Bulletin of the **S**ection of **L**ogic
Volume 53/4 (2024), pp. 491–510
https://doi.org/10.18778/0138-0680.2024.16

Natnael Teshale Amare

$\mathcal{L}$-WEAKLY 1-ABSORBING PRIME IDEALS AND FILTERS

Abstract

In this manuscript, we have presented the concept of $\mathcal{L}$-weakly 1-absorbing prime ideals and $\mathcal{L}$-weakly 1-absorbing prime filters within an ADL. Mainly, we illustrate the connections between $\mathcal{L}$-weakly prime ideals (filters) and $\mathcal{L}$-weakly 1-absorbing prime ideals (filters), as well as between $\mathcal{L}$-weakly 1-absorbing prime ideals (filters) and $\mathcal{L}$-weakly 2-absorbing ideals (filters). Lastly, we have shown that both the image and inverse image of $\mathcal{L}$-weakly 1-absorbing prime ideals (filters) result in $\mathcal{L}$-weakly 1-absorbing prime ideals (filters).

Keywords: Weakly 1-absorbing prime ideal, weakly 1-absorbing prime filter, $\mathcal{L}$-weakly 2-absorbing ideal, $\mathcal{L}$-weakly 1-absorbing prime ideal, $\mathcal{L}$-weakly 2-absorbing filter, $\mathcal{L}$-weakly 1-absorbing prime filter.

2020 Mathematical Subject Classification: 06D72, 06F15, 08A72.

1. Introduction

The idea of prime ideals(filters) is vital in the study of structure theory of distributive lattices in general and in particular, that of Boolean algebras. Badawi [7] introduced the concept of 2-absorbing ideals in commutative rings, extending the idea of prime ideals from [11]. Chuadhari [9] further extended 2-absorbing ideals to semi-rings. Badawi and Darani [8] introduced weakly 2-absorbing ideals in commutative rings, a generalization of weakly prime ideals by Anderson and Smith [6]. Wasakidar and Gaikerad [24] extended the concepts of 2-absorbing and weakly 2-absorbing ideals to

Presented by: Janusz Ciuciura
Received: March 16, 2024
Published online: November 14, 2024

© Copyright by Author(s), Łódź 2024
© Copyright for this edition by the University of Lodz, Łódź 2024

lattices. Natnael TA [23, 2, 1] introduced weakly 2-absorbing ideals and weakly 2-absorbing filters, along with 1-absorbing prime filters in an ADL. L.A. Zadeh [25] defined a fuzzy subset of a set X as a function mapping elements to real numbers in $[0, 1]$. Goguen [12] expanded this concept by using a complete lattice $\mathcal{L}$ instead of the valuation set $[0, 1]$, aiming for a more comprehensive exploration of fuzzy set theory through fuzzy sets. Darani and Ghasemi [10], as well as Mandal [14], introduced fuzzy 2-absorbing ideals and 2-absorbing fuzzy ideals for commutative rings, respectively, generalizing the concept of fuzzy prime ideals in rings explored by June [13] and Sharma [18]. Nimbhorkar and Patil [15, 16] introduced fuzzy weakly 2-absorbing ideals in lattices. In our previous work [20, 21], we introduced the concepts of fuzzy ideals and filters within an ADL, serving as the basis for our research. Natnael [5, 2] later expanded on this by introducing the concept of fuzzy 2-absorbing ideals and filters in an ADL.

In this paper, we have introduced the concept of $\mathcal{L}$-weakly $1A$-prime ideals and filters in an ADL, aiming to extend the idea of $\mathcal{L}$-prime ideals and filters in an ADL as presented in [17, 19]. Initially, we define $\mathcal{L}$-weakly $1A$-prime ideals, which are less stringent than $\mathcal{L}$-prime ideals. Also, we study on $\mathcal{L}$-weakly $1A$-prime filters in an ADL which is weaker than that $\mathcal{L}$-prime filters. Our main emphasis is on investigating the connections between $\mathcal{L}$-prime ideals and $\mathcal{L}$-weakly $1A$-prime ideals, as well as the relationships between $\mathcal{L}$-weakly $1A$-prime ideals and $\mathcal{L}$-$2A$-ideals. Also, we investigating the connections between $\mathcal{L}$-prime filters and $\mathcal{L}$-weakly $1A$-prime filters, and $\mathcal{L}$-weakly $1A$-prime filters and $\mathcal{L}$-$2A$-filters. Counter examples are provided to demonstrate that the converses of these relationships do not hold. Furthermore, we demonstrate that the direct product of any two $\mathcal{L}$-prime ideals ($\mathcal{L}$-prime filters) results in an $\mathcal{L}$-weakly $1A$-prime ideal ($\mathcal{L}$-weakly $1A$-prime filter) in an ADL. However, it is important to note that the product of $\mathcal{L}$-weakly $1A$-prime ideals ($\mathcal{L}$-weakly $1A$-prime filters) may not necessarily yield an $\mathcal{L}$-weakly $1A$-prime ideal ($\mathcal{L}$-weakly $1A$-prime filter) in an ADL. Additionally, we establish that both the image and pre-image of any $\mathcal{L}$-weakly $1A$-prime ideals ($\mathcal{L}$-weakly $1A$-prime filters) are again $\mathcal{L}$-weakly $1A$-prime ideals ($\mathcal{L}$-weakly $1A$-prime filters).

2. Preliminaries

In this portion, we revisit certain definitions and fundamental findings primarily sourced from [20, 17, 22].

DEFINITION 2.1. An algebra $R = (R, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is referred to as an ADL if it meets the subsequent conditions for all r, s and t in R.

1. $0 \wedge r = 0$
2. $r \vee 0 = r$
3. $r \wedge (s \vee t) = (r \wedge s) \vee (r \wedge t)$
4. $r \vee (s \wedge t) = (r \vee s) \wedge (r \vee t)$
5. $(r \vee s) \wedge t = (r \wedge t) \vee (s \wedge t)$
6. $(r \vee s) \wedge s = s$.

Every distributive lattice with a lower bound is categorized as an ADL.

Example 2.2. For any nonempty set A, it's possible to transform it into an ADL that doesn't constitute a lattice by selecting any element 0 from A and fixing an arbitrary element $u_0 \in R$. For every $u, v \in R$, define $\wedge$ and $\vee$ on R as follows:

$$u \wedge v = \begin{cases} v & \text{if } u \neq u_0 \\ u_0 & \text{if } u = u_0 \end{cases} \qquad \text{and} \qquad u \vee v = \begin{cases} u & \text{if } u \neq u_0 \\ v & \text{if } u = u_0 \end{cases}$$

Then $(A, \wedge, \vee, u_0)$ is an ADL (called the **discrete ADL**) with u_0 as its zero element.

DEFINITION 2.3. Consider $R = (R, \wedge, \vee, 0)$ be an ADL. For any r and $s \in R$, establish $r \leq s$ if $r = r \wedge s$ (which is equivalent to $r \vee s = s$). Then $\leq$ is a partial order on R with respect to which 0 is the smallest element in R.

THEOREM 2.4. *The following conditions are valid for any r, s and t in an ADL R.*

(1) $r \wedge 0 = 0 = 0 \wedge r$ *and* $r \vee 0 = r = 0 \vee r$

(2) $r \wedge r = r = r \vee r$

(3) $r \wedge s \leq s \leq s \vee r$

(4) $r \wedge s = r$ *iff* $r \vee s = s$

(5) $r \wedge s = s$ *iff* $r \vee s = r$

(6) $(r \wedge s) \wedge t = r \wedge (s \wedge t)$ *(in other words, $\wedge$ is associative)*

(7) $r \vee (s \vee r) = r \vee s$

(8) $r \leq s \Rightarrow r \wedge s = r = s \wedge r$ $\big($ *iff* $r \vee s = s = s \vee r\big)$

(9) $(r \wedge s) \wedge t = (s \wedge r) \wedge t$

(10) $(r \vee s) \wedge t = (s \vee r) \wedge t$

(11) $r \wedge s = s \wedge r$ *iff* $r \vee s = s \vee r$

(12) $r \wedge s = \inf\{r, s\}$ *iff* $r \wedge s = s \wedge r$ *iff* $r \vee s = \sup\{r, s\}$.

Definition 2.5. Let R and G be ADLs and form the set $R \times G = \{(r, g) : r \in R \text{ and } g \in G\}$. For all $(r_1, g_1), (r_2, g_2) \in R \times G$, define $\wedge$ and $\vee$ in $R \times G$ by $(r_1, g_1) \wedge (r_2, g_2) = (r_1 \wedge r_2, g_1 \wedge g_2)$ and $(r_1, g_1) \vee (r_2, g_2) = (r_1 \vee r_2, g_1 \vee g_2)$. Then $(R \times G, \wedge, \vee, 0)$ is an ADL under the pointwise operations and $0 = (0, 0)$ is the zero element in $R \times G$.

Definition 2.6. A non-empty subset, denoted as F in an ADL R is termed an ideal (filter) in R if it satisfies the conditions: if u and v belong to F, then $u \vee v$ $(u \wedge v)$ is also in F, and for every element r in R, the $u \wedge r$ $(r \vee u)$ is in F.

Definition 2.7. A proper ideal(filter) F in R is a prime ideal (filter) if for any u and v belongs R, $u \wedge v$ $(u \vee v)$ belongs F, then either u belongs F or v belongs F.

Definition 2.8. Let R and G be ADLs. A mapping $k : R \to G$ is called a homomorphism if the following are satisfied, for any $r, s, t \in R$.

(1) $k(r \wedge s \wedge t) = k(r) \wedge k(s) \wedge k(t)$

(2) $k(r \vee s \vee t) = k(r) \vee k(s) \vee k(t)$

(3) $k(0) = 0$.

Definition 2.9. An $\mathcal{L}$-subset Φ^w is defined as a mapping from R to a complete lattice L that adheres to the infinite meet distributive law. When the lattice L is represented by the unit interval $[0,1]$ of real numbers, these $\mathcal{L}$-subsets correspond to the conventional notion of $\mathcal{L}$-subsets in R.

Definition 2.10. An $\mathcal{L}$-subset Φ^w is an $\mathcal{L}$-ideal(filter) in R, if $\Phi^w(0) = 1\big(\Phi^w(u) = 1$, for any maximal element u in $R\big)$ and $\Phi^w(r \vee s) = \Phi^w(r) \wedge \Phi^w(s)$ $\big(\Phi^w(r \wedge s) = \Phi^w(r) \wedge \Phi^w(s)\big)$, for all r and s belongs to R.

Theorem 2.11. *Let Φ^w be an $\mathcal{L}$-ideal and $\emptyset \neq F \subseteq R$. Then for any r and s belongs to R, we have the following:*

(1) *If $r \leq s$, then $\Phi^w(s) \leq \Phi^w(r)$*

(2) *If r is an associate with s, then $\Phi^w(r) = \Phi^w(s)$*

(3) $\Phi^w(r \wedge s) = \Phi^w(s \wedge r)$ *and* $\Phi^w(r \vee s) = \Phi^w(s \vee r)$

(4) *If $r \in \langle F]$, then $\bigwedge_{i=1}^{n} \Phi^w(x_i) \leq \Phi^w(r)$, for some $x_1, x_2, ..., x_n \in F$*

(5) *If $r \in \langle s]$, then $\Phi^w(s) \leq \Phi^w(r)$*

(6) *If u is maximal in R, then $\Phi^w(u) \leq \Phi^w(r)$*

(7) $\Phi^w(u) = \Phi^w(v)$, *for any maximal elements u and v in R.*

Theorem 2.12. *Let Φ^w be an $\mathcal{L}$-filter and $\emptyset \neq F \subseteq R$. Then for any $r, s \in R$, we have the following.*

(1) *If $r \leq s$, then $\Phi^w(r) \leq \Phi^w(s)$*

(2) *If $r \sim s$, then $\Phi^w(r) = \Phi^w(s)$*

(3) $\Phi^w(r \vee s) = \Phi^w(s \vee r)$

(4) *If $r \in [F\rangle$, then $\bigwedge_{i=1}^{n} \Phi^w(x_i) \leq \Phi^w(r)$, for some $x_1, x_2, ..., x_n \in F$*

(5) *If $r \in [s\rangle$, then $\Phi^w(s) \leq \Phi^w(r)$.*

Definition 2.13. A proper $\mathcal{L}$-ideal(filter) Φ^w is referred to as a prime $\mathcal{L}$-ideal(filter) if $\psi \wedge \eta \leq \Phi^w$ implies either $\psi \leq \Phi^w$ or $\eta \leq \Phi^w$, for any $\mathcal{L}$-ideals(filters) ψ and η in R.

Definition 2.14. A proper $\mathcal{L}$-ideal(filter) Φ^w is an $\mathcal{L}$-prime ideal(filter) in R if $\Phi^w(r \wedge s)\Big(\Phi^w(r \vee s)\Big)$ equals either $\Phi^w(r)$ or $\Phi^w(s)$, for any r and s in R.

3. $\mathcal{L}$-weakly $1A$-prime ideals

In the subsequent discussion, we present the concepts of $\mathcal{L}$-weakly $1-$absorbing prime ideals in an ADL R and their characterizations. Initially, let us revisit the definition outlined in [23], indicating that a proper ideal H in R is a weakly $1-$absorbing prime ideal (in short, a weakly $1A$-prime ideal) in R if, for all elements r, s, and t in R such that $r \wedge s \wedge t \neq 0$, the condition $r \wedge s \wedge t$ belonging to H implies either $r \wedge s$ belonging to H or t belonging to H. Now, we aim to extend this outcome to the realm of $\mathcal{L}$-weakly $1A$-prime ideals as elucidated below.

Definition 3.1. A proper $\mathcal{L}$-ideal Φ^w in R is referred to as an $\mathcal{L}$-weakly $1A$-prime ideal in R if for any elements r,s and t belongs to R such that $r \wedge s \wedge t \neq 0$, the inequality $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$ remains valid.

Example 3.2. Let $R = \{0, r, s, t\}$ and the chain L consisting of four elements $\{0, \gamma, \beta, 1\}$, where $0 < \gamma < \beta < 1$ and let $\vee$ and $\wedge$ be binary operations on R defined by:

$\vee$	0	r	s	t
0	0	r	s	t
r	r	r	r	r
s	s	s	s	s
t	t	r	s	t

$\wedge$	0	r	s	t
0	0	0	0	0
r	0	r	s	t
s	0	r	s	t
t	0	t	t	t

Define an $\mathcal{L}$-subset Φ^w in R as follows: $\Phi^w(0) = 1$, $\Phi^w(r) = \gamma = \Phi^w(s)$ and $\Phi^w(t) = \beta$. It is evident that Φ^w is an $\mathcal{L}$-ideal in R. Furthermore, for any elements r, s and $t \in R$ such that $r \wedge s \wedge t = t \neq 0$, we observe that $\Phi^w(r \wedge s \wedge t) = \beta = \gamma \vee \beta = \Phi^w(r \wedge s) \vee \Phi^w(t)$. Consequently, Φ^w qualifies as an $\mathcal{L}$-weakly $1A$-prime ideal in R.

Following that, we define the concept of an $\mathcal{L}$-weakly $1A$-prime ideal with respect to β-cut, where $\Phi^w_\beta = \{r \in R : \beta \leq \Phi^w(r)\}$.

Theorem 3.3. *Let Φ^w be an $\mathcal{L}$-ideal in R. Then an ideal Φ^w_β is a weakly $1A$-prime ideal in R, for all $\beta \in L$ iff Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R.*

Proof: Assume Φ^w_β is a weakly $1A$-prime ideal, for all $\beta \in L$. In this case, for any elements $r, s, t \in R$ such that $r \wedge s \wedge t \neq 0$, it is ensured that either $r \wedge s \in \Phi^w_{\Phi^w(r \wedge s \wedge t)}$ or $t \in \Phi^w_{\Phi^w(r \wedge s \wedge t)}$, leading to $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s)$ or

$\Phi^w(t)$. Consequently, $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. Conversely, if Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal, consider $r, s, t \in R$ such that $r \wedge s \wedge t \in \Phi^w_\beta$, for all $\beta \in L$. This implies $\beta \leq \Phi^w(r \wedge s \wedge t)$, which further leads to $\beta \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. Consequently, either $\beta \leq \Phi^w(r \wedge s)$ or $\beta \leq \Phi^w(t)$. Hence, either $r \wedge s \in \Phi^w_\beta$ or $t \in \Phi^w_\beta$. Therefore, Φ^w_β is a weakly $1A$-prime ideal in R. □

Corollary 3.4. An ideal P in R is classified as a weakly $1A$-prime ideal in R iff its characteristic set χ_P is an $\mathcal{L}$-weakly $1A$-prime ideal in R.

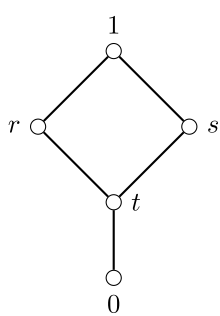
In the upcoming theorems, we establish the connections between $\mathcal{L}$-weakly $1A$-prime ideals and both $\mathcal{L}$-weakly prime ideals and $\mathcal{L}$-weakly $2A$-ideals within the context of an ADL.

Theorem 3.5. *Let Φ^w be an $\mathcal{L}$-ideal in R. Then Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R only if Φ^w is an $\mathcal{L}$-weakly prime ideal in R.*

Proof: Assume Φ^w is an $\mathcal{L}$-weakly prime ideal in R. For any elements $r, s, t \in R$ such that $r \wedge s \wedge t \neq 0$, it follows that $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r) \vee \Phi^w(s \wedge t)$, or $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. This establishes the conclusion. □

In the provided example, we demonstrate that every $\mathcal{L}$-weakly $1A$-prime ideal in R does not qualify as an $\mathcal{L}$-weakly prime ideals in R.

Example 3.6. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, r, s, t, 1\}$ be the lattice represented by the Hasse diagram given below:



Examine the set $D \times L = \{(y, z) \mid y \in D \text{ and } z \in L\}$. Then, the structure $(D \times L, \wedge, \vee, 0)$ forms an ADL, employing pointwise operations $\wedge$ and $\vee$ on

$D \times L$, where 0 is defined as $(0,0)$. Consider $P = \{0, t\}$. It is evident that P is an ideal in L. Now define $\Phi^w : D \times L \to [0,1]$ by

$$\Phi^w(y,z) = \begin{cases} 1 & \text{if } (y,z) = (0,0) \\ 3/4 & \text{if } y \neq 0 \textit{ and } z \in P \\ 0 & \text{otherwise} \end{cases}$$

for all $(y,z) \in D \times L$. Moreover, Φ^w is identified as an $\mathcal{L}$-ideal. Consequently, Φ^w qualifies as an $\mathcal{L}$-weakly $1A$-prime ideal, while Φ^w does not meet the criteria for an $\mathcal{L}$-weakly prime ideal in $D \times L$. This distinction arises from the fact that $\Phi^w((u,r) \wedge (v,s)) = 3/4 \nleq 0$ whereas $\Phi^w(u,r) \vee \Phi^w(v,s)$ results in 0.

Definition 3.7 ([4]). A proper $\mathcal{L}$-ideal Φ^w in R is an $\mathcal{L}$-weakly $2A$-ideal in R if for any elements r,s and $t \in R$ such that $r \wedge s \wedge t \neq 0$, $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(r \wedge t) \vee \Phi^w(s \wedge t)$.

Theorem 3.8. *Let Φ^w be an $\mathcal{L}$-ideal in R. If Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R, then Φ^w is an $\mathcal{L}$-weakly $2A$-ideal in R. The converse of this result is not true.*

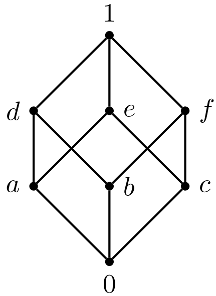
Proof: Assume Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R. Then for all $r,s,t \in R$ such that $r \wedge s \wedge t \neq 0$, it follows that $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. By theorem 2.11(1) and (3), we deduce $\Phi^w(t) \leq \Phi^w(t \wedge s) = \Phi^w(s \wedge t)$ and $\Phi^w(t) \leq \Phi^w(t \wedge r) = \Phi^w(r \wedge t)$. Consequently, $\Phi^w(t) \leq \Phi^w(s \wedge t) \vee \Phi^w(r \wedge t)$. This implies, $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(s \wedge t) \vee \Phi^w(r \wedge t)$. Hence, Φ^w qualifies as an $\mathcal{L}$-weakly $2A$-ideal in R. □

Example 3.9. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below. Let $Q = \{0, b, c, f\}$. Clearly Q is an ideal in L. Define $\mathcal{L}$-subset $\Phi^w : R \to [0, 1]$ by

$$\Phi^w(x,y) = \begin{cases} 1 & \text{if } x = 0 \textit{ and } y \in Q \\ 1/3 & \text{otherwise} \end{cases}$$

for all $(x,y) \in D \times L$. It is evident that Φ^w qualifies as an $\mathcal{L}$-ideal in R. Consequently, Φ^w is an $\mathcal{L}$-weakly $2A$-ideal in R. However, it does not meet the criteria for being an $\mathcal{L}$-weakly $1A$-prime ideal in $D \times L$, as illustrated by the instance

$$\begin{aligned}\Phi^w((0,d)\wedge(u,e)\wedge(v,f)) &= 1\\ &\nleq 1/3\\ &= \Phi^w((0,d)\wedge(u,e))\vee\Phi^w(v,f).\end{aligned}$$



The product of $\mathcal{L}$-subsets Φ^w and Ψ^w in R and G respectively is denoted by $\Phi^w \times \Psi^w$ and defined by $(\Phi^w \times \Psi^w)(a,b) = \Phi^w(a) \wedge \Psi^w(b)$, for all $(a,b) \in R \times G$.

THEOREM 3.10. *Let Φ^w and Ψ^w be $\mathcal{L}$-ideals in R and G respectively. If $\Phi^w \times \Psi^w$ is an $\mathcal{L}$-weakly $1A$-prime ideal of $R \times G$, then Φ^w and Ψ^w are $\mathcal{L}$-weakly $1A$-prime ideals in R and G respectively.*

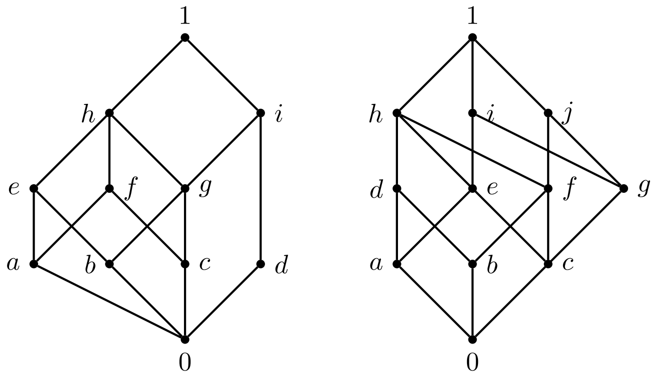
PROOF: Suppose that $\Phi^w \times \Psi^w$ is an $\mathcal{L}$-weakly $1A$-prime ideal of $R \times G$. Let $r,s,t \in R$ and $x,y,z \in G$ such that $r \wedge s \wedge t \neq 0$ and $x \wedge y \wedge z \neq 0$. Consider,

$$\begin{aligned}\Phi^w(r\wedge s\wedge t)\wedge\Psi^w(x\wedge y\wedge z) &= (\Phi^w\times\Psi^w)(r\wedge s\wedge t, x\wedge y\wedge z)\\ &= (\Phi^w\times\Psi^w)\big((r,x)\wedge(s,y)\wedge(t,z)\big)\\ &\leq (\Phi^w\times\Psi^w)\big((r,x)\wedge(s,y)\big)\vee(\Phi^w\times\Psi^w)(t,z)\\ &= \big(\Phi^w(r\wedge s)\wedge\Psi^w(x\wedge y)\big)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\\ &= \Big(\Phi^w(r\wedge s)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\ &\quad\wedge\Big(\Psi^w(x\wedge y)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\ &= \big(\Phi^w(r\wedge s)\vee\Phi^w(t)\big)\wedge\big(\Phi^w(r\wedge s)\vee\Psi^w(z)\big)\\ &\quad\wedge\big(\Psi^w(x\wedge y)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\wedge y)\vee\Psi^w(z)\big)\\ &\leq \big(\Phi^w(r\wedge s)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\wedge y)\vee\Psi^w(z)\big).\end{aligned}$$

Hence the result. □

The direct product of any two $\mathcal{L}$-weakly $1A$-prime ideals in R may not result in an $\mathcal{L}$-weakly $1A$-prime ideal in R; an illustrative example can be considered.

Example 3.11. Let $R = \{0, a, b, c, d, e, f, g, h, i, 1\}$ and $G = \{0, a, b, c, d, 6e, f, g, h, i, j, 1\}$ be the lattice represented by the Hasse diagram respectively given below:



Define $\mathcal{L}$-subsets $\Phi^w : R \to [0, 1]$ and $\Psi^w : G \to [0, 1]$, respectively as follows: $\Phi^w(0) = \Phi^w(b) = \Phi^w(c) = \Phi^w(g) = 1, \Phi^w(a) = 0.5, \Phi^w(d) = \Phi^w(e) = \Phi^w(f) = \Phi^w(h) = \Phi^w(i) = \Phi^w(1) = 0$ and $\Psi^w(0) = \Psi^w(a) = \Psi^w(b) = 1, \Psi^w(c) = \Psi^w(e) = 0.75, \Psi^w(d) = \Psi^w(f) = \Psi^w(g) = \Psi^w(h) = \Psi^w(i) = \Psi^w(j) = \Psi^w(1) = 0$. Clearly both Φ^w and Ψ^w are $\mathcal{L}$-weakly $1A$-prime ideals in R and G respectively. However, $\Phi^w \times \Psi^w$ is not $\mathcal{L}$-weakly $1A$-prime ideal in $R \times G$. This is demonstrated by considering,

$$\begin{aligned}(\Phi^w \times \Psi^w)(e \wedge f \wedge g, h \wedge i \wedge j) &= (\Phi^w \times \Psi^w)(0, c)\\ &= \Phi^w(0) \wedge \Psi^w(c)\\ &= 0.75\\ &\nleq 0.5\\ &= (\Phi^w \times \Psi^w)(e \wedge f, h \wedge i) \vee (\Phi^w \times \Psi^w)(g, j).\end{aligned}$$

Corollary 3.12. Let Φ^w and Ψ^w be $\mathcal{L}$-ideals in R and G respectively. Then Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R if and only if $\Phi^w_\beta = \Psi^w_\beta \times G$ or $\Phi^w_\beta = R \times \Psi^w_\beta$, for all $\beta \in L$.

THEOREM 3.13. *Assume R and G are ADLs, and $k : R \to G$ is a lattice homomorphism. If Ψ^w represents an $\mathcal{L}$-weakly $1A$-prime ideal in G, then $k^{-1}(\Psi^w)$ is an $\mathcal{L}$-weakly $1A$-prime ideal in R. Additionally, in the case of k being an epimorphism and Φ^w being an $\mathcal{L}$-weakly $1A$-prime ideal in R, it follows that $k(\Phi^w)$ is an $\mathcal{L}$-weakly $1A$-prime ideal in G.*

PROOF: Suppose that Ψ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in G and let k be a lattice homomorphism. Then, for all $r, s, t \in G$ such that $r \wedge s \wedge t \neq 0$,

$$\begin{aligned} k^{-1}(\Psi^w)(r \wedge s \wedge t) &= \Psi^w\big(k(r \wedge s \wedge t)\big) \\ &= \Psi^w\big(k(r) \wedge k(s) \wedge k(t)\big) \\ &\leq \Psi^w\big(k(r) \wedge k(s)\big) \vee \Psi^w(k(t)) \\ &= \Psi^w\big(k(r \wedge s)\big) \vee \Psi^w(k(t)) \\ &= k^{-1}(\Psi^w)(r \wedge s) \vee k^{-1}(\Psi^w)(t). \end{aligned}$$

Thus $k^{-1}(\Psi^w)$ is an $\mathcal{L}$-weakly $1A$-prime ideal in R. Also, let k be an isomorphism and suppose that Φ^w is an $\mathcal{L}$-weakly $1A$-prime ideal in R. Let $a, b, c \in R$ such that $a \wedge b \wedge c \neq 0$. Now, consider,

$$\begin{aligned} k(\Phi^w)(a \wedge b) \vee k(\Phi^w)(c) &= \Big[\bigvee_{a\wedge b \in k^{-1}(x\wedge y)} \Phi^w(a \wedge b)\Big] \vee \Big[\bigvee_{c \in k^{-1}(z)} \Phi^w(c)\Big] \\ &\geq \Big[\bigvee_{a\wedge b\wedge c \in k^{-1}(x\wedge y\wedge z)} \Phi^w(a \wedge b \wedge c)\Big] \\ &= k(\Phi^w)(a \wedge b \wedge c). \end{aligned}$$

Thus, $k(\Phi^w)$ is an $\mathcal{L}$-weakly $1A$-prime ideal in G. □

4. $\mathcal{L}$-weakly 1A-Prime Filters

In the subsequent discussion, we present the concepts of $\mathcal{L}$-weakly 1-absorbing prime filters and their characterizations. To begin with, let's review the definition provided in [1], stating that a proper filter H in R is a 1-absorbing prime filter (referred to as a weakly $1A$-prime filter) if, for all elements $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, the condition $r \vee s \vee t$ belonging to H implies either $r \vee s$ belonging to H or t belonging to H. Now, we aim to extend this outcome to the realm of L-weakly $1A$-prime filters as elaborated below.

Definition 4.1. A proper $\mathcal{L}$-filter Φ^w in R is an $\mathcal{L}$-weakly $1A$-prime filter in R when, for any elements r, s and t in R such that $r \vee s \vee t \neq 1$, the condition $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$ is satisfied.

Example 4.2. Let R be an ADL defined in example 3.2 with elements $\{0, r, s, t\}$, and $L = [0, 1]$. Define an $\mathcal{L}$-subset $\Phi^w : R \to L$ as follows: $\Phi^w(0) = 0$, $\Phi^w(r) = 1, \Phi^w(s) = 3/4$ and $\Phi^w(t) = 1/2$. It is evident that Φ^w is an $\mathcal{L}$-filter. Now, consider any elements $a, b, c \in R$ such that $a \vee b \vee c \neq 1$. Then $\Phi^w(a \vee b \vee c) \leq \Phi^w(a \vee b) \vee \Phi^w(c)$. Consequently, Φ^w qualifies as an $\mathcal{L}$-weakly $1A$-prime filter in R.

Subsequently, we elaborate on the notion of an $\mathcal{L}$-weakly $1A$-prime filter concerning the γ-cut.

Theorem 4.3. *Suppose Φ^w is an $\mathcal{L}$-filter in R. A filter Φ^w_γ is a weakly $1A$-prime filter in R, for all $\gamma \in L$ if and only if Φ^w qualifies as an $\mathcal{L}$-weakly $1A$-prime filter in R.*

Proof: Assume that Φ^w_γ is a weakly $1A$-prime filter for all $\gamma \in L$. In this case, for any elements $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, it follows that either $r \vee s$ is an element of $\Phi^w_{\Phi^w(r \vee s \vee t)}$ or t is an element of $\Phi^w_{\Phi^w(r \vee s \vee t)}$. This implies $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s)$ or $\Phi^w(t)$. Consequently, $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$, leading to the desired result. Conversely, assume Φ^w is an $\mathcal{L}$-weakly $1A$-prime filter. Consider $r, s, t \in R$ such that $r \vee s \vee t \neq 1$. If $r \vee s \vee t$ is an element of Φ^w_γ, then $\gamma \leq \Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$, which implies that either $\gamma \leq \Phi^w(r \vee s)$ or $\gamma \leq \Phi^w(t)$. This, in turn, means that either $r \vee s \in \Phi^w_\gamma$ or $t \in \Phi^w_\gamma$. Therefore, Φ^w_γ is a weakly $1A$-prime filter in R. □

Corollary 4.4. A filter F in R is classified as a weakly $1A$-prime filter in R iff χ_F is an $\mathcal{L}$-weakly $1A$-prime filter in R.

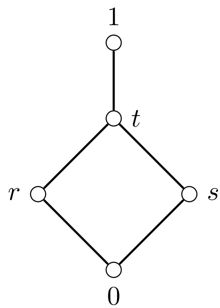
In the following discourse, we clarify the relationships between $\mathcal{L}$-weakly prime filters and $\mathcal{L}$-weakly $1A$-prime filters within an ADL.

Theorem 4.5. *Suppose Φ^w is an $\mathcal{L}$-filter in R. Then Φ^w is an $\mathcal{L}$-weakly $1A$-prime filter in R only if Φ^w is an $\mathcal{L}$-weakly prime filter in R.*

Proof: It is clear. □

In the forthcoming example, we illustrate the presence of $\mathcal{L}$-weakly $1A$-prime filters in an ADL R that do not meet the criteria for being $\mathcal{L}$-weakly prime filters in R.

Example 4.6. Consider the discrete ADL $D = \{0, u, v\}$ with 0 as its zero element, as defined in 2.2. Let $L = \{0, r, s, t, 1\}$ represent the lattice depicted in the given Hasse diagram:



Consider $D \times L = \{(d, e) \mid d \in D \text{ and } e \in L\}$. Then, the structure $(D \times L, \wedge, \vee, 0)$ forms an ADL through point-wise operations $\wedge$ and $\vee$ on $D \times L$, where 0 is represented by $(0, 0)$, the zero element in $D \times L$. Define $F = \{t, 1\}$. It is evident that F is a filter in L. Now define $\Phi^w : D \times L \to [0, 1]$ by

$$\Phi^w(d, e) = \begin{cases} 0 & \text{if } (d, e) = (0, 0) \\ 1 & \text{if } d \neq 0 \textit{ and } e \in F \\ 0.55 & \text{otherwise} \end{cases}$$

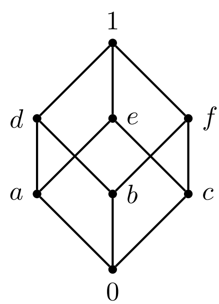
for all $(d, e) \in D \times L$. Additionally, Φ^w is an $\mathcal{L}$-filter of $D \times L$. Then $\Phi^w_1 = \{(u, t), (v, t), (u, 1), (v, 1)\}$. Consequently, Φ^w emerges as an $\mathcal{L}$-weakly $1A$-prime filter of $D \times L$. However, Φ^w does not qualify as an $\mathcal{L}$-weakly prime filter of $D \times L$, as Φ^w_1 is a weakly $1A$-prime filter of $D \times L$ but not weakly prime filter. This is demonstrated by considering, $(u, r), (v, s)$ in $D \times L$, where $(u, r) \vee (v, s) = (v, t)$ belongs to Φ^w_1 implying $(u, r) \notin \Phi^w_1$ and $(v, s) \notin \Phi^w_1$.

Definition 4.7 ([3]). A proper $\mathcal{L}$-filter Φ^w in R is an $\mathcal{L}$-weakly $2A$-filter in R if for any elements r,s and $t \in R$ such that $r \vee s \vee t \neq 1$, $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$.

Theorem 4.8. *Suppose Φ^w is an $\mathcal{L}$-filter in R. If Φ^w is an $\mathcal{L}$-weakly $1A$-prime filter in R, then Φ^w is an $\mathcal{L}$-weakly $2A$-filter in R. The converse of this result is not true.*

Proof: Let Φ^w be an $\mathcal{L}$-weakly $1A$-prime filter in R. Then, for all $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, it holds that $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$. By utilizing Theorem 2.12(1) and (3), we can deduce that $\Phi^w(t) \leq \Phi^w(t \vee s) = \Phi^w(s \vee t)$ and $\Phi^w(t) = \Phi^w(t \vee r) = \Phi^w(r \vee t)$, given that $t \leq t \vee s$ and $t \leq t \vee r$. Consequently, $\Phi^w(t) \leq \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$. This leads to the conclusion that $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$, thus establishing the desired result. □

Example 4.9. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below:



Define $\mathcal{L}$-filter $\Phi^w : R \to [0, 1]$ by

$$\Phi^w(y, z) = \begin{cases} 0 & \text{if } (y, z) = (0, 0) \\ 3/4 & \text{if } y = u \text{ and } z = 1 \\ 1/2 & \text{otherwise} \end{cases}$$

for all $(y, z) \in D \times L$. It is evident that Φ^w qualifies as an $\mathcal{L}$-weakly filter of $D \times L$. Let $H = \Phi^w_{3/4} = \{(u, 1)\}$. Notably, H emerges as a filter in $D \times L$. Consequently, Φ^w identified as an $\mathcal{L}$-weakly $2A$-filter of $D \times L$, albeit not $\mathcal{L}$-weakly $1A$-prime filter. This is demonstrated by considering any elements $(0, a), (u, c), (v, b) \in D \times L$, where $(0, a) \vee (u, c) \vee (v, b)$ belongs to H, implying $(0, a) \vee (u, c) = (u, e) \notin H$ and $(v, b) \notin H$.

Theorem 4.10. *Consider $\mathcal{L}$-weakly filters Φ^w and Ψ^w be in R and G, respectively. If the product $\Phi^w \times \Psi^w$ forms an $\mathcal{L}$-weakly $1A$-prime filter in*

$R \times G$, then both Φ^w and Ψ^w individually constitute $\mathcal{L}$-weakly $1A$-prime filters in R and G, respectively.

Proof: Assume that $\Phi^w \times \Psi^w$ is an $\mathcal{L}$-weakly $1A$-prime filter. Take $r, s, t \in R$ and $x, y, z \in G$ such that $r \vee s \vee t \neq 1$ and $x \vee y \vee z \neq 1$. Then,

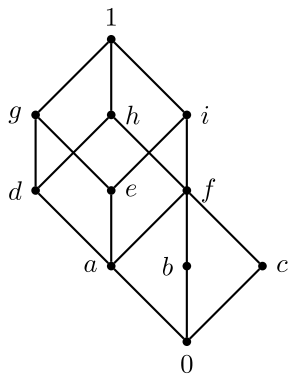
$$\begin{aligned}
\Phi^w(r \vee s \vee t) \wedge \Psi^w(x \vee y \vee z) &= (\Phi^w \times \Psi^w)(r \vee s \vee t, x \vee y \vee z) \\
&= (\Phi^w \times \Psi^w)\big((r,x) \vee (s,y) \vee (t,z)\big) \\
&\leq (\Phi^w \times \Psi^w)\big((r,x) \vee (s,y)\big) \vee (\Phi^w \times \Psi^w)(t,z) \\
&= \Big(\Phi^w(r \vee s) \wedge \Psi^w(x \vee y)\Big) \vee \Big(\Phi^w(t) \wedge \Psi^w(z)\Big) \\
&= \Big(\Phi^w(r \vee s) \vee \big(\Phi^w(t) \wedge \Psi^w(z)\big)\Big) \\
&\quad \wedge \Big(\Psi^w(x \vee y) \vee \big(\Phi^w(t) \wedge \Psi^w(z)\big)\Big) \\
&= \big(\Phi^w(r \vee s) \vee \Phi^w(t)\big) \wedge \big(\Phi^w(r \vee s) \vee \Psi^w(z)\big) \\
&\quad \wedge \big(\Psi^w(x \vee y) \vee \Phi^w(t)\big) \wedge \big(\Psi^w(x \vee y) \vee \Psi^w(z)\big) \\
&\leq \big(\Phi^w(r \vee s) \vee \Phi^w(t)\big) \wedge \big(\Psi^w(x \vee y) \vee \Psi^w(z)\big).
\end{aligned}$$

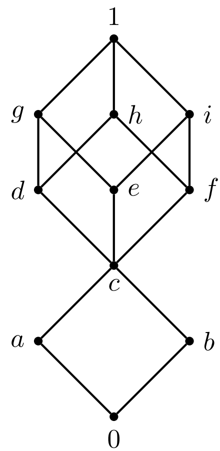
Hence the result. □

The presence of $\mathcal{L}$-weakly $1A$-prime filters does not guarantee that their direct product will be an $\mathcal{L}$-weakly $1A$-prime filter. An example demonstrating this is provided below.

Example 4.11. Let $R = \{0, a, b, c, d, e, f, g, h, i, 1\}$ and $G = \{0, a, b, c, d, e, f, g, h, i, 1\}$ be the lattice represented by the Hasse diagram respectively given below:

Define $\mathcal{L}$-subsets Φ^w and Ψ^w in R and G, respectively such that for Φ^w: $\Phi^w(0) = \Phi^w(a) = 0$, $\Phi^w(b) = 1/3$, $\Phi^w(c) = 0$, $\Phi^w(d) = \Phi^w(e) = \Phi^w(g) = 3/5, \Phi^w(f) = 1, \Phi^w(h) = 3/5, \Phi^w(i) = 3/5, \Phi^w(1) = 1$ and for Ψ^w: $\Psi^w(0) = \Psi^w(a) = \Psi^w(b) = 0$, $\Psi^w(c) = \Psi^w(d) = \Psi^w(e) = \Psi^w(f) = 1/2, \Psi^w(i) = \Psi^w(g) = \Psi^w(h) = \Psi^w(1) = 1$. Clearly, both Φ^w and Ψ^w are $\mathcal{L}$-weakly $1A$-prime filters in R and G, respectively. However, the direct product $\Phi^w \times \Psi^w$ is not $\mathcal{L}$-weakly $1A$-prime filter in $R \times G$, as evidenced by the example where





$$\begin{aligned}(\Phi^w \times \Psi^w)(d \vee e \vee f, d \vee e \vee f) &= (\Phi^w \times \Psi^w)(1,1)\\ &= 1\\ &\nleq 3/5\\ &= (\Phi^w \times \Psi^w)(d \vee e, d \vee e) \vee (\Phi^w \times \Psi^w)(f,f).\end{aligned}$$

Corollary 4.12. Let Φ^w and Ψ^w be $\mathcal{L}$-filters in R and G, respectively, and for all $\beta \in L$. Then Φ^w is an $\mathcal{L}$-weakly $1A$-prime filter in R if and only if $\Phi^w_\beta = \Psi^w_\beta \times G$ or $\Phi^w_\beta = R \times \Psi^w_\beta$, where Φ^w_β and Ψ^w_β are weakly $1A$-prime filter in R and G respectively.

Lastly, we explore the homomorphism of $\mathcal{L}$-weakly $1A$-prime filters in ADLs.

Theorem 4.13. *Consider ADLs R and G, with a lattice homomorphism $k : R \to G$. Then $k^{-1}(\Psi^w)$ is an $\mathcal{L}$-weakly $1A$-prime filter in R only if Ψ^w is an $\mathcal{L}$-weakly $1A$-prime filter in G. Additionally, if k is an epimorphism and Φ^w is an $\mathcal{L}$-weakly $1A$-prime filter in R, then $k(\Phi^w)$ is an $\mathcal{L}$-weakly $1A$-prime filter in G.*

Proof: Let $k : R \to G$ be a lattice homomorphism. Suppose that Ψ^w is an $\mathcal{L}$-weakly $1A$-prime filter in G. For all $r, s, t \in G$ such that $r \vee s \vee t \neq 1$. Then

$$\begin{aligned} k^{-1}(\Psi^w)(r \vee s \vee t) &= \Psi^w\big(k(r \vee s \vee t)\big) \\ &= \Psi^w\big(k(r) \vee k(s) \vee k(t)\big) \\ &\leq \Psi^w\big(k(r) \vee k(s)\big) \vee \Psi^w(k(t)) \\ &= \Psi^w\big(k(r \vee s)\big) \vee \Psi^w(k(t)) \\ &= k^{-1}(\Psi^w)(r \vee s) \vee k^{-1}(\Psi^w)(t). \end{aligned}$$

Thus $k^{-1}(\Psi^w)$ is an $\mathcal{L}$-weakly $1A$-prime filter in R. Let k be an isomorphism and suppose that Φ^w be an $\mathcal{L}$-weakly $1A$-prime filter in R. For all $a, b, c \in R$ such that $a \vee b \vee c \neq 1$. Now, consider,

$$\begin{aligned} k(\Phi^w)(a \vee b) \vee k(\Phi^w)(c) &= \Big[\bigvee_{a \vee b \in k^{-1}(x \wedge y)} \Phi^w(a \vee b)\Big] \vee \Big[\bigvee_{c \in k^{-1}(z)} \Phi^w(c)\Big] \\ &\geq \Big[\bigvee_{a \vee b \vee c \in k^{-1}(x \wedge y \wedge z)} \Phi^w(a \vee b \vee c)\Big] \\ &= k(\Phi^w)(a \vee b \vee c). \end{aligned}$$

Thus, $g(\Phi^w)$ is an $\mathcal{L}$-weakly $1A$-prime filter in G. $\square$

5. Conclusion

This study concentrates on investigating $\mathcal{L}$-weakly $1A$-prime ideals and filters within an ADL, constituting a pivotal aspect of our research. We delve into the characteristics of these elements, exploring their properties. Furthermore, we elucidate the connection between $\mathcal{L}$-weakly prime filters (ideals) and $\mathcal{L}$-weakly $1A$-prime filters (ideals) in ADLs. Notably, we offer examples to illustrate instances where the converse relationship may not be applicable.

Author contribution statement. I affirm that I am the exclusive author of this work, and I have not consulted any sources other than those explicitly cited in the references. Additionally, I confirm that this manuscript has not been submitted to any other journal for publication.

Data availability. No data were used to support this study.

Conflicts of interest. The author(s) declare(s) that there are no conflicts of interest regarding the publication of this paper.

Funding statement. No grants or support received.

Acknowledgements. The author wishes to express their sincere appreciation to the reviewers for their insightful comments and suggestions, which have greatly enhanced the paper's presentation.

References

[1] N. T. Amare, *On Weakly 1-absorbing Prime Filters in ADLs*, **Southeast Asian Bulletin of Mathematics**, (2023), accepted.

[2] N. T. Amare, *Fuzzy 2-absorbing Filters in an ADL*, **Advances in Fuzzy Systems**, (2024), communicated.

[3] N. T. Amare, *L-fuzzy Weakly 2-absorbing filters in an ADL*, **International Journal of Mathematics and Mathematical Sciences**, (2024), communicated.

[4] N. T. Amare, N. Rafi, S. Rao, *L-fuzzy Weakly 2-absorbing Ideals in an Almost Distributive Lattice*, **Palestine Journal of Mathematics**, (2024), accepted.

[5] N. T. Amare, N. Rao, *L-fuzzy 2-absorbing Ideals in an ADL*, **Journal of Applied Mathematics and Informatics**, (2024), accepted.

[6] D. D. Anderson, E. Smith, *Weakly prime ideals*, **Houston Journal of Mathematics**, vol. 29(4) (2003), p. 831.

[7] A. Badawi, *On 2-absorbing ideals of commutative rings*, **Bulletin of the Australian Mathematical Society**, vol. 75(3) (2007), pp. 417–429, DOI: https://doi.org/10.1017/S0004972700039344.

[8] A. Badawi, A. Y. Darani, *On weakly 2-absorbing ideals of commutative rings*, **Houston Journal of Mathematics**, vol. 39(2) (2013), pp. 441–452.

[9] J. N. Chaudhari, *2-absorbing ideals in semirings*, **International Journal of Algebra**, vol. 6(6) (2012), pp. 265–270, URL: https://www.m-hikari.com/ija/ija-2012/ija-5-8-2012/chaudhariIJA5-8-2012-1.pdf.

[10] A. Y. Darani, G. Ghasemi, *On L-fuzzy 2-absorbing ideals*, **Italian Journal of Pure and Applied Mathematics**, vol. 36 (2016), pp. 147–154, URL: https://ijpam.uniud.it/online_issue/201636/12-DaraniGhasemi.pdf.

[11] M. K. Dubey, *Prime and weakly prime ideals in semirings*, **Quasigroups and Related Systems**, vol. 20 (2012), pp. 151–156, URL: https://www.math.md/files/qrs/v20-n2/v20-n2-(pp197-202).pdf.

[12] J. A. Goguen, *L-fuzzy sets*, **Journal of Mathematical Analysis and Applications**, vol. 18(1) (1967), pp. 145–174, DOI: https://doi.org/10.1016/0022-247X(67)90189-8.

[13] Y. B. Jun, *On fuzzy prime ideals of* Γ*-rings*, **Soochow Journal of Mathematics**, vol. 21(1) (1995), pp. 41–48.

[14] D. Mandal, *On 2-absorbing fuzzy ideals of commutative semirings*, **TWMS Journal of Applied and Engineering Mathematics**, vol. 11(2) (2021), p. 368, URL: https://jaem.isikun.edu.tr/web/images/articles/vol.11.no.2/06.pdf.

[15] S. K. Nimbhorkar, Y. S. Patil, *Fuzzy weakly 2-absorbing ideals of a lattice*, **Discussiones Mathematicae – General Algebra and Applications**, vol. 42(2) (2022), pp. 255–277, DOI: https://doi.org/10.7151/dmgaa.1389.

[16] S. K. Nimbhorkar, Y. S. Patil, *Fuzzy Weakly 2-Absorbing Ideals of a Lattice*, **Discussiones Mathematicae – General Algebra and Applications**, vol. 42(2) (2022), pp. 255–277, DOI: https://doi.org/10.7151/dmgaa.1389.

[17] C. S. S. Raj, N. T. Amare, U. M. Swamy, *Fuzzy Prime Ideals of ADL's*, **International Journal of Computing Science and Applied Mathematics**, vol. 4(2) (2018), pp. 32–36, URL: https://iptek.its.ac.id/index.php/ijcsam/article/download/3187/2924.

[18] T. R. Sharma, R. Sharma, *Completely Prime Fuzzy Ideals of* Γ*-Semi-rings*, **International Journal of Pure and Applied Mathematical Sciences**, vol. 16(1) (2023), pp. 31–41, DOI: https://doi.org/10.37622/IJPAMS/16.1.2023.31-41.

[19] C. S. Sundar Raj, N. T. Amare, U. Swamy, *Prime and Maximal Fuzzy Filters of ADLs*, **Palestine Journal of Mathematics**, vol. 9(2) (2020), URL: https://pjm.ppu.edu/sites/default/files/papers/PJM_May2020_730_to_739.pdf.

[20] U. Swamy, C. S. S. Raj, A. N. Teshale, *Fuzzy ideals of almost distributive lattices*, **Annals of Fuzzy Mathematics and Informatics**, vol. 14(4) (2017), pp. 371–379, URL: http://hdl.handle.net/10603/367770.

[21] U. Swamy, C. S. S. Raj, A. N. Teshale, *L-fuzzy filters of almost distributive lattices*, **International Journal of Mathematical Sciences and Computing**, vol. 8(1) (2018), pp. 35–43, DOI: https://doi.org/10.26708/IJMSC.2017.1.8.05.

[22] U. M. Swamy, G. C. Rao, *Almost distributive lattices*, **Journal of the Australian Mathematical Society**, vol. 31(1) (1981), pp. 77–91, DOI: https://doi.org/10.1017/S1446788700018498.

[23] N. Teshale Amare, *Weakly 2-Absorbing Ideals in Almost Distributive Lattices*, **Journal of Mathematics**, vol. 2022(1) (2022), p. 9252860, DOI: https://doi.org/10.1155/2022/9252860.

[24] M. P. Wasadikar, K. T. Gaikwad, *On 2-absorbing and weakly 2-absorbing ideals of lattices*, **Mathematical Sciences International Research Journal**, vol. 4(2) (2015), pp. 82–85.

[25] L. A. Zadeh, *Fuzzy sets*, **Information and Control**, vol. 8(3) (1965), pp. 338–353, DOI: https://doi.org/10.1016/S0019-9958(65)90241-X.

Natnael Teshale Amare
University of Gondar
Department of Mathematics
Ethiopia
e-mail: yenatnaelteshale@gmail.com