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INTRODUCTION: BILATERALISM AND PROOF-THEORETIC SEMANTICS (PART II)

Most of the papers contained in this special issue[1] are results from contributions at a conference on this topic, which took place at the Ruhr University Bochum in March 2022. Since the topic of proof-theoretic semantics (PTS) can by now be considered as well-established in the logic community and has been exclusively dealt with at several conferences and in many publications[2], this introduction's focus will be on the part of logical bilateralism. Before summarizing the content of this special issue, a brief overview of the development in the field will be given, though this is not meant and does not aim to be an exhaustive account of the existing literature.[3]

There are rather different approaches branded as *bilateralism* in the literature, whose differences are mostly not made explicit, though. Although the origin of bilateralism is Rumfitt's [20] seminal paper in the sense that the concrete term and idea are introduced therein and spelled out thoroughly, there are some predecessors to the general idea that are frequently cited, like [12], [22], and [8].[4] The most frequent characterization that is

[1] For editorial reasons it was decided to have actually two issues on this topic, which is why this introduction will appear in both parts and only differ in the presentation of the papers contained in the respective issue.

[2] See, e.g., [21, 4, 9, 11].

[3] Parts of the following paragraphs can also be found in a joint paper by Heinrich Wansing and myself on the topic of multilateralism [27]. In its introductory part we give an overview of the literature on bilateralism as well as of the existing but scarce literature extending this concept to multilateralism.

[4] A paper which is *not* often mentioned in this context, probably due to the fact that it was written in German, but which deserves recognition in this context is [24]. Von Kutschera is concerned with the relation between the notions of proof and refutation

used for bilateralism is that it is a theory of meaning displaying a symmetry between certain notions (or often rather: conditions governing these notions), which have not been considered being on a par by 'conventional' theories of meaning. The relevant notions are most often *assertion* and *denial*, or *assertibility* and *deniability*, sometimes also *acceptance* and *rejection*.[5] While the former are usually taken to describe speech acts, the latter are usually – though not always (see [19] for a thorough distinction) – considered to describe the corresponding internal cognitive states or attitudes. 'Assertibility' and 'deniability', on the other hand, are of a third kind, since they can be seen to describe something like properties of propositions. The symmetry between these respective concepts is often described with expressions like "both being primitive", "not reducible to each other", "being on a par", and "of equal importance". Another point to characterize bilateralism, which is often mentioned, though not as frequent or central as the former point,[6] is that in a bilateral approach the denial of A is not interpreted in terms of, or *as* the assertion of the negation of A but that it is the other way around: In bilateralism rejection and/or denial are usually considered as conceptually prior to negation.

Ripley [18, 19] distinguishes two camps of bilateral theories of meaning in terms of "what kinds of condition on assertion and denial they appeal to" [19, p. 50]: a warrant-based approach and a coherence-based approach, for the latter of which he himself argues [17] and which was firstly devised by Restall [13, 14].[7] As references for the first camp, which Ripley calls the 'orthodox' bilateralism, [12], [22], and [20] are given. Warrant-based bilateralism takes the relevant conditions to be the ones under which propositions can be *warrantedly* asserted or denied. Coherence-based bilateralism,

and claims, e.g., that it is not necessary to define the latter in terms of the former but that it could just as well be done the other way around, or, although in the paper he does differently, that both could be seen as primitive. Thus, it seems that he voices quite bilateralist ideas.

[5] To give some examples of references using a characterization of essentially this flavor: [5, 7, 10, 16, 20, 26].

[6] The following use this as an additional characterization (while also using the essential characterization that the references in fn. 4 use): [2, 3, 17, 23]. This is not to say that this point does not occur in other works on bilateralism but that it is not used as a *characterizing feature* of bilateralism there.

[7] In [19] this one is called the "bounds-based bilateralism". Interestingly, Restall does not use the expression "bilateralism" at all in the cited works, only later does this term become part of his terminology, e.g., in [15].

on the other hand, takes the relevant conditions to be the conditions under which *collections* of propositions can be *coherently* asserted and/or denied together.

What the two approaches have in common is that they were both meant, as they were originally devised, to motivate a PTS approach using *classical* instead of intuitionistic logic. What they tend to differ in, though, is their design and interpretations of proof systems. Rumfitt [20] uses a natural deduction system with signed formulas for assertion and denial, i.e., rules do not apply to propositions but to speech acts. He argues that the shortcomings that a classical natural deduction calculus has from a PTS point of view are overcome once we consider a calculus containing introduction and elimination rules determining not only the assertion conditions for formulas containing the connective in question but also the denial conditions. Thus, he means to give a motivation how the rules of classical logic lay down the meaning of the connectives.[8]

Restall [13], opting for the coherence-based approach, does the same but comes from another direction in suggesting a bilateral reading of classical sequent calculus (i.e., with multiple conclusions) incorporating the speech acts of assertion and denial. In a nutshell, he proposes that having the derivation of a sequent $\Gamma \vdash \Delta$, means that the position of asserting each of the members of Γ while simultaneously denying each of the members of Δ would be 'out of bounds'. In a recent paper, though, Restall [15] seems convinced by Steinberger's [23] criticism of multiple-conclusion systems as not adhering to our natural inferential practice and he considers an approach using a natural deduction system instead, which does not employ signed formulas but rather uses different positions for certain commitments from which the inference is drawn to the conclusion.[9]

What Ripley [19] mentions in a footnote is that there are also other kinds of bilateralism, which do not fit into either camp because they do not consider speech acts (i.e., assertion and denial) as the primary notions to act upon in the context of PTS but rather notions being on a par with proof, provability, or verification, i.e., refutation, refutability, or falsification, respectively. The point of interest is, thus, to implement different

[8] For critical assessments of that paper, see, e.g., [6, 2, 10, 5].

[9] The motivation is still to make a case for classical logic being usable in a PTS framework, although Restall does not seem too dogmatic about anything being 'the best' logic. He also wants to show how such a system can be used for substructural logics.

derivability relations in a proof-theoretic framework expressing a duality between different inferential relationships, which has been devised, e.g., in [25, 26].

These different varieties of bilateralism depicted above are actually very well represented in this special issue. It is even the majority of the contributions dealing with what can be called – in one way or another – 'unorthodox' bilateralism.

In the paper "Fractional-valued modal logic and soft bilateralism" Mario Piazza, Gabriele Pulcini and Matteo Tesi outline yet another, unorthodox variety of bilateralism, which they call *soft bilateralism* to demarcate their approach from more traditional conceptions. It is 'bilateral' because the rules in the calculi they introduce are meant to deal with both derivability and underivability. It is only 'softly' bilateral due to their conception of the speech act of denial, namely as rejection in the sense of proving the unprovability of a formula rather than in the sense of the stronger notion of directly refuting that formula. Based on this approach they argue for considering *fractional semantics* – a semantics whose values are the rational numbers in the closed interval [0,1] – for a family of modal logics and investigate and prove certain properties for these systems.

There are also papers, though, which deal with issues of 'orthodox' bilateralism. Nils Kürbis' paper "Supposition: A problem for bilateralism" spells out an important objection that can be raised against a system of natural deduction with signed formulas to be interpreted as speech acts in Rumfitt-style. The argument against such a system is as simple as it is compelling: Natural deduction systems work with assumptions. Making an assumption is also to be considered as a kind of speech act. Embedding speech acts within other speech acts is – as it is widely agreed upon – not possible. Thus, we cannot make sense of the use of assumptions in a proof system which implements bilateralism in such a way.

Leonardo Ceragioli's paper "Bilateral rules as complex rules" deals with the same kind of proof system and more specifically, two objections raised in [5] about issues caused by the so-called *coordination principles*, which are needed in such a bilateralist system besides the operational rules. The first objection is that in a bilateralist framework the notorious connective `tonk` cannot be ruled out by the criterion of harmony as it can be usually done in a unilateralist framework and that thus, there can be (at least on a certain understanding of the term) a reduction procedure for `tonk`, which indeed would be highly undesirable from the viewpoint of PTS. The

second objection concerns a bilateralist version of rules for a paradoxical zero-ary connective, which Gabbay [5] presents and which he claims to be in harmony. However, together with the coordination principles they would trivialize the system, i.e., they should not be admitted, although they seem fine by the harmony criterion. Ceragioli's proposed solution to these two issues is based on reinterpreting bilateral systems as systems with *complex rules* and applies the results existing on such systems to the special case of bilateralism.

Last but not least, the paper by Pedro del Valle-Inclan "Harmony and normalisation in bilateral logic" builds upon former work by the author and co-author Julian Schlöder [1] in which they propose a specific notion of proof-theoretic harmony for bilateralist contexts. In the present paper del Valle-Inclan argues that this notion also leads to a special notion of normal form. Based on this, he goes on to prove normalization results for two (Rumfitt-style) bilateralist calculi for classical logic, which are subsequently compared to other existing results in this area.

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