

PREFACE: NON-CLASSICAL LOGICS. THEORY AND APPLICATIONS (PART I)

The articles in the present and forthcoming issues are revised and extended versions of papers presented at the conference *Non-Classical Logics. Theory and Applications*, held in Łódź on 4–8 September 2024.¹

Non-Classical Logics. Theory and Applications (NCL) is an international conference series devoted to novel results and survey work in broadly understood non-classical logics and their applications. The first two editions took place in Łódź, Poland, in 2008 and 2009. Subsequently, the conference was held alternately in Toruń (2010, 2012, 2015, 2018) and Łódź (2011, 2013, 2016, 2022). The tenth edition, organised by the University of Lodz in 2022, was the first to publish its proceedings in *Electronic Proceedings in Theoretical Computer Science*. This practice was continued in the most recent, eleventh edition, with all accepted long papers again appearing in an EPTCS volume. The 2024 edition was supported by the European Research Council as part of the project *Coming to Terms: Proof Theory for Definite Descriptions and Other Terms* (ExtenDD), and featured four

¹Due to the high number of accepted post-conference submissions, the editors decided to divide them into two sets, to be published in two separate issues.



invited talks, eighteen contributed talks, and eighteen short presentations accepted through a light reviewing process.

Given that non-classical logics form a broad and diverse area of research within logic, the contributions collected in this issue address a wide range of topics. They include, among others, proof-theoretic properties of connexive implication, term-modalities, **until**-free fragments of linear temporal logic, and subminimal intuitionistic negation.

The article “Cut-Elimination and Normalization Theorems for Connexive Logics over Wansing’s C1” by Norihiro Kamide develops a unified Gentzen-style proof-theoretical framework for a family of connexive logics based on Wansing’s constructive connexive logic C. Within this framework, the author introduces sequent calculi and natural deduction systems (including variants with general elimination rules) for C and its extensions—C3, MC, and CN—obtained by adding the law of excluded middle, Peirce’s law, or both. The paper proves cut-elimination theorems for the sequent calculi, normalisation theorems for the corresponding natural deduction systems, and establishes their equivalence. In addition, similar results are obtained for a family of paraconsistent logics (N-family) over Nelson’s four-valued logic N4. Compared to earlier work, the article extends and refines proofs, corrects errors from a prior conference version, and provides detailed technical developments that yield an integrated proof-theoretical treatment of connexive and related paraconsistent logics. The article by Takahiro Sawasaki, “Semantic Incompleteness of Liberman et al. (2020)’s Hilbert-Style Systems for Term-Modal Logics with Equality and Non-Rigid Terms”, establishes that the Hilbert-style systems proposed by Liberman et al. [1] for term-modal logics are semantically incomplete when extended with standard modal axioms (T, D, 4, 5). Term-modal logic, which allows modal operators indexed by first-order terms, is important in epistemic and deontic contexts, but the author shows that certain formulas valid in the intended Kripke semantics are unprovable in these systems. In particular, the validity of the formula $x = c \rightarrow (P(x) \rightarrow P(c))$ highlights the gap. To demonstrate this, the paper develops a non-standard Kripke semantics in which the interpretation of constants and function symbols depends on the relations they occur with, thereby exposing the systems’ limitations.

The paper also corrects a mischaracterised frame correspondence in Liberman et al.’s paper, providing refined technical results on the relationship between syntax, semantics, and completeness in this family of logics.

Norihiro Kamide and Sara Negri’s paper “Unified Sequent Calculi and Natural Deduction Systems for Until-free Linear-time Temporal Logics” introduces a unified Gentzen-style proof-theoretic framework for until-free propositional linear-time temporal logic (LTL) and its intuitionistic variant. It develops both single-succedent sequent calculi and natural deduction systems that extend Gentzen’s classical (LK, NK) and intuitionistic (LI, NI) calculi in a uniform way. The main results establish the equivalence between the proposed sequent calculi and natural deduction systems, prove cut-elimination theorems for the calculi, and show normalisation theorems for the deduction systems. By doing so, the article provides a modular, consistent, and proof-theoretically robust foundation for reasoning in these temporal logics, clarifying their structural properties and relations to classical proof theory.

Finally, the article “Continua of Logics Related to Intuitionistic and Minimal Logics” by Kaito Ichikura investigates the landscape of logical systems lying between and around intuitionistic and minimal logics. Building on Vakarelov’s work on co-minimal and subminimal logics [2], the paper reformulates earlier approaches in a uniform framework and introduces a simpler characterisation of the intersection of minimal and co-minimal logics. Using algebraic semantics (Wroński’s method) rather than neighborhood semantics, the author demonstrates the existence of continua of distinct logics situated between these systems, thereby extending classical results on the cardinality of intermediate logics. The study not only clarifies the relations among various subminimal systems (such as SUBMIN, CO-MIN, and their fragments), but also provides simpler proofs of known results and shows how algebraic tools can yield new insights into the fine-grained structure of the logical space below intuitionistic logic.

References

- [1] A. O. Liberman, A. Achen, R. K. Rendsvig, *Dynamic Term-modal Logics for First-order Epistemic Planning*, **Artificial Intelligence**, vol. 286 (2020), p. 103305, DOI: <https://doi.org/10.1016/j.artint.2020.103305>.
- [2] D. Vakarelov, *Nelson's negation on the base of weaker versions of intuitionistic negation*, **Studia Logica**, vol. 80 (2005), pp. 393–430, DOI: <https://doi.org/10.1007/s11225-005-8476-5>.

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