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INTRODUCTION: BILATERALISM AND PROOF-THEORETIC SEMANTICS (PART I)

Most of the papers contained in this special issue[1] are results from contributions at a conference on this topic, which took place at the Ruhr University Bochum in March 2022. Since the topic of proof-theoretic semantics (PTS) can by now be considered as well-established in the logic community and has been exclusively dealt with at several conferences and in many publications[2], this introduction's focus will be on the part of logical bilateralism. Before summarizing the content of this special issue, a brief overview of the development in the field will be given, though this is not meant and does not aim to be an exhaustive account of the existing literature.[3]

There are rather different approaches branded as *bilateralism* in the literature, whose differences are mostly not made explicit, though. Although the origin of bilateralism is Rumfitt's [19] seminal paper in the sense that the concrete term and idea are introduced therein and spelled out thoroughly, there are some predecessors to the general idea that are frequently cited, like [11], [21], and [7].[4] The most frequent characterization that is

[1] For editorial reasons it was decided to have actually two issues on this topic, which is why this introduction will appear in both parts and only differ in the presentation of the papers contained in the respective issue.

[2] See, e.g., [20, 3, 8, 10].

[3] Parts of the following paragraphs can also be found in a joint paper by Heinrich Wansing and myself on the topic of multilateralism [26]. In its introductory part we give an overview of the literature on bilateralism as well as of the existing but scarce literature extending this concept to multilateralism.

[4] A paper which is *not* often mentioned in this context, probably due to the fact that it was written in German, but which deserves recognition in this context is [23]. Von Kutschera is concerned with the relation between the notions of proof and refutation and claims, e.g., that it is not necessary to define the latter in terms of the former but

used for bilateralism is that it is a theory of meaning displaying a symmetry between certain notions (or often rather: conditions governing these notions), which have not been considered being on a par by 'conventional' theories of meaning. The relevant notions are most often *assertion* and *denial*, or *assertibility* and *deniability*, sometimes also *acceptance* and *rejection*.[5] While the former are usually taken to describe speech acts, the latter are usually – though not always (see [18] for a thorough distinction) – considered to describe the corresponding internal cognitive states or attitudes. 'Assertibility' and 'deniability', on the other hand, are of a third kind, since they can be seen to describe something like properties of propositions. The symmetry between these respective concepts is often described with expressions like "both being primitive", "not reducible to each other", "being on a par", and "of equal importance". Another point to characterize bilateralism, which is often mentioned, though not as frequent or central as the former point,[6] is that in a bilateral approach the denial of A is not interpreted in terms of, or *as* the assertion of the negation of A but that it is the other way around: In bilateralism rejection and/or denial are usually considered as conceptually prior to negation.

Ripley [17, 18] distinguishes two camps of bilateral theories of meaning in terms of "what kinds of condition on assertion and denial they appeal to" [18, p. 50]: a warrant-based approach and a coherence-based approach, for the latter of which he himself argues [16] and which was firstly devised by Restall [12, 13].[7] As references for the first camp, which Ripley calls the 'orthodox' bilateralism, [11], [21], and [19] are given. Warrant-based bilateralism takes the relevant conditions to be the ones under which propositions can be *warrantedly* asserted or denied. Coherence-based bilateralism, on the other hand, takes the relevant conditions to be the conditions under

that it could just as well be done the other way around, or, although in the paper he does differently, that both could be seen as primitive. Thus, it seems that he voices quite bilateralist ideas.

[5] To give some examples of references using a characterization of essentially this flavor: [4, 6, 9, 15, 19, 25].

[6] The following use this as an additional characterization (while also using the essential characterization that the references in fn. 4 use): [1, 2, 16, 22]. This is not to say that this point does not occur in other works on bilateralism but that it is not used as a *characterizing feature* of bilateralism there.

[7] In [18] this one is called the "bounds-based bilateralism". Interestingly, Restall does not use the expression "bilateralism" at all in the cited works, only later does this term become part of his terminology, e.g., in [14].

which *collections* of propositions can be *coherently* asserted and/or denied together.

What the two approaches have in common is that they were both meant, as they were originally devised, to motivate a PTS approach using *classical* instead of intuitionistic logic. What they tend to differ in, though, is their design and interpretations of proof systems. Rumfitt [19] uses a natural deduction system with signed formulas for assertion and denial, i.e., rules do not apply to propositions but to speech acts. He argues that the shortcomings that a classical natural deduction calculus has from a PTS point of view are overcome once we consider a calculus containing introduction and elimination rules determining not only the assertion conditions for formulas containing the connective in question but also the denial conditions. Thus, he means to give a motivation how the rules of classical logic lay down the meaning of the connectives.[8]

Restall [12], opting for the coherence-based approach, does the same but comes from another direction in suggesting a bilateral reading of classical sequent calculus (i.e., with multiple conclusions) incorporating the speech acts of assertion and denial. In a nutshell, he proposes that having the derivation of a sequent $\Gamma \vdash \Delta$, means that the position of asserting each of the members of Γ while simultaneously denying each of the members of Δ would be 'out of bounds'. In a recent paper, though, Restall [14] seems convinced by Steinberger's [22] criticism of multiple-conclusion systems as not adhering to our natural inferential practice and he considers an approach using a natural deduction system instead, which does not employ signed formulas but rather uses different positions for certain commitments from which the inference is drawn to the conclusion.[9]

What Ripley [18] mentions in a footnote is that there are also other kinds of bilateralism, which do not fit into either camp because they do not consider speech acts (i.e., assertion and denial) as the primary notions to act upon in the context of PTS but rather notions being on a par with proof, provability, or verification, i.e., refutation, refutability, or falsification, respectively. The point of interest is, thus, to implement different derivability relations in a proof-theoretic framework expressing a duality

[8] For critical assessments of that paper, see, e.g., [5, 1, 9, 4].

[9] The motivation is still to make a case for classical logic being usable in a PTS framework, although Restall does not seem too dogmatic about anything being 'the best' logic. He also wants to show how such a system can be used for substructural logics.

between different inferential relationships, which has been devised, e.g., in [24, 25].

These different varieties of bilateralism depicted above are actually very well represented in this special issue. It is even the majority of the contributions dealing with what can be called – in one way or another – 'unorthodox' bilateralism.

Greg Restall's paper "Structural rules in natural deduction with alternatives" explores features of a special kind of bilateralist natural deduction system, namely with *alternatives*. These are 'negative assumptions' with which a natural deduction system of Gentzen-Prawitz-style is extended; otherwise, the rules for the connectives are not changed from the usual ones of such a system, i.e., as Restall notes, the extension is of purely structural nature. What is shown for this system is that the rule of explosion and the rule of allowing vacuous discharge, both being principles introducing irrelevance to the system, can actually be seen as corresponding principles. Restall shows how with the shift to what he calls a *mildly* bilateralist system this extension of Gentzen-Prawitz-style natural deduction can not only be used to give an account for classical logic but also for substructural systems, such as linear, relevant and affine logic. It is only 'mildly' bilateralist because neither is every formula in the system signed to be of either some positive or negative force nor are any operational rules added to the system, as it is done in one way or another in what he calls 'fully' bilateralist systems.

The paper "Core type theory" by Emma van Dijk, David Ripley and Julian Gutierrez also deals with a system which may not strike one as 'obviously' bilateralist but which nevertheless can be seen as one in an interesting way. In the paper a slightly modified version of Tennant's natural deduction proof system for his core logic is presented and used as a type theory. It is shown that strong normalization can be proven for this system, while it cannot for Tennant's original system. Although there are no signed formulas or derivability relations in this system, it is bilateralist in the sense that it is a system in which both proofs and refutations can be constructed and neither concept is taken to be reducible to the other. For this reason, the authors connect the spirit of bilateralism inherent in core logic to the type of bilateralism that is put forth in [24, 25].

Implementing bilateralism on the level of derivational constructions is also advocated in the paper "On synonymy in proof-theoretic semantics.

The case of `2Int`" by myself and Heinrich Wansing. We present a `G3`-style sequent calculus, `SC2Int`, for the bi-intuitionistic logic `2Int`, which is bilateral in that two kinds of signed sequents are used, one representing proofs, the other representing refutations and for which the structural rules are shown to be admissible. Then, by defining and using so-called *interaction rules*, which allow switching from proofs to refutations, and vice versa, an approach to propositional synonymy in a bilateralist PTS setting is devised. This concept relies on the notion of *inherited identity* between derivations and, applied to `SC2Int`, leads to notions of positive and negative synonymy of formulas.

Another special form of PTS and bilateralism is explored by Alexander V. Gheorghiu and David J. Pym in "Definite formulae, negation-as-failure, and the base-extension semantics of intuitionistic propositional logic". They analyze a base-extension semantics for intuitionistic propositional logic – that is, a semantics building upon sets of inference rules for atomic sentences – in the context of logic programming. The bases are interpreted as programs, i.e., collections of definite formulas, and investigated using an operational reading. The paper recovers the completeness of intuitionistic propositional logic through this perspective. Significantly, in logic programming, assertion and denial are understood in terms of the success and failure to find a proof. Using the *negation-as-failure protocol*, the paper provides an interpretation of negation in a PTS for intuitionistic propositional logic as denial, meaning that the latter is – in accordance with a bilateralist conception – conceptionally prior to the former.

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