic*, **Theoria**, Vol. 23:7 (1961), pp. 107–118.

[3] M. C. Fitting, **First-Order Logic and Automated Theorem Proving**, Springer, 1996 (first edition, 1990).

[4] R. Hilpinen, **Deontic Logic: Introductory and Systematic Readings** (Synthese Library), Dordrecht, D. Reidel Publishing Company, 2013.

[5] J. Kalinowski, *Teoria zdań normatywnych* (in Polish), **Studia Logica** 1 (1953), pp. 113–146 (French translation: Jerzy Kalinowski, *Theorie des propositions normatives*, **Studia Logica** 1 (1953), pp. 147–182).

[6] P. Kulicki, *A Note on the Adequacy of Jerzy Kalinowski's K1 Logic*, **Bulletin of the Section of Logic** 43:3/4 (2014), pp. 183–190.

[7] P. Kulicki and R. Trypuz, *Doing the right things – trivalence in deontic action logic*, available at https://philpapers.org/rec/KULDTR

[8] L. L. Royakkers, **Extending Deontic Logic for the Formalisation of Legal Rules**, Kluwer Academic Publishers, Dordrecht, 1998.

[9] W. Suchoń, *Studia nad logiką deontyczną* (in Polish), **Rozprawy Habilitacyjne** Nr 76, Uniwersytet Jagieloński, Kraków, 1983.

[10] R. Trypuz and P. Kulicki, *Jerzy Kalinowski's Logic of Normative Sentences Revisited*, **Studia Logica** 103(2), pp. 389–412.

[11] P. McNamara, *Deontic Logic*, [in:] **Stanford Encyclopedia of Philosophy**, available at http://plato.stanford.edu/entries/logic-deontic/

[12] G. H. von Wright, *Problems and prospects of deontie logic: A survey*, [in:] **Modern Logic – A Survey**, Dordrecht, D. Reidel Publishing Company, 1981, pp. 399–423.

[13] G. H. von Wright, *Deontic logic*, **Mind**, Vol. 60:237 (1951), pp. 1–15.

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