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TABLEAUX FOR SOME DEONTIC LOGICS WITH THE EXPLICIT PERMISSION OPERATOR

Abstract

In this paper we present a tableau system for deontic logics with the operator of explicit permission. By means of this system the decidability of the considered logics can be proved. We will sketch how these logics are semantically defined by means of relating semantics and how they provide a simple solution to the free choice permission problem. In short, these logics employ relating implication and a certain propositional constant. These two are in turn used to define deontic operators similarly as in Andersonian-Kangerian reduction, which uses different intensional implications and constants.

Keywords: Explicit permission, free choice, relating semantics.

1. Introduction

The present paper provides a decidability method for the logics proposed in [8]. The intuitive justification along with an axiomatic system for these logics are presented there in details. Here we will just sketch the motivation behind the system.

The logical framework in question focuses on three central ideas. First, it provides an analysis and a solution to the problem of free choice permission. Secondly, it introduces the idea of deontic legitimisation in the

Presented by: Andrzej Indrzejczak

Received: April 9, 2022

Published online: June 23, 2022

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context of permission. Finally, it employs the idea of an Andersonian-Kangerian reduction in order to define permission(s). These three elements are put together in the proposed analysis of explicit free choice permission via relating semantics. The resulting framework allows for a fine-grained understanding of permission, which is both an explicit permission and a free choice permission.

Free choice permission (see [10, p. 166–167], [9, p. 206–218]) can be expressed in the following way:

$$P(\varphi \vee \psi) \supset (P\varphi \wedge P\psi) \quad (\text{FCP})$$

As an example, consider:

You may have tea or coffee.

By means of such a permission we usually offer somebody a choice:

You may have tea and you may have coffee.

that can be entailed by (FCP).

However, in Standard Deontic Logic (SDL), i.e., a counterpart of normal modal logic D with operator of obligation interpreted as necessity and the operator of permission interpreted as possibility not only (FCP) is not valid but cannot be added without the cost of getting a contradictory system, a trivial system, in which any formula is valid:

- | | |
|---|-------------------------------------|
| 1. $O(p \vee \neg p)$ | Classical logic, Necessitation Rule |
| 2. $O(p \vee \neg p) \equiv \neg P\neg(p \vee \neg p)$ | Definition of O |
| 3. $O(p \vee \neg p) \supset \neg P\neg(p \vee \neg p)$ | Classical logic, 2, Modus Ponenes |
| 4. $\neg P\neg(p \vee \neg p)$ | 1, 3, Modus Ponenes |
| 5. $O(\neg(p \vee \neg p) \vee (p \vee \neg p))$ | Classical logic, Necessitation Rule |
| 6. $O(\neg(p \vee \neg p) \vee (p \vee \neg p)) \supset P(\neg(p \vee \neg p) \vee (p \vee \neg p))$ | Axiom (D) |
| 7. $P(\neg(p \vee \neg p) \vee (p \vee \neg p))$ | 5, 6, Modus Ponens |
| 8. $P(\neg(p \vee \neg p) \vee (p \vee \neg p)) \supset (P\neg(p \vee \neg p) \wedge P(p \vee \neg p))$ | (FCP) |
| 9. $P\neg(p \vee \neg p) \wedge P(p \vee \neg p)$ | 7, 8, Modus Ponens |
| 10. $P\neg(p \vee \neg p)$ | Classical logic, 9, Modus Ponens |
| 11. \perp | Classical logic, 4, 10 |

Of course, the apparent mismatch between the formalism and our linguistic intuitions has led to many suggested solutions to the problem of free choice permission (see, for example, [9]).

Concerning the latter two central themes, the solution presented in [8] combines Andersonian-Kangerian [1, 2, 17] reduction of the permission operator to a special constant and an intensional relation between some permitted states of affairs or actions and their normative justification. To capture the idea that a state is relevant to the permissions that are issued, the notion of *legitimation* is employed. Legitimation can be understood as a kind of normative justification (for a general justification logic see [4]). Legitimation is understood as a general relation between actions, events, or states of affairs that normatively justifies one of them on the basis of another one. For instance, taking the partner's last name when getting married legitimises one's usage of this surname henceforth. Paying for a chocolate bar legitimises one in walking out of the store with the mentioned bar and eating it. And being explicitly permitted to have tea or coffee legitimises one (at least) in opting for tea (or coffee). A relation in which an explicit permission is the legitimating argument is a special case of legitimation in general.

As a formal tool for defining the properties of legitimation relating semantics is used. The semantics [12, 15] has been earlier successfully applied in the area of relatedness logic [7], connexive logic [16], and in the context of deontic logic [14], with a specific focus on obligations and prohibitions rather than permissions. The approach introduced in [8] employs the framework to (explicit) free choice permission.

The present paper is of a technical nature. We introduce tableaux adequate to the family of systems from [8]. In section 2 we reproduce the given formal framework along with the definition of permission using notion of legitimation. In section 3 we present the main results of the paper: the tableau systems, and in section 4 we summarise our contribution and point out some directions for future research.

2. Formal language and semantics

Let us now reproduce the basic tenets of the formal framework intuitively and informally sketched above. The language consists of the propositional variables $p_1, p_2, p_3 \dots$; the (deontic) propositional constant **permit**; the

classical propositional operators: \neg (negation), \wedge (conjunction), \vee (disjunction), \supset (material implication), relating implication: \rightarrow^w , and brackets. The set of propositional variables is denoted by Var . The set of formulas of the language is defined in the standard way and denoted by For .

Except for **permit** and \rightarrow^w all operators are standard and understood classically. The intended meaning of **permit** is the combined content of all permissions that are issues with the proviso that at least one permission is issued. The connective \rightarrow^w is a non-classical implication whose role is to express the normative relation between its arguments. Technically, it is a member of the family of relating operators discussed in [13] and [12]. Its formal meaning is given by its semantics which is shown and explained below. We read the sentence $\varphi \rightarrow^w \psi$ as follows: ‘if φ , then ψ that legitimises φ ’.

The operator of permission is then defined in the following way (+ stresses that the permission is explicit):

$$P^+\varphi := (\varphi \rightarrow^w \text{permit}) \wedge (\neg\varphi \supset \text{permit}). \quad (P^+)$$

Definition (P^+) says that φ is permitted ($P^+\varphi$) iff (1) if φ then **permit**, that legitimises φ and (2) if $\neg\varphi$, then still **permit**. These conditions express the thought that no matter whether φ is true or not (in the latter case $\neg\varphi$ is true), a permission has been issued, i.e. **permit** is true, and this legitimises φ . Exploiting the classical understanding of \neg and \supset , and assuming that \rightarrow^w is closed under Modus Ponens, (P^+) might be reduced to the following definition, which will be used henceforth:

$$P^+\varphi := (\varphi \rightarrow^w \text{permit}) \wedge \text{permit}. \quad (\text{Def } P^+)$$

Various constraints can be imposed on it, and in turn shape the given system. The following three choice conditions discussed in [8] are at the basis of the formalisation:

$$P^+(\varphi \vee \psi) \supset (P^+\varphi \wedge P^+\psi) \quad (\text{FCP-I})$$

$$P^+(\varphi \vee \psi) \supset P^+(\varphi \wedge \psi) \quad (\text{FCP-II})$$

$$P^+(\varphi \vee \psi) \supset \neg P^+(\varphi \wedge \psi). \quad (\text{FCP-III})$$

Informally, (FCP-I) states the basic free choice property. Namely, that if $\varphi \vee \psi$ is permitted then both φ and ψ are permitted. (FCP-II) and (FCP-III) are alternative extensions of that property concerning the con-

junction of φ and ψ that can be construed as a joint realisation of them. (FCP-II) states that under assumption that $\varphi \vee \psi$ is permitted so is the conjunction of φ and ψ and (FCP-III) – the opposite: that it is not true that their conjunction is permitted.

After applying the definition (Def P^+) to (FCP-I)–(FCP-III), we reduce them to the following schemas:

$$\begin{aligned} & (((\varphi \vee \psi) \rightarrow^w \text{permit}) \wedge \text{permit}) \\ & \quad \supset (((\varphi \rightarrow^w \text{permit}) \wedge (\psi \rightarrow^w \text{permit})) \wedge \text{permit}) \quad (\text{RFCP-I}) \end{aligned}$$

$$\begin{aligned} & (((\varphi \vee \psi) \rightarrow^w \text{permit}) \wedge \text{permit}) \\ & \quad \supset (((\varphi \wedge \psi) \rightarrow^w \text{permit}) \wedge \text{permit}) \quad (\text{RFCP-II}) \end{aligned}$$

$$\begin{aligned} & (((\varphi \vee \psi) \rightarrow^w \text{permit}) \wedge \text{permit}) \\ & \quad \supset \neg(((\varphi \wedge \psi) \rightarrow^w \text{permit}) \wedge \text{permit}). \quad (\text{RFCP-III}) \end{aligned}$$

The semantics proposed in [13] and [12] is used to interpret the meaning of the constants of this language formally. To begin with, a *model* is an ordered pair $\langle v, R \rangle$ such that:

- $v: \text{Var} \cup \{\text{permit}\} \mapsto \{1, 0\}$ is a valuation of propositional variables and **permit**
- R is a binary relation defined on $\text{For} \times \text{For}$.

Note that in a given model, $\langle v, R \rangle$, $v(\text{permit}) = 1$ or $v(\text{permit}) = 0$, so a permission has or has not been issued. The relation R in the model involves pairs of sentences that are related by legitimisation.

The truth conditions for formulas are as follows. Let $\mathfrak{M} = \langle v, R \rangle$ and $\varphi \in \text{For}$. φ is *true in* \mathfrak{M} ($\mathfrak{M} \models \varphi$) iff for any $\psi, \chi \in \text{For}$ ($\mathfrak{M} \not\models \varphi$ means that φ is false):

$v(\varphi) = 1,$	if $\varphi \in \text{Var} \cup \{\text{permit}\}$
$\mathfrak{M} \not\models \psi,$	if $\varphi = \neg\psi$
$\mathfrak{M} \models \psi$ and $\mathfrak{M} \models \chi,$	if $\varphi = \psi \wedge \chi$
$\mathfrak{M} \models \psi$ or $\mathfrak{M} \models \chi,$	if $\varphi = \psi \vee \chi$
$\mathfrak{M} \not\models \psi$ or $\mathfrak{M} \models \chi,$	if $\varphi = \psi \supset \chi$
$(\mathfrak{M} \not\models \psi$ or $\mathfrak{M} \models \chi)$ and $R_{\mathfrak{M}}(\psi, \chi),$	if $\varphi = \psi \rightarrow^w \chi.$

When $\mathfrak{M} = \langle v, R \rangle$, we will sometimes write $v_{\mathfrak{M}} (R_{\mathfrak{M}})$ instead of $v (R)$. According to the definition of truth in model, the classical connectives have a standard meaning. The constant **permit** is true in the model when $v(\mathbf{permit}) = 1$. The relating implication $\varphi \rightarrow^w \psi$ is true in the model when the predecessor φ is false or the successor ψ is true, and both arguments are related, i.e.: $R(\varphi, \psi)$. The intended reading we have given to $\varphi \rightarrow^w \psi$ is: ‘if φ , then ψ that legitimises φ ’. In this situation, the relation R from the model is the converse of legitimisation. It is assumed in [8] that legitimisation is irreflexive and transitive. Thus, for all $\varphi, \psi, \chi \in \mathbf{For}$ we have:

$$\sim R(\varphi, \varphi) \tag{Ir}$$

$$(R(\varphi, \psi) \text{ and } R(\psi, \chi)) \implies R(\varphi, \chi). \tag{Tr}$$

We thus only consider models with relations that meet these two conditions. The schemas (FCP-I)–(FCP-III) in turn help to capture different variants of free choice permission. Taking into account that R is the converse of legitimisation we can transform them, respectively, in a rather straightforward manner. For all $\varphi, \psi \in \mathbf{For}$ we have:

$$R(\varphi \vee \psi, \mathbf{permit}) \implies (R(\varphi, \mathbf{permit}) \text{ and } R(\psi, \mathbf{permit})) \tag{R1}$$

$$R(\varphi \vee \psi, \mathbf{permit}) \implies R(\varphi \wedge \psi, \mathbf{permit}) \tag{R2}$$

$$R(\varphi \vee \psi, \mathbf{permit}) \implies \sim R(\varphi \wedge \psi, \mathbf{permit}). \tag{R3}$$

The following fact is proved in [8]:

FACT 2.1. Let $\langle v, R \rangle$ be a model. Then:

- (1) if R satisfies (R1), then $\langle v, R \rangle \models (\mathbf{RFCP-I})$
- (2) if R satisfies (R2), then $\langle v, R \rangle \models (\mathbf{RFCP-II})$
- (3) if R satisfies (R3), then $\langle v, R \rangle \models (\mathbf{RFCP-III})$.

It is worth noting that the converse implications of fact 2.1 are false. To prove this, it suffices to consider models with false **permit**.

Let \bar{m} be the powerset of $\{1, 2, 3\}$, the empty set excluded. The following seven classes of models can be defined: \mathbf{M}_x , where $x \in \bar{m}$ and \mathbf{M}_x contains all relations satisfying the (R*i*) condition, where $i \in x$. Moreover, all relations in the models are irreflexive and transitive, as we assumed earlier. The logic $\models_{\mathbf{M}_x}$, where $x \in \bar{m}$, is determined in a standard way

as the semantic consequence relation defined on the Cartesian product of the powerset of **For** and **For** by the appropriate class of models \mathbf{M}_i . Thus, $\Phi \models_{\mathbf{M}_i} \psi$ iff for all models $\mathfrak{M} \in \mathbf{M}_x$, if for all $\chi \in \Phi, \mathfrak{M} \models \chi$, then $\mathfrak{M} \models \psi$. Consequently we obtain the logics in which our variants of free choice permission occur (by fact 2.1), two of which do not behave well. We mean the logics defined by the classes of models: $\models_{\mathbf{M}_{\{1,2,3\}}}$ and $\models_{\mathbf{M}_{\{2,3\}}}$. This is because conditions (R2) and (R3) remain in a logical conflict: from $P^+(\varphi \vee \psi)$ (a.k.a. $((\varphi \vee \psi) \rightarrow^w \text{permit}) \wedge \text{permit}$) in these logics we can conclude anything. The conflict is explained by another fact proved in [8]:

FACT 2.2. Let $\models_{\mathbf{M}_x}$ be a logic determined by such a class of models \mathbf{M}_x that $2, 3 \in x$. Let $\varphi, \psi, \chi \in \text{For}$. Then: $((\varphi \vee \psi) \rightarrow^w \text{permit}) \wedge \text{permit} \models_{\mathbf{M}_x} \chi$.

Fact 2.2 shows that examining only the logics defined in terms of the five classes of models makes sense: $\mathbf{M}_{\{1\}}, \mathbf{M}_{\{2\}}, \mathbf{M}_{\{3\}}, \mathbf{M}_{\{1,2\}}, \mathbf{M}_{\{1,3\}}$. Indeed, since (FCP-I) is the basic free choice condition that we have considered, we are mainly interested in models that include condition 1, i.e., $\mathbf{M}_{\{1\}}, \mathbf{M}_{\{1,2\}}, \mathbf{M}_{\{1,3\}}$, that define basic, non-exclusive and exclusive free choice permission respectively.

3. Tableau systems for the presented logics

When presenting tableaux, we will be guided by the strategy and results presented in the articles [11], [15], and [14]. To define the tableau systems for the given five logics, we need to define some additional notions.

The language of the tableau systems is the language of **For** extended by auxiliary expressions. The *set of auxiliary expressions* (formally: **Ae**) is the least set Σ such that: if $\varphi, \psi \in \text{For}$, then $\varphi \mathbf{r} \psi, \varphi \bar{\mathbf{r}} \psi \in \Sigma$. Expressions of the form $\varphi \mathbf{r} \psi$ and $\varphi \bar{\mathbf{r}} \psi$ are supposed to represent relations in the tableau language and state that φ, ψ are or are not related, respectively. The *set of tableau expressions* is set $\text{Ex} = \text{For} \cup \text{Ae}$.

In addition, we need the notion of tableau inconsistency. Let $\Sigma \subseteq \text{Ex}$. Σ is *tableau inconsistent* (for short: *t-inconsistent*) iff at least one of the following conditions is satisfied: there is $\varphi \in \text{For}$ such that $\varphi, \neg\varphi \in \Sigma$ or there are $\varphi, \psi \in \text{For}$ such that $\varphi \mathbf{r} \psi, \varphi \bar{\mathbf{r}} \psi \in \Sigma$. Σ is *tableau consistent* (for short: *t-consistent*) iff Σ is not t-inconsistent.

$$\begin{array}{l}
(R_{\wedge}) \frac{\varphi \wedge \psi}{\varphi, \psi} \quad (R_{\vee}) \frac{\varphi \vee \psi}{\varphi \mid \psi} \quad (R_{\supset}) \frac{\varphi \supset \psi}{\neg\varphi \mid \psi} \\
(R_{\rightarrow^w}) \frac{\varphi \rightarrow^w \psi}{\neg\varphi, \varphi \mathbf{r} \psi \mid \psi, \varphi \mathbf{r} \psi} \quad (R_{\neg\neg}) \frac{\neg\neg\varphi}{\varphi} \quad (R_{\neg\wedge}) \frac{\neg(\varphi \wedge \psi)}{\neg\varphi \mid \neg\psi} \\
(R_{\neg\vee}) \frac{\neg(\varphi \vee \psi)}{\neg\varphi, \neg\psi} \quad (R_{\neg\supset}) \frac{\neg(\varphi \supset \psi)}{\varphi, \neg\psi} \quad (R_{\neg\rightarrow^w}) \frac{\neg(\varphi \rightarrow^w \psi)}{\varphi, \neg\psi \mid \varphi \bar{\mathbf{r}} \psi}
\end{array}$$

Figure 1. Rules of logical connectives

We propose a set of tableau rules constructed with expressions Ex . The expressions in the numerator of a given rule will be called *input*, and expressions in the denominator will be called *output*. Some rules can have more outputs than one; see, for example, the rule (R_{\rightarrow^w}) in Figure 1. The standard elimination rules for the classical connectives and the rule for relating implication are introduced in the mentioned figure. The set containing all these rules along with the rules (R_{Ir}) and (R_{Tr}) in Figure 2 will be denoted TR . The remaining tableau rules in Figure 2 are optional – used only for some of the considered systems. By TR_x , where $x \subseteq \{1, 2, 3\}$, we denote the set of tableau rules $\text{TR} \cup \{(R_{R_i}) : i \in x\}$.

$$\begin{array}{l}
(R_{\text{Ir}}) \frac{\varphi \mathbf{r} \varphi}{\varphi \bar{\mathbf{r}} \varphi} \quad (R_{\text{Tr}}) \frac{\varphi \mathbf{r} \psi, \psi \mathbf{r} \chi}{\varphi \mathbf{r} \chi} \quad (R_{\text{R1}}) \frac{\varphi \vee \psi \mathbf{r} \text{ permit}}{\varphi \mathbf{r} \text{ permit}, \psi \mathbf{r} \text{ permit}} \\
(R_{\text{R2}}) \frac{\varphi \vee \psi \mathbf{r} \text{ permit}}{\varphi \wedge \psi \mathbf{r} \text{ permit}} \quad (R_{\text{R3}}) \frac{\varphi \vee \psi \mathbf{r} \text{ permit}}{\varphi \wedge \psi \bar{\mathbf{r}} \text{ permit}}
\end{array}$$

Figure 2. Rules of legitimisation relation

Now we define the notion of closure under the set of tableau rules TR_x . Let $\Sigma, \Gamma \subseteq \text{Ex}$. The set Γ is a *closure of Σ under the set of tableau rules TR_x* iff (1) $\Sigma \subseteq \Gamma$, (2) Γ is either t-inconsistent, or Γ is such a minimal set that for all tableau rules r in TR_x , if an input of r is contained in Γ , so is at least one output. Having a set of tableau rules TR_x , we would finally like to define the concept of tableau operation consequence. Formula ϕ is

a *tableau consequence* of set of formulas Σ in respect of TR_x (formally: $\Sigma \triangleright_{\text{TR}_x} \phi$) iff there exists such a finite $\Sigma' \subseteq \Sigma$ that all closures of $\Sigma' \cup \{\neg\phi\}$ under the set of tableau rules TR_x are t-inconsistent.

In order to illustrate how the tableaux works we present a proof that (FCP) is a tableau consequence of the empty set (see Figure 3).

To prove the metatheoretical relationships between tableaux and models, we need the notion of suitability. Let $\mathfrak{M} = \langle v, R \rangle$ be a model and $\Sigma \subseteq \text{Ex}$. \mathfrak{M} is *suitable* for Σ iff for all $\varphi, \psi \in \text{For}$:

- if $\varphi \in \Sigma$, then $\mathfrak{M} \models \varphi$
- if $\varphi \text{ r } \psi \in \Sigma$ then, $R_{\mathfrak{M}}(\varphi, \psi)$
- if $\varphi \bar{\text{r}} \psi \in \Sigma$, then $\sim R_{\mathfrak{M}}(\varphi, \psi)$.

LEMMA 3.1. *Let $\Sigma \subseteq \text{Ex}$ and $\mathfrak{M} = \langle v, R \rangle \in \mathbf{M}_x$ be suitable for Σ . For any tableau rule $r \in \text{TR}_x$, if r has been applied to Σ , then \mathfrak{M} is suitable for the union of Σ and at least one output obtained by the application of rule r .*

PROOF: The proof of 3.1 is by inspection of the tableau rules. For the rules from TR the proof is presented in [15]. For (R_{R1}) we assume that $\varphi \vee \psi \text{ r permit} \in \Sigma$ and since \mathfrak{M} is suitable for Σ , $R_{\mathfrak{M}}(\varphi \vee \psi, \text{permit})$. If the rule was applied, the output $\{\varphi \text{ r permit}, \psi \text{ r permit}\}$ was obtained. Since the model \mathfrak{M} satisfies the condition (R1), $R_{\mathfrak{M}}(\varphi, \text{permit})$ and $R_{\mathfrak{M}}(\psi, \text{permit})$, and by the definition of a suitable model, \mathfrak{M} is suitable for $\Sigma \cup \{\varphi \text{ r permit}, \psi \text{ r permit}\}$. The remaining cases (R_{R2}) and (R_{R3}) are similar. \square

Finally, to show completeness of the tableau system, we will introduce the notion of generated model. Let TR_x be a set of tableau rules. Let $\Sigma \subseteq \text{Ex}$ be a t-consistent closure under TR_x . A *model generated by Σ* (for short: Σ -model) is a model $\langle v_{\Sigma}, R_{\Sigma} \rangle$ such that:

- for any $\varphi \in \text{Var}$:

$$v_{\Sigma}(\varphi) = \begin{cases} 1, & \text{if } \varphi \in \Sigma \\ 0, & \text{if } \varphi \notin \Sigma \end{cases}$$

- for any $\varphi, \psi \in \text{For}$:

$$R_{\Sigma}(\varphi, \psi) \text{ iff } \varphi \text{ r } \psi \in \Sigma.$$

Now, we have another lemma:

LEMMA 3.2. *Let TR_x be a set of tableau rules and Σ be a t -consistent closure under TR_x . Then, there is a model \mathfrak{M} such that:*

- (1) $\mathfrak{M} \in \mathbf{M}_x$
- (2) for any $\varphi \in \text{For}$, if $\varphi \in \Sigma$ then $\mathfrak{M} \models \varphi$.

PROOF: Again, the proof of 3.2 is made by inspection of the tableau rules and by induction. As a model in the thesis we take Σ -model $\mathfrak{M} = \langle v_\Sigma, R_\Sigma \rangle$. For (1): Suppose that $(R_{R3}) \in \text{TR}_x$. So, $3 \in x$. Let $\varphi, \psi \in \text{For}$ and $R_{\mathfrak{M}}(\varphi \vee \psi, \text{permit})$. By the definition of generated model, $\varphi \vee \psi \text{ r permit} \in \Sigma$.

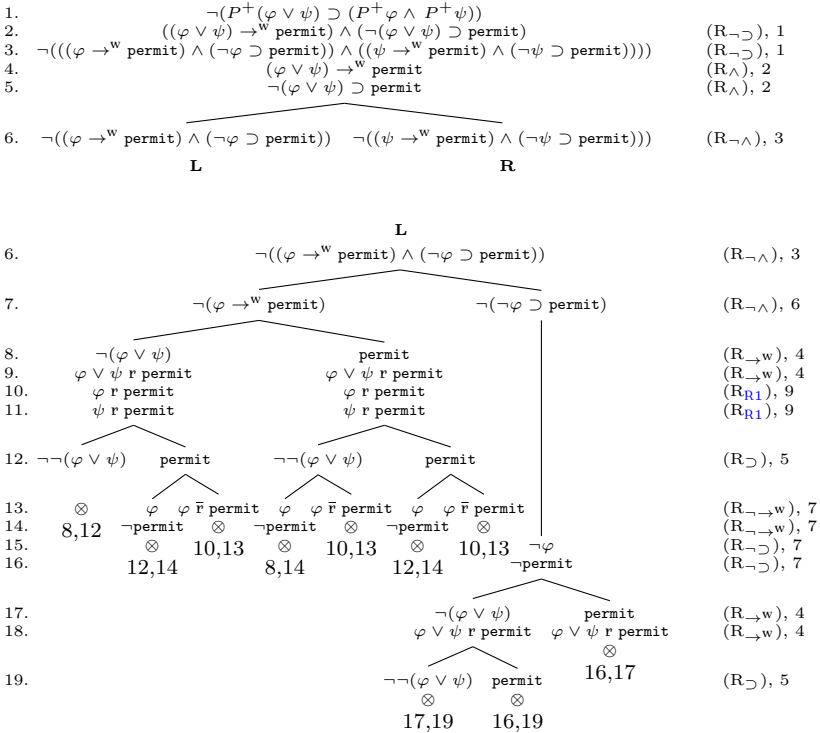


Figure 3. A tableau proof of (FCP) (left branch)

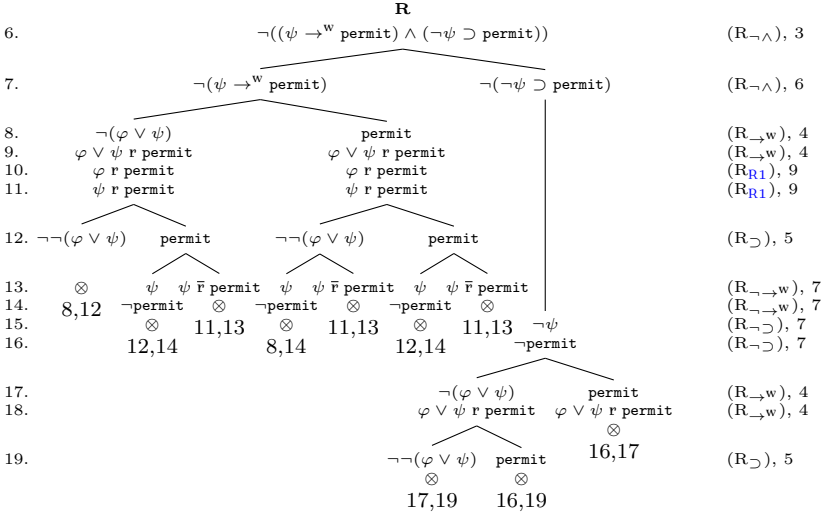


Figure 3 (continued). A tableau proof of (FCP) (right branch)

Since Σ is a closure under TR_x , $\varphi \wedge \psi \bar{\text{r}} \text{ permit} \in \Sigma$. Thus, by the definition of generated model, $\sim R(\varphi \wedge \psi, \text{r})$, since Σ is t-consistent. Finally, the model \mathfrak{M} belongs to the class of models satisfying the condition (R3). The remaining cases are either similar ((R R_1), (R R_2)) or were already examined in [15] ((R I), (R T)). For the point (2) we start from the atomic cases, using the definition of generated model, and then we examine the decomposition rules which was done in [13] and [15]. □

Now we can obtain the soundness and completeness of our tableau systems.

THEOREM 3.3. *Let TR_x be a set of tableau rules and $\Sigma \cup \{\varphi\} \subseteq \text{For}$. Then, $\Sigma \models_{\text{M}_x} \varphi$ iff $\Sigma \triangleright_{\text{TR}_x} \varphi$.*

PROOF: Assume all the hypotheses. For ‘left to right’ suppose $\Sigma \not\triangleright_{\text{TR}_x} \varphi$. So, for any finite $\Gamma \subseteq \Sigma$ there is a t-consistent closure Δ of $\Gamma \cup \{\neg\varphi\}$ under TR_x , such that $\Gamma \cup \{\neg\varphi\} \subseteq \Delta$. Hence, there is a t-consistent closure Δ' of $\Sigma \cup \{\neg\varphi\}$ under TR_x such that $\Sigma \cup \{\neg\varphi\} \subseteq \Delta'$. Otherwise, any such a closure would contain some t-inconsistency. But by definition of closure

this would mean that for some finite $\Gamma \subseteq \Sigma$ no closure of $\Gamma \cup \{\neg\varphi\}$ under TR_x is t-consistent. As a consequence, by lemma 3.2, there is a Δ' -model $\mathfrak{M} \in \mathbf{M}_x$ such that $\mathfrak{M} \models \Sigma \cup \{\neg\varphi\}$. Therefore, $\Sigma \not\models_{\mathbf{M}_x} \varphi$.

For ‘right to left’ suppose $\Sigma \triangleright_{\text{TR}_x} \varphi$. Hence, there is a finite $\Gamma \subseteq \Sigma$ such that any closure of $\Gamma \cup \{\neg\varphi\}$ under TR_x is t-inconsistent. Suppose that there is a model $\mathfrak{M} \in \mathbf{M}_x$ such that $\mathfrak{M} \models \Sigma$ and $\mathfrak{M} \models \neg\varphi$. By definition of a suitable model, \mathfrak{M} is suitable for $\Gamma \cup \{\neg\varphi\}$. By lemma 3.1 there is a closure $\Gamma \cup \{\neg\varphi\}$ under TR_x , for which \mathfrak{M} is suitable. However, such a closure must be t-inconsistent. Thus in the closure either there is $\psi \in \text{For}$ such that $\mathfrak{M} \models \psi$ and $\mathfrak{M} \not\models \psi$ or there are $\psi, \chi \in \text{For}$ such that $\text{R}_{\mathfrak{M}}(\psi, \chi)$ and $\sim \text{R}_{\mathfrak{M}}(\psi, \chi)$. Hence, for any model $\mathfrak{M} \in \mathbf{M}_x$, $\mathfrak{M} \models \Sigma$ implies $\mathfrak{M} \models \varphi$. Therefore, $\Sigma \models_{\mathbf{M}_x} \varphi$. \square

4. Concluding remarks

The paper introduces tableau systems for the family of logics of free choice permission introduced in [8].

The presented tableau systems constitute an effective decision procedure for these logics, because all branches in proofs are of a finite length.

The formalism from [8] can be further developed in many directions. Including other normative notions into the system seems to be a natural step here. Applying Andersonian-Kangerian approach, with `violation` constant for prohibition and obligation would allow to reuse the legitimisation relation for the purpose of understanding these notions. We believe that the tableau approach proposed in the present paper can be quite naturally applied to such extensions of the systems.

Acknowledgements. This work was supported by the Slovak Research and Development Agency under the contract no. APVV-17-0057 and VEGA 2/0125/22 and by National Science Centre, Poland (UMO-2017/26/M/HS1/01092). This work was presented at the conference Non-classical Logic 2022. We are grateful to the audience for their valuable feedback. We would also like to thank Luis Estrada-González for his remarks and suggestions. The contents of this article are the result of a joint research work of the four authors.

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