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$\mathcal{L}$-WEAKLY 1-ABSORBING PRIME IDEALS AND FILTERS

Abstract

In this manuscript, we have presented the concept of $\mathcal{L}-$weakly 1-absorbing prime ideals and $\mathcal{L}-$weakly 1-absorbing prime filters within an ADL. Mainly, we illustrate the connections between $\mathcal{L}-$weakly prime ideals (filters) and $\mathcal{L}-$weakly 1-absorbing prime ideals (filters), as well as between $\mathcal{L}-$weakly 1-absorbing prime ideals (filters) and $\mathcal{L}-$weakly 2$-$absorbing ideals (filters). Lastly, we have shown that both the image and inverse image of $\mathcal{L}-$weakly 1-absorbing prime ideals (filters) result in $\mathcal{L}-$weakly 1-absorbing prime ideals (filters).

Keywords: Weakly 1-absorbing prime ideal, weakly 1$-$absorbing prime filter, $\mathcal{L}-$weakly 2$-$absorbing ideal, $\mathcal{L}-$weakly 1$-$absorbing prime ideal, $\mathcal{L}-$weakly 2$-$absorbing filter, $\mathcal{L}-$weakly 1$-$absorbing prime filter.

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1. Introduction

The idea of prime ideals(filters) is vital in the study of structure theory of distributive lattices in general and in particular, that of Boolean algebras. Badawi [7] introduced the concept of 2-absorbing ideals in commutative rings, extending the idea of prime ideals from [11]. Chuadhari [9] further extended 2-absorbing ideals to semi-rings. Badawi and Darani [8] introduced weakly 2-absorbing ideals in commutative rings, a generalization of weakly prime ideals by Anderson and Smith [6]. Wasakidar and Gaikerad [24] extended the concepts of 2-absorbing and weakly 2-absorbing ideals

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to lattices. Natnael TA [23, 2, 1] introduced weakly 2-absorbing ideals and weakly 2-absorbing filters, along with 1-absorbing prime filters in an ADL. L.A. Zadeh [25] defined a fuzzy subset of a set X as a function mapping elements to real numbers in $[0, 1]$. Goguen [12] expanded this concept by using a complete lattice $\mathcal{L}$ instead of the valuation set $[0, 1]$, aiming for a more comprehensive exploration of fuzzy set theory through fuzzy sets. Darani and Ghasemi [10], as well as Mandal [14], introduced fuzzy 2-absorbing ideals and 2-absorbing fuzzy ideals for commutative rings, respectively, generalizing the concept of fuzzy prime ideals in rings explored by June [13] and Sharma [18]. Nimbhorkar and Patil [15, 16] introduced fuzzy weakly 2-absorbing ideals in lattices. In our previous work [20, 21], we introduced the concepts of fuzzy ideals and filters within an ADL, serving as the basis for our research. Natnael [5, 2] later expanded on this by introducing the concept of fuzzy 2-absorbing ideals and filters in an ADL.

In this paper, we have introduced the concept of $\mathcal{L}-$weakly $1A-$prime ideals and filters in an ADL, aiming to extend the idea of $\mathcal{L}-$prime ideals and filters in an ADL as presented in [17, 19]. Initially, we define $\mathcal{L}-$weakly $1A-$prime ideals, which are less stringent than $\mathcal{L}-$prime ideals. Also, we study on $\mathcal{L}-$weakly $1A-$prime filters in an ADL which is weaker than that $\mathcal{L}-$prime filters. Our main emphasis is on investigating the connections between $\mathcal{L}-$prime ideals and $\mathcal{L}-$weakly $1A-$prime ideals, as well as the relationships between $\mathcal{L}-$weakly $1A-$prime ideals and $\mathcal{L}-2A-$ideals. Also, we investigating the connections between $\mathcal{L}-$prime filters and $\mathcal{L}-$weakly $1A-$prime filters, and $\mathcal{L}-$weakly $1A-$prime filters and $\mathcal{L}-2A-$filters. Counter examples are provided to demonstrate that the converses of these relationships do not hold. Furthermore, we demonstrate that the direct product of any two $\mathcal{L}-$prime ideals ($\mathcal{L}-$prime filters) results in an $\mathcal{L}-$weakly $1A-$prime ideal($\mathcal{L}-$weakly $1A-$prime filter) in an ADL. However, it is important to note that the product of $\mathcal{L}-$weakly $1A-$prime ideals($\mathcal{L}-$weakly $1A-$prime filters) may not necessarily yield an $\mathcal{L}-$weakly $1A-$prime ideal($\mathcal{L}-$weakly $1A-$prime filter) in an ADL. Additionally, we establish that both the image and pre-image of any $\mathcal{L}-$weakly $1A-$prime ideals ($\mathcal{L}-$weakly $1A-$prime filters) are again $\mathcal{L}-$weakly $1A-$prime ideals($\mathcal{L}-$weakly $1A-$prime filters).

2. Preliminaries

In this portion, we revisit certain definitions and fundamental findings primarily sourced from [20, 17, 22].

Definition 2.1. An algebra $R = (R, \wedge, \vee, 0)$ of type $(2, 2, 0)$ is referred to as an ADL if it meets the subsequent conditions for all r, s and t in R.

1. $0 \wedge r = 0$
2. $r \vee 0 = r$
3. $r \wedge (s \vee t) = (r \wedge s) \vee (r \wedge t)$
4. $r \vee (s \wedge t) = (r \vee s) \wedge (r \vee t)$
5. $(r \vee s) \wedge t = (r \wedge t) \vee (s \wedge t)$
6. $(r \vee s) \wedge s = s$.

Every distributive lattice with a lower bound is categorized as an ADL.

Example 2.2. For any nonempty set A, it's possible to transform it into an ADL that doesn't constitute a lattice by selecting any element 0 from A and fixing an arbitrary element $u_0 \in R$. For every $u, v \in R$, define $\wedge$ and $\vee$ on R as follows:

$$u \wedge v = \begin{cases} v & \text{if } u \neq u_0 \\ u_0 & \text{if } u = u_0 \end{cases} \qquad \text{and} \qquad u \vee v = \begin{cases} u & \text{if } u \neq u_0 \\ v & \text{if } u = u_0 \end{cases}$$

Then $(A, \wedge, \vee, u_0)$ is an ADL (called the **discrete ADL**) with u_0 as its zero element.

Definition 2.3. Consider $R = (R, \wedge, \vee, 0)$ be an ADL. For any r and $s \in R$, establish $r \leq s$ if $r = r \wedge s$ (which is equivalent to $r \vee s = s$). Then $\leq$ is a partial order on R with respect to which 0 is the smallest element in R.

Theorem 2.4. *The following conditions are valid for any r, s and t in an ADL R.*

(1) $r \wedge 0 = 0 = 0 \wedge r$ *and* $r \vee 0 = r = 0 \vee r$

(2) $r \wedge r = r = r \vee r$

(3) $r \wedge s \leq s \leq s \vee r$

(4) $r \wedge s = r$ *iff* $r \vee s = s$

(5) $r \wedge s = s$ *iff* $r \vee s = r$

(6) $(r \wedge s) \wedge t = r \wedge (s \wedge t)$ *(in other words, $\wedge$ is associative)*

(7) $r \vee (s \vee r) = r \vee s$

(8) $r \leq s \Rightarrow r \wedge s = r = s \wedge r$ $\big($ *iff* $r \vee s = s = s \vee r\big)$

(9) $(r \wedge s) \wedge t = (s \wedge r) \wedge t$

(10) $(r \vee s) \wedge t = (s \vee r) \wedge t$

(11) $r \wedge s = s \wedge r$ *iff* $r \vee s = s \vee r$

(12) $r \wedge s = \inf\{r, s\}$ *iff* $r \wedge s = s \wedge r$ *iff* $r \vee s = \sup\{r, s\}$.

Definition 2.5. Let R and G be ADLs and form the set $R \times G = \{(r, g) : r \in R \text{ and } g \in G\}$. For all $(r_1, g_1), (r_2, g_2) \in R \times G$, define $\wedge$ and $\vee$ in $R \times G$ by $(r_1, g_1) \wedge (r_2, g_2) = (r_1 \wedge r_2, g_1 \wedge g_2)$ and $(r_1, g_1) \vee (r_2, g_2) = (r_1 \vee r_2, g_1 \vee g_2)$. Then $(R \times G, \wedge, \vee, 0)$ is an ADL under the pointwise operations and $0 = (0, 0)$ is the zero element in $R \times G$.

Definition 2.6. A non-empty subset, denoted as F in an ADL R is termed an ideal (filter) in R if it satisfies the conditions: if u and v belong to F, then $u \vee v$ ($u \wedge v$) is also in F, and for every element r in R, the $u \wedge r$ ($r \vee u$) is in F.

Definition 2.7. A proper ideal(filter) F in R is a prime ideal (filter) if for any u and v belongs R, $u \wedge v$ ($u \vee v$) belongs F, then either u belongs F or v belongs F.

Definition 2.8. Let R and G be ADLs. A mapping $k : R \to G$ is called a homomorphism if the following are satisfied, for any $r, s, t \in R$.
(1). $k(r \wedge s \wedge t) = k(r) \wedge k(s) \wedge k(t)$
(2). $k(r \vee s \vee t) = k(r) \vee k(s) \vee k(t)$
(3). $k(0) = 0$.

DEFINITION 2.9. An $\mathcal{L}-$subset Φ^w is defined as a mapping from R to a complete lattice L that adheres to the infinite meet distributive law. When the lattice L is represented by the unit interval $[0,1]$ of real numbers, these $\mathcal{L}-$subsets correspond to the conventional notion of $\mathcal{L}-$subsets in R.

DEFINITION 2.10. An $\mathcal{L}-$subset Φ^w is an $\mathcal{L}-$ideal(filter) in R, if $\Phi^w(0) = 1\big(\Phi^w(u) = 1$, for any maximal element u in $R\big)$ and $\Phi^w(r \vee s) = \Phi^w(r) \wedge \Phi^w(s)$ $\big(\Phi^w(r \wedge s) = \Phi^w(r) \wedge \Phi^w(s)\big)$, for all r and s belongs to R.

THEOREM 2.11. *Let Φ^w be an $\mathcal{L}-$ideal and $\emptyset \neq F \subseteq R$. Then for any r and s belongs to R, we have the following:*

(1) *If $r \leq s$, then $\Phi^w(s) \leq \Phi^w(r)$*

(2) *If r is an associate with s, then $\Phi^w(r) = \Phi^w(s)$*

(3) $\Phi^w(r \wedge s) = \Phi^w(s \wedge r)$ *and* $\Phi^w(r \vee s) = \Phi^w(s \vee r)$

(4) *If $r \in \langle F]$, then $\bigwedge\limits_{i=1}^{n} \Phi^w(x_i) \leq \Phi^w(r)$, for some $x_1, x_2, ..., x_n \in F$*

(5) *If $r \in \langle s]$, then $\Phi^w(s) \leq \Phi^w(r)$*

(6) *If u is maximal in R, then $\Phi^w(u) \leq \Phi^w(r)$*

(7) $\Phi^w(u) = \Phi^w(v)$, *for any maximal elements u and v in R.*

THEOREM 2.12. *Let Φ^w be an $\mathcal{L}-$filter and $\emptyset \neq F \subseteq R$. Then for any $r, s \in R$, we have the following.*

(1) *If $r \leq s$, then $\Phi^w(r) \leq \Phi^w(s)$*

(2) *If $r \sim s$, then $\Phi^w(r) = \Phi^w(s)$*

(3) $\Phi^w(r \vee s) = \Phi^w(s \vee r)$

(4) *If $r \in [F\rangle$, then $\bigwedge\limits_{i=1}^{n} \Phi^w(x_i) \leq \Phi^w(r)$, for some $x_1, x_2, ..., x_n \in F$*

(5) *If $r \in [s\rangle$, then $\Phi^w(s) \leq \Phi^w(r)$.*

DEFINITION 2.13. A proper $\mathcal{L}-$ideal(filter) Φ^w is referred to as a prime $\mathcal{L}-$ideal(filter) if $\psi \wedge \eta \leq \Phi^w$ implies either $\psi \leq \Phi^w$ or $\eta \leq \Phi^w$, for any $\mathcal{L}-$ideals(filters) ψ and η in R.

DEFINITION 2.14. A proper $\mathcal{L}-$ideal(filter) Φ^w is an $\mathcal{L}-$prime ideal(filter) in R if $\Phi^w(r \wedge s)\Big(\Phi^w(r \vee s)\Big)$ equals either $\Phi^w(r)$ or $\Phi^w(s)$, for any r and s in R.

3. $\mathcal{L}-$weakly $1A-$prime ideals

In the subsequent discussion, we present the concepts of $\mathcal{L}-$weakly $1-$absorbing prime ideals in an ADL R and their characterizations. Initially, let us revisit the definition outlined in [23], indicating that a proper ideal H in R is a weakly $1-$absorbing prime ideal (in short, a weakly $1A-$prime ideal) in R if, for all elements r, s, and t in R such that $r \wedge s \wedge t \neq 0$, the condition $r \wedge s \wedge t$ belonging to H implies either $r \wedge s$ belonging to H or t belonging to H. Now, we aim to extend this outcome to the realm of $\mathcal{L}-$weakly $1A-$prime ideals as elucidated below.

Definition 3.1. A proper $\mathcal{L}-$ideal Φ^w in R is referred to as an $\mathcal{L}-$weakly $1A-$prime ideal in R if for any elements r,s and t belongs to R such that $r \wedge s \wedge t \neq 0$, the inequality $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$ remains valid.

Example 3.2. Let $R = \{0, r, s, t\}$ and the chain L consisting of four elements $\{0, \gamma, \beta, 1\}$, where $0 < \gamma < \beta < 1$ and let $\vee$ and $\wedge$ be binary operations on R defined by:

$\vee$	0	r	s	t
0	0	r	s	t
r	r	r	r	r
s	s	s	s	s
t	t	r	s	t

$\wedge$	0	r	s	t
0	0	0	0	0
r	0	r	s	t
s	0	r	s	t
t	0	t	t	t

Define an $\mathcal{L}-$subset Φ^w in R as follows: $\Phi^w(0) = 1$, $\Phi^w(r) = \gamma = \Phi^w(s)$ and $\Phi^w(t) = \beta$. It is evident that Φ^w is an $\mathcal{L}-$ideal in R. Furthermore, for any elements r, s and $t \in R$ such that $r \wedge s \wedge t = t \neq 0$, we observe that $\Phi^w(r \wedge s \wedge t) = \beta = \gamma \vee \beta = \Phi^w(r \wedge s) \vee \Phi^w(t)$. Consequently, Φ^w qualifies as an $\mathcal{L}-$weakly $1A-$prime ideal in R.

Following that, we define the concept of an $\mathcal{L}-$weakly $1A-$prime ideal with respect to β-cut, where $\Phi^w_\beta = \{r \in R : \beta \leq \Phi^w(r)\}$.

Theorem 3.3. *Let Φ^w be an $\mathcal{L}-$ideal in R. Then an ideal Φ^w_β is a weakly $1A-$prime ideal in R, for all $\beta \in L$ iff Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R.*

Proof: Assume Φ^w_β is a weakly $1A-$prime ideal, for all $\beta \in L$. In this case, for any elements $r, s, t \in R$ such that $r \wedge s \wedge t \neq 0$, it is ensured that either $r \wedge s \in \Phi^w_{\Phi^w(r \wedge s \wedge t)}$ or $t \in \Phi^w_{\Phi^w(r \wedge s \wedge t)}$, leading to $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s)$ or

$\Phi^w(t)$. Consequently, $\Phi^w(r\wedge s\wedge t) \leq \Phi^w(r\wedge s)\vee\Phi^w(t)$. Conversely, if Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal, consider $r, s, t \in R$ such that $r\wedge s\wedge t \in \Phi^w_\beta$, for all $\beta \in L$. This implies $\beta \leq \Phi^w(r \wedge s \wedge t)$, which further leads to $\beta \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. Consequently, either $\beta \leq \Phi^w(r \wedge s)$ or $\beta \leq \Phi^w(t)$. Hence, either $r \wedge s \in \Phi^w_\beta$ or $t \in \Phi^w_\beta$. Therefore, Φ^w_β is a weakly $1A-$prime ideal in R. □

Corollary 3.4. An ideal P in R is classified as a weakly $1A-$prime ideal in R iff its characteristic set χ_P is an $\mathcal{L}-$weakly $1A-$prime ideal in R.

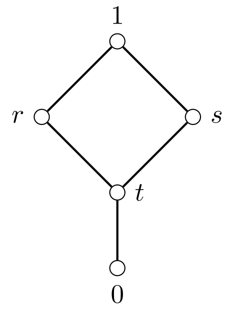
In the upcoming theorems, we establish the connections between $\mathcal{L}-$weakly $1A-$prime ideals and both $\mathcal{L}-$weakly prime ideals and $\mathcal{L}-$weakly $2A-$ideals within the context of an ADL.

Theorem 3.5. *Let Φ^w be an $\mathcal{L}-$ideal in R. Then Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R only if Φ^w is an $\mathcal{L}-$weakly prime ideal in R.*

Proof: Assume Φ^w is an $\mathcal{L}-$weakly prime ideal in R. For any elements $r, s, t \in R$ such that $r \wedge s \wedge t \neq 0$, it follows that $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r) \vee \Phi^w(s \wedge t)$, or $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. This establishes the conclusion. □

In the provided example, we demonstrate that every $\mathcal{L}-$weakly $1A-$prime ideal in R does not qualify as an $\mathcal{L}-$weakly prime ideals in R.

Example 3.6. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, r, s, t, 1\}$ be the lattice represented by the Hasse diagram given below:



Examine the set $D \times L = \{(y, z) \mid y \in D \text{ and } z \in L\}$. Then, the structure $(D \times L, \wedge, \vee, 0)$ forms an ADL, employing pointwise operations $\wedge$ and $\vee$ on

$D \times L$, where 0 is defined as $(0,0)$. Consider $P = \{0, t\}$. It is evident that P is an ideal in L. Now define $\Phi^w : D \times L \to [0,1]$ by

$$\Phi^w(y,z) = \begin{cases} 1 & \text{if } (y,z) = (0,0) \\ 3/4 & \text{if } y \neq 0 \textit{ and } z \in P \\ 0 & \text{otherwise} \end{cases}$$

for all $(y,z) \in D \times L$. Moreover, Φ^w is identified as an $\mathcal{L}-$ideal. Consequently, Φ^w qualifies as an $\mathcal{L}-$weakly $1A-$prime ideal, while Φ^w does not meet the criteria for an $\mathcal{L}-$weakly prime ideal in $D \times L$. This distinction arises from the fact that $\Phi^w((u,r) \wedge (v,s)) = 3/4 \nleq 0$ whereas $\Phi^w(u,r) \vee \Phi^w(v,s)$ results in 0.

Definition 3.7 ([4]). A proper $\mathcal{L}-$ideal Φ^w in R is an $\mathcal{L}-$weakly $2A-$ideal in R if for any elements r,s and $t \in R$ such that $r \wedge s \wedge t \neq 0$, $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(r \wedge t) \vee \Phi^w(s \wedge t)$.

Theorem 3.8. *Let Φ^w be an $\mathcal{L}-$ideal in R. If Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R, then Φ^w is an $\mathcal{L}-$weakly $2A-$ideal in R. The converse of this result is not true.*

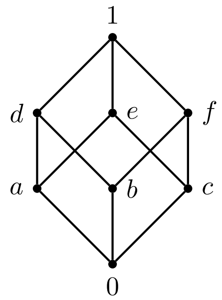
Proof: Assume Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R. Then for all $r, s, t \in R$ such that $r \wedge s \wedge t \neq 0$, it follows that $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(t)$. By theorem 2.11(1) and (3), we deduce $\Phi^w(t) \leq \Phi^w(t \wedge s) = \Phi^w(s \wedge t)$ and $\Phi^w(t) \leq \Phi^w(t \wedge r) = \Phi^w(r \wedge t)$. Consequently, $\Phi^w(t) \leq \Phi^w(s \wedge t) \vee \Phi^w(r \wedge t)$. This implies, $\Phi^w(r \wedge s \wedge t) \leq \Phi^w(r \wedge s) \vee \Phi^w(s \wedge t) \vee \Phi^w(r \wedge t)$. Hence, Φ^w qualifies as an $\mathcal{L}-$weakly $2A-$ideal in R. □

Example 3.9. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below. Let $Q = \{0, b, c, f\}$. Clearly Q is an ideal in L. Define $\mathcal{L}-$subset $\Phi^w : R \to [0,1]$ by

$$\Phi^w(x,y) = \begin{cases} 1 & \text{if } x = 0 \textit{ and } y \in Q \\ 1/3 & \text{otherwise} \end{cases}$$

for all $(x,y) \in D \times L$. It is evident that Φ^w qualifies as an $\mathcal{L}-$ideal in R. Consequently, Φ^w is an $\mathcal{L}-$weakly $2A-$ideal in R. However, it does not meet the criteria for being an $\mathcal{L}-$weakly $1A-$prime ideal in $D \times L$, as illustrated by the instance

$$\Phi^w((0,d)\wedge(u,e)\wedge(v,f)) = 1 \nleq 1/3 = \Phi^w((0,d)\wedge(u,e))\vee\Phi^w(v,f).$$



The product of $\mathcal{L}-$subsets Φ^w and Ψ^w in R and G respectively is denoted by $\Phi^w\times\Psi^w$ and defined by $(\Phi^w\times\Psi^w)(a,b)=\Phi^w(a)\wedge\Psi^w(b)$, for all $(a,b)\in R\times G$.

THEOREM 3.10. *Let Φ^w and Ψ^w be $\mathcal{L}-$ideals in R and G respectively. If $\Phi^w\times\Psi^w$ is an $\mathcal{L}-$weakly $1A-$prime ideal of $R\times G$, then Φ^w and Ψ^w are $\mathcal{L}-$weakly $1A-$prime ideals in R and G respectively.*

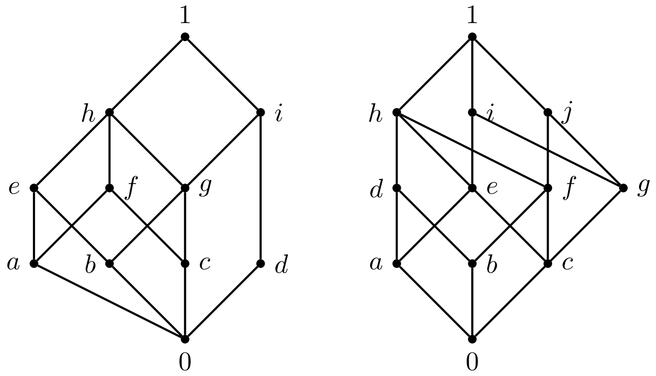
PROOF: Suppose that $\Phi^w\times\Psi^w$ is an $\mathcal{L}-$weakly $1A-$prime ideal of $R\times G$. Let $r,s,t\in R$ and $x,y,z\in G$ such that $r\wedge s\wedge t\neq 0$ and $x\wedge y\wedge z\neq 0$. Consider,

$$\begin{aligned}
\Phi^w(r\wedge s\wedge t)\wedge\Psi^w(x\wedge y\wedge z) =&(\Phi^w\times\Psi^w)(r\wedge s\wedge t, x\wedge y\wedge z)\\
=&(\Phi^w\times\Psi^w)\big((r,x)\wedge(s,y)\wedge(t,z)\big)\\
\leq&(\Phi^w\times\Psi^w)\big((r,x)\wedge(s,y)\big)\vee(\Phi^w\times\Psi^w)(t,z)\\
=&\big(\Phi^w(r\wedge s)\wedge\Psi^w(x\wedge y)\big)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\\
=&\Big(\Phi^w(r\wedge s)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\
&\wedge\Big(\Psi^w(x\wedge y)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\
=&\big(\Phi^w(r\wedge s)\vee\Phi^w(t)\big)\wedge\big(\Phi^w(r\wedge s)\vee\Psi^w(z)\big)\\
&\wedge\big(\Psi^w(x\wedge y)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\wedge y)\vee\Psi^w(z)\big)\\
\leq&\big(\Phi^w(r\wedge s)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\wedge y)\vee\Psi^w(z)\big).
\end{aligned}$$

Hence the result. □

The direct product of any two $\mathcal{L}-$weakly $1A-$prime ideals in R may not result in an $\mathcal{L}-$weakly $1A-$prime ideal in R; an illustrative example can be considered.

Example 3.11. Let $R = \{0, a, b, c, d, e, f, g, h, i, 1\}$ and $G = \{0, a, b, c, d,$ $6e, f, g, h, i, j, 1\}$ be the lattice represented by the Hasse diagram respectively given below:



Define $\mathcal{L}-$subsets $\Phi^w : R \to [0, 1]$ and $\Psi^w : G \to [0, 1]$, respectively as follows: $\Phi^w(0) = \Phi^w(b) = \Phi^w(c) = \Phi^w(g) = 1, \Phi^w(a) = 0.5, \Phi^w(d) = \Phi^w(e) = \Phi^w(f) = \Phi^w(h) = \Phi^w(i) = \Phi^w(1) = 0$ and $\Psi^w(0) = \Psi^w(a) = \Psi^w(b) = 1, \Psi^w(c) = \Psi^w(e) = 0.75, \Psi^w(d) = \Psi^w(f) = \Psi^w(g) = \Psi^w(h) = \Psi^w(i) = \Psi^w(j) = \Psi^w(1) = 0$. Clearly both Φ^w and Ψ^w are $\mathcal{L}-$weakly $1A-$prime ideals in R and G respectively. However, $\Phi^w \times \Psi^w$ is not $\mathcal{L}-$weakly $1A-$prime ideal in $R \times G$. This is demonstrated by considering,

$$\begin{aligned}
(\Phi^w \times \Psi^w)(e \wedge f \wedge g, h \wedge i \wedge j) &= (\Phi^w \times \Psi^w)(0, c) \\
&= \Phi^w(0) \wedge \Psi^w(c) \\
&= 0.75 \\
&\nleq 0.5 \\
&= (\Phi^w \times \Psi^w)(e \wedge f, h \wedge i) \vee (\Phi^w \times \Psi^w)(g, j).
\end{aligned}$$

COROLLARY 3.12. Let Φ^w and Ψ^w be $\mathcal{L}-$ideals in R and G respectively. Then Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R if and only if $\Phi^w_\beta = \Psi^w_\beta \times G$ or $\Phi^w_\beta = R \times \Psi^w_\beta$, for all $\beta \in L$.

THEOREM 3.13. *Assume R and G are ADLs, and $k : R \to G$ is a lattice homomorphism. If Ψ^w represents an $\mathcal{L}-$weakly $1A-$prime ideal in G, then $k^{-1}(\Psi^w)$ is an $\mathcal{L}-$weakly $1A-$prime ideal in R. Additionally, in the case of k being an epimorphism and Φ^w being an $\mathcal{L}-$weakly $1A-$prime ideal in R, it follows that $k(\Phi^w)$ is an $\mathcal{L}-$weakly $1A-$prime ideal in G.*

PROOF: Suppose that Ψ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in G and let k be a lattice homomorphism. Then, for all $r, s, t \in G$ such that $r \wedge s \wedge t \neq 0$,

$$\begin{aligned} k^{-1}(\Psi^w)(r \wedge s \wedge t) &= \Psi^w\big(k(r \wedge s \wedge t)\big) \\ &= \Psi^w\big(k(r) \wedge k(s) \wedge k(t)\big) \\ &\leq \Psi^w\big(k(r) \wedge k(s)\big) \vee \Psi^w(k(t)) \\ &= \Psi^w\big(k(r \wedge s)\big) \vee \Psi^w(k(t)) \\ &= k^{-1}(\Psi^w)(r \wedge s) \vee k^{-1}(\Psi^w)(t). \end{aligned}$$

Thus $k^{-1}(\Psi^w)$ is an $\mathcal{L}-$weakly $1A-$prime ideal in R. Also, let k be an isomorphism and suppose that Φ^w is an $\mathcal{L}-$weakly $1A-$prime ideal in R. Let $a, b, c \in R$ such that $a \wedge b \wedge c \neq 0$. Now, consider,

$$\begin{aligned} k(\Phi^w)(a \wedge b) \vee k(\Phi^w)(c) &= \Big[\bigvee_{a \wedge b \in k^{-1}(x \wedge y)} \Phi^w(a \wedge b)\Big] \vee \Big[\bigvee_{c \in k^{-1}(z)} \Phi^w(c)\Big] \\ &\geq \Big[\bigvee_{a \wedge b \wedge c \in k^{-1}(x \wedge y \wedge z)} \Phi^w(a \wedge b \wedge c)\Big] \\ &= k(\Phi^w)(a \wedge b \wedge c). \end{aligned}$$

Thus, $k(\Phi^w)$ is an $\mathcal{L}-$weakly $1A-$prime ideal in G. □

4. $\mathcal{L}-$weakly 1A-Prime Filters

In the subsequent discussion, we present the concepts of $\mathcal{L}$-weakly 1-absorbing prime filters and their characterizations. To begin with, let's review the definition provided in [1], stating that a proper filter H in R is a 1-absorbing prime filter (referred to as a weakly $1A$-prime filter) if, for all elements $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, the condition $r \vee s \vee t$ belonging to H implies either $r \vee s$ belonging to H or t belonging to H. Now, we aim to extend this outcome to the realm of L-weakly $1A$-prime filters as elaborated below.

DEFINITION 4.1. A proper $\mathcal{L}-$filter Φ^w in R is an $\mathcal{L}-$weakly $1A-$prime filter in R when, for any elements r, s and t in R such that $r \vee s \vee t \neq 1$, the condition $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$ is satisfied.

Example 4.2. Let R be an ADL defined in example 3.2 with elements $\{0, r, s, t\}$, and $L = [0, 1]$. Define an $\mathcal{L}-$subset $\Phi^w : R \to L$ as follows: $\Phi^w(0) = 0$, $\Phi^w(r) = 1, \Phi^w(s) = 3/4$ and $\Phi^w(t) = 1/2$. It is evident that Φ^w is an $\mathcal{L}-$filter. Now, consider any elements $a, b, c \in R$ such that $a \vee b \vee c \neq 1$. Then $\Phi^w(a \vee b \vee c) \leq \Phi^w(a \vee b) \vee \Phi^w(c)$. Consequently, Φ^w qualifies as an $\mathcal{L}-$weakly $1A-$prime filter in R.

Subsequently, we elaborate on the notion of an $\mathcal{L}-$weakly $1A-$prime filter concerning the γ-cut.

THEOREM 4.3. *Suppose Φ^w is an $\mathcal{L}-$filter in R. A filter Φ^w_γ is a weakly $1A-$prime filter in R, for all $\gamma \in L$ if and only if Φ^w qualifies as an $\mathcal{L}-$weakly $1A-$prime filter in R.*

PROOF: Assume that Φ^w_γ is a weakly $1A-$prime filter for all $\gamma \in L$. In this case, for any elements $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, it follows that either $r \vee s$ is an element of $\Phi^w_{\Phi^w(r \vee s \vee t)}$ or t is an element of $\Phi^w_{\Phi^w(r \vee s \vee t)}$. This implies $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s)$ or $\Phi^w(t)$. Consequently, $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$, leading to the desired result. Conversely, assume Φ^w is an $\mathcal{L}-$weakly $1A-$prime filter. Consider $r, s, t \in R$ such that $r \vee s \vee t \neq 1$. If $r \vee s \vee t$ is an element of Φ^w_γ, then $\gamma \leq \Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$, which implies that either $\gamma \leq \Phi^w(r \vee s)$ or $\gamma \leq \Phi^w(t)$. This, in turn, means that either $r \vee s \in \Phi^w_\gamma$ or $t \in \Phi^w_\gamma$. Therefore, Φ^w_γ is a weakly $1A-$prime filter in R. □

COROLLARY 4.4. A filter F in R is classified as a weakly $1A-$prime filter in R iff χ_F is an $\mathcal{L}-$weakly $1A-$prime filter in R.

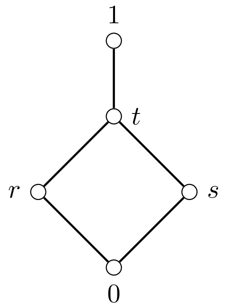
In the following discourse, we clarify the relationships between $\mathcal{L}-$weakly prime filters and $\mathcal{L}-$weakly $1A-$prime filters within an ADL.

THEOREM 4.5. *Suppose Φ^w is an $\mathcal{L}-$filter in R. Then Φ^w is an $\mathcal{L}-$weakly $1A-$prime filter in R only if Φ^w is an $\mathcal{L}-$weakly prime filter in R .*

Proof: It is clear. □

In the forthcoming example, we illustrate the presence of $\mathcal{L}-$weakly $1A-$prime filters in an ADL R that do not meet the criteria for being $\mathcal{L}-$weakly prime filters in R.

Example 4.6. Consider the discrete ADL $D = \{0, u, v\}$ with 0 as its zero element, as defined in 2.2. Let $L = \{0, r, s, t, 1\}$ represent the lattice depicted in the given Hasse diagram:



Consider $D \times L = \{(d, e) \mid d \in D \text{ and } e \in L\}$. Then, the structure $(D \times L, \wedge, \vee, 0)$ forms an ADL through point-wise operations $\wedge$ and $\vee$ on $D \times L$, where 0 is represented by $(0, 0)$, the zero element in $D \times L$. Define $F = \{t, 1\}$. It is evident that F is a filter in L. Now define $\Phi^w : D \times L \to [0, 1]$ by

$$\Phi^w(d,e) = \begin{cases} 0 & \text{if } (d,e) = (0,0) \\ 1 & \text{if } d \neq 0 \textit{ and } e \in F \\ 0.55 & \text{otherwise} \end{cases}$$

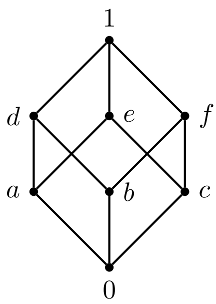
for all $(d, e) \in D \times L$. Additionally, Φ^w is an $\mathcal{L}-$filter of $D \times L$. Then $\Phi^w_1 = \{(u, t), (v, t), (u, 1), (v, 1)\}$. Consequently, Φ^w emerges as an $\mathcal{L}-$weakly $1A-$prime filter of $D \times L$. However, Φ^w does not qualify as an $\mathcal{L}-$weakly prime filter of $D \times L$, as Φ^w_1 is a weakly $1A-$prime filter of $D \times L$ but not weakly prime filter. This is demonstrated by considering, $(u, r), (v, s)$ in $D \times L$, where $(u, r) \vee (v, s) = (v, t)$ belongs to Φ^w_1 implying $(u, r) \notin \Phi^w_1$ and $(v, s) \notin \Phi^w_1$.

DEFINITION 4.7 ([3]). A proper $\mathcal{L}-$filter Φ^w in R is an $\mathcal{L}-$weakly $2A-$filter in R if for any elements r,s and $t \in R$ such that $r \vee s \vee t \neq 1$, $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$.

THEOREM 4.8. *Suppose Φ^w is an $\mathcal{L}-$filter in R. If Φ^w is an $\mathcal{L}-$weakly $1A-$prime filter in R, then Φ^w is an $\mathcal{L}-$weakly $2A-$filter in R. The converse of this result is not true.*

PROOF: Let Φ^w be an $\mathcal{L}-$weakly $1A-$prime filter in R. Then, for all $r, s, t \in R$ such that $r \vee s \vee t \neq 1$, it holds that $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(t)$. By utilizing Theorem 2.12(1) and (3), we can deduce that $\Phi^w(t) \leq \Phi^w(t \vee s) = \Phi^w(s \vee t)$ and $\Phi^w(t) = \Phi^w(t \vee r) = \Phi^w(r \vee t)$, given that $t \leq t \vee s$ and $t \leq t \vee r$. Consequently, $\Phi^w(t) \leq \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$. This leads to the conclusion that $\Phi^w(r \vee s \vee t) \leq \Phi^w(r \vee s) \vee \Phi^w(r \vee t) \vee \Phi^w(s \vee t)$, thus establishing the desired result. □

Example 4.9. Let $D = \{0, u, v\}$ be a discrete ADL with 0 as its zero element defined in 2.2 and $L = \{0, a, b, c, d, e, f, 1\}$ be a lattice whose Hasse diagram is given below:



Define $\mathcal{L}-$filter $\Phi^w : R \to [0, 1]$ by

$$\Phi^w(y, z) = \begin{cases} 0 & \text{if } (y, z) = (0, 0) \\ 3/4 & \text{if } y = u \text{ and } z = 1 \\ 1/2 & \text{otherwise} \end{cases}$$

for all $(y, z) \in D \times L$. It is evident that Φ^w qualifies as an $\mathcal{L}-$weakly filter of $D \times L$. Let $H = \Phi^w_{3/4} = \{(u, 1)\}$. Notably, H emerges as a filter in $D \times L$. Consequently, Φ^w identified as an $\mathcal{L}-$weakly $2A-$filter of $D \times L$, albeit not $\mathcal{L}-$weakly $1A-$prime filter. This is demonstrated by considering any elements $(0,a),(u,c),(v,b) \in D\times L$, where $(0,a)\vee(u,c)\vee(v,b)$ belongs to H, implying $(0,a)\vee(u,c)=(u,e)\notin H$ and $(v,b)\notin H$.

Theorem 4.10. *Consider $\mathcal{L}-$weakly filters Φ^w and Ψ^w be in R and G, respectively. If the product $\Phi^w\times\Psi^w$ forms an $\mathcal{L}-$weakly $1A-$prime filter in $R\times G$, then both Φ^w and Ψ^w individually constitute $\mathcal{L}-$weakly $1A-$prime filters in R and G, respectively.*

Proof: Assume that $\Phi^w\times\Psi^w$ is an $\mathcal{L}-$weakly $1A-$prime filter. Take $r,s,t\in R$ and $x,y,z\in G$ such that $r\vee s\vee t\neq 1$ and $x\vee y\vee z\neq 1$. Then,

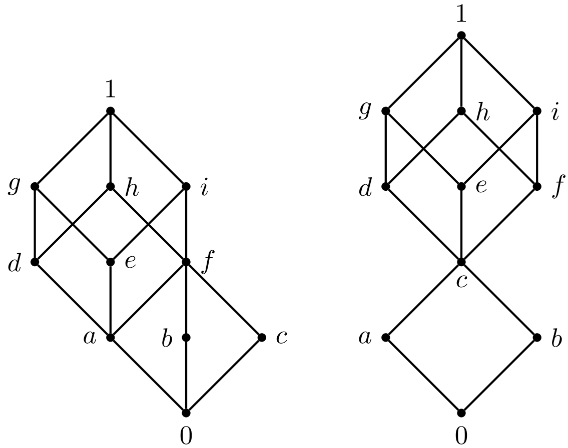
$$\begin{aligned}
\Phi^w(r\vee s\vee t)\wedge\Psi^w(x\vee y\vee z) &= (\Phi^w\times\Psi^w)(r\vee s\vee t, x\vee y\vee z)\\
&= (\Phi^w\times\Psi^w)\big((r,x)\vee(s,y)\vee(t,z)\big)\\
&\leq (\Phi^w\times\Psi^w)\big((r,x)\vee(s,y)\big)\vee(\Phi^w\times\Psi^w)(t,z)\\
&= \Big(\Phi^w(r\vee s)\wedge\Psi^w(x\vee y)\Big)\vee\Big(\Phi^w(t)\wedge\Psi^w(z)\Big)\\
&= \Big(\Phi^w(r\vee s)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\
&\quad\wedge\Big(\Psi^w(x\vee y)\vee\big(\Phi^w(t)\wedge\Psi^w(z)\big)\Big)\\
&= \big(\Phi^w(r\vee s)\vee\Phi^w(t)\big)\wedge\big(\Phi^w(r\vee s)\vee\Psi^w(z)\big)\\
&\quad\wedge\big(\Psi^w(x\vee y)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\vee y)\vee\Psi^w(z)\big)\\
&\leq \big(\Phi^w(r\vee s)\vee\Phi^w(t)\big)\wedge\big(\Psi^w(x\vee y)\vee\Psi^w(z)\big).
\end{aligned}$$

Hence the result. □

The presence of $\mathcal{L}-$weakly $1A-$prime filters does not guarantee that their direct product will be an $\mathcal{L}-$weakly $1A-$prime filter. An example demonstrating this is provided below.

Example 4.11. Let $R=\{0,a,b,c,d,e,f,g,h,i,1\}$ and $G=\{0,a,b,c,d,e,f,g,h,i,1\}$ be the lattice represented by the Hasse diagram respectively given below:

Define $\mathcal{L}-$subsets Φ^w and Ψ^w in R and G, respectively such that for Φ^w: $\Phi^w(0)=\Phi^w(a)=0$, $\Phi^w(b)=1/3$, $\Phi^w(c)=0$, $\Phi^w(d)=\Phi^w(e)=\Phi^w(g)=3/5, \Phi^w(f)=1, \Phi^w(h)=3/5, \Phi^w(i)=3/5, \Phi^w(1)=1$ and for Ψ^w: $\Psi^w(0)=\Psi^w(a)=\Psi^w(b)=0$, $\Psi^w(c)=\Psi^w(d)=\Psi^w(e)=\Psi^w(f)=$



$1/2, \Psi^w(i) = \Psi^w(g) = \Psi^w(h) = \Psi^w(1) = 1$. Clearly, both Φ^w and Ψ^w are $\mathcal{L}-$weakly $1A-$prime filters in R and G, respectively. However, the direct product $\Phi^w \times \Psi^w$ is not $\mathcal{L}-$weakly $1A-$prime filter in $R \times G$, as evidenced by the example where

$$\begin{aligned}(\Phi^w \times \Psi^w)(d\vee e\vee f, d\vee e\vee f) &= (\Phi^w \times \Psi^w)(1,1)\\ &= 1\\ &\nleq 3/5\\ &= (\Phi^w \times \Psi^w)(d\vee e, d\vee e)\vee(\Phi^w \times \Psi^w)(f,f).\end{aligned}$$

Corollary 4.12. Let Φ^w and Ψ^w be $\mathcal{L}-$filters in R and G, respectively, and for all $\beta \in L$. Then Φ^w is an $\mathcal{L}-$weakly $1A-$prime filter in R if and only if $\Phi^w_\beta = \Psi^w_\beta \times G$ or $\Phi^w_\beta = R \times \Psi^w_\beta$, where Φ^w_β and Ψ^w_β are weakly $1A-$prime filter in R and G respectively.

Lastly, we explore the homomorphism of $\mathcal{L}-$weakly $1A-$prime filters in ADLs.

Theorem 4.13. *Consider ADLs R and G, with a lattice homomorphism $k : R \to G$. Then $k^{-1}(\Psi^w)$ is an $\mathcal{L}-$weakly $1A-$prime filter in R only if Ψ^w is an $\mathcal{L}-$weakly $1A-$prime filter in G. Additionally, if k is an epimorphism and Φ^w is an $\mathcal{L}-$weakly $1A-$prime filter in R, then $k(\Phi^w)$ is an $\mathcal{L}-$weakly $1A-$prime filter in G.*

PROOF: Let $k : R \to G$ be a lattice homomorphism. Suppose that Ψ^w is an $\mathcal{L}-$weakly $1A-$prime filter in G. For all $r, s, t \in G$ such that $r \vee s \vee t \neq 1$. Then

$$\begin{aligned} k^{-1}(\Psi^w)(r \vee s \vee t) &= \Psi^w\big(k(r \vee s \vee t)\big) \\ &= \Psi^w\big(k(r) \vee k(s) \vee k(t)\big) \\ &\leq \Psi^w\big(k(r) \vee k(s)\big) \vee \Psi^w(k(t)) \\ &= \Psi^w\big(k(r \vee s)\big) \vee \Psi^w(k(t)) \\ &= k^{-1}(\Psi^w)(r \vee s) \vee k^{-1}(\Psi^w)(t). \end{aligned}$$

Thus $k^{-1}(\Psi^w)$ is an $\mathcal{L}-$weakly $1A-$prime filter in R. Let k be an isomorphism and suppose that Φ^w be an $\mathcal{L}-$weakly $1A-$prime filter in R. For all $a, b, c \in R$ such that $a \vee b \vee c \neq 1$. Now, consider,

$$\begin{aligned} k(\Phi^w)(a \vee b) \vee k(\Phi^w)(c) &= \Big[\bigvee_{a \vee b \in k^{-1}(x \wedge y)} \Phi^w(a \vee b)\Big] \vee \Big[\bigvee_{c \in k^{-1}(z)} \Phi^w(c)\Big] \\ &\geq \Big[\bigvee_{a \vee b \vee c \in k^{-1}(x \wedge y \wedge z)} \Phi^w(a \vee b \vee c)\Big] \\ &= k(\Phi^w)(a \vee b \vee c). \end{aligned}$$

Thus, $g(\Phi^w)$ is an $\mathcal{L}-$weakly $1A-$prime filter in G. □

5. Conclusion

This study concentrates on investigating $\mathcal{L}-$weakly $1A-$prime ideals and filters within an ADL, constituting a pivotal aspect of our research. We delve into the characteristics of these elements, exploring their properties. Furthermore, we elucidate the connection between $\mathcal{L}-$weakly prime filters (ideals) and $\mathcal{L}-$weakly $1A-$prime filters (ideals) in ADLs. Notably, we offer examples to illustrate instances where the converse relationship may not be applicable.

Author contribution statement. I affirm that I am the exclusive author of this work, and I have not consulted any sources other than those explicitly cited in the references. Additionally, I confirm that this manuscript has not been submitted to any other journal for publication.

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