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FIRST-ORDER MODAL SEMANTICS AND EXISTENCE PREDICATE

Abstract

In the article we study the existence predicate ε in the context of semantics for first-order modal logic. For a formula φ we define φ^{ε}—the so called *existence relativization*. We point to a gap in the work of Fitting and Mendelsohn [1] concerning the relationship between the truth of φ and φ^{ε} in classes of varying- and constant-domain models. We introduce operations on models which allow us to fill the gap and provide a more general perspective on the issue. As a result we obtain a series of theorems describing the logical connection between the notion of truth of a formula with the existence predicate in constant-domain models and the notion of truth of a formula without the existence predicate in varying-domain models.

Keywords: First-order modal logic, constant-domain model, varying-domain model, existence predicate.

Introduction

Semantic theory for first-order modal logic makes use of two philosophically important notions of varying- and constant-domain models which may shape the discussion about the role of existence predicate in modal logic and the meaning of quantifying over non-existing entities. Models with constant domains correspond to quantifying over merely-possible objects in addition to actually existent entities, while models with varying domains are in consonance with the actualistic interpretation of the quantifier, restraining quantification to that what actually exists. Relationship between

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the two approaches is often being studied via incorporating the existence predicate in the first-order language and examination of the translation of formulas without such a predicate into formulas containing it.

The question whether existence is a property of individuals or even whether it is a property at all has baffled philosophers and logicians for centuries, starting with Immanuel Kant and his *Critique of Pure Reason* in which he argued that existence is not a genuine attribute of things. This idea, defended in its particular form by Frege [2], is built into the very foundation of modern mathematical logic. It manifests itself in the use of the existential quantifier *instead of* the existence predicate. To say that there exists a root of the equation $x^2 - 3x = 0$ is to say that the propositional function '$x^2 - 3x = 0$' is satisfied by *some* number and that is to say that the proposition $\exists x(x^2 - 3x = 0)$ is true.

However, some philosophers, like Alexius Meinong [4], have felt the need for having the existence predicate in addition to the existential quantifier. One obvious way of introducing such a predicate in a first-order language is to define 'x exists' as $\exists y(x = y)$. The problem is that in classical first-order logic individual variables always denote something, and the formula $\exists y(x = y)$ is satisfied in every model. Another possibility is to introduce the existence predicate as a primitive symbol. Assuming the existence predicate is a unary predicate ε the question arises: what does and what does not exist? And this depends on the quantifiers. (For some discussion of these issues you can see [3].) For if the quantifiers quantify over existent objects only, the proposition $\forall x \varepsilon(x)$ is logically true and for any formula φ, $\forall x(\varepsilon(x) \wedge \varphi(x))$ and $\forall x \varphi(x)$ are equivalent, making the existence predicate redundant. If, on the other hand, the scope of quantification includes objects which do not exist but are possible, the existence predicate can do its job and select among all entities those which actually exist. This is exactly the idea standing behind the constant-domain models. Moreover, if the existence predicate seems redundant when quantifiers are actualistic, for then everything exists, but turns out to be useful when quantifiers are possibilistic, surely there must be some kind of connection between these two ways of doing logic. And, indeed, there is.

1. Preliminaries

All crucial definitions and elementary facts can be found in [1]. For readers' convenience let us remind basic concepts. The language with which we will deal is the standard first-order language with individual variables as the only terms with the addition of $\Box$ as the modal operator. We will take $\Box$, $\neg$, $\wedge$ and $\exists$ as primitive.

Two of the most commonly used on the next pages will be notions of *constant-* and *varying-domain* models. We will treat constant-domain models as a special case of varying-domain models (as they actually are). So for us 'model' and 'varying-domain model' will mean pretty much the same.

A (varying-domain) model $\mathcal{M}$ is a four-tuple $(\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I})$ such that $\mathcal{U}$ is a non-empty set (its elements we will also call 'worlds' or 'points'), $\mathcal{R} \subseteq \mathcal{U} \times \mathcal{U}$ is a binary relation (called the *accessibility relation*), $\mathcal{D}$ is a function which maps elements of $\mathcal{U}$ to non-empty sets—to each element u of $\mathcal{U}$ it assigns a non-empty set $\mathcal{D}(u)$ which we call *a domain of* u, and by $\mathcal{D}(\mathcal{M})$ we mean the sum of all $\mathcal{D}(u)$. $\mathcal{I}$ is an interpretation of predicates. Strictly speaking, $\mathcal{I}$ is a mapping such that $\mathcal{I}(r, u) \subseteq \mathcal{D}(\mathcal{M})^{\tau(r)}$, where r is a predicate and $\tau(r)$ is arity of r.

A *valuation* is a map $v \colon Var \to \mathcal{D}(\mathcal{M})$, where Var is a set of all individual variables. For $a \in \mathcal{D}(\mathcal{M})$ and $x \in Var$, by $v(a/x)$ we mean a valuation such that $v(a/x)(x) = a$ and for any variable y distinct from x, $v(a/x)(y) = v(y)$.

The satisfaction relation $\Vdash$ is defined recursively in the standard way as follows.

Definition 1.1. Take a model $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}), u \in \mathcal{U}$, valuation v, and predicate r of arity n. For a formula φ we define the expression

$$(\mathcal{M}, u) \Vdash \varphi[v],$$

which we read as φ *is satisfied at* u *in model* $\mathcal{M}$ *under valuation* v:

(i) $(\mathcal{M}, u) \Vdash r(x_1, \ldots, x_n)[v] \iff \langle v(x_1), \ldots, v(x_n)\rangle \in \mathcal{I}(r, u)$,

(ii) $(\mathcal{M}, u) \Vdash \neg\varphi[v] \iff (\mathcal{M}, u) \nVdash \varphi[v]$,

(iii) $(\mathcal{M}, u) \Vdash (\varphi \wedge \psi)[v] \iff (\mathcal{M}, u) \Vdash \varphi[v]$ and $(\mathcal{M}, u) \Vdash \psi[v]$,

(iv) $(\mathcal{M}, u) \Vdash \Box\varphi[v] \iff$ for any $t \in \mathcal{U}$, if $u\mathcal{R}t$, then $(\mathcal{M}, t) \Vdash \varphi[v]$,

(v) $(\mathcal{M}, u) \Vdash \exists x\varphi[v] \iff$ there is $a \in \mathcal{D}(u)$ and $(\mathcal{M}, u) \Vdash \varphi[v(a/x)]$.

A formula φ is satisfied by a class of models K, $K \Vdash \varphi$ in symbols, when $(\mathcal{M}, t) \Vdash \varphi[v]$, for any $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}) \in K$, any $t \in \mathcal{U}$, and any valuation v. By $\mathbb{VD}$ we denote the class of all (varying-domain) models. Moreover, let $\mathbb{CD}$ stand for the class of all models $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I})$ such that $\mathcal{D}(u) = \mathcal{D}(w)$, for any $u, w \in \mathcal{U}$. Elements of $\mathbb{CD}$ are called *constant-domain models*.

Definition 1.2. Let ε be a unary predicate. Following Fitting and Mendelsohn, for any φ we define φ^ε as follows:

(i) For an atomic formula, $r(x_1, \ldots, x_n)^\varepsilon = r(x_1, \ldots, x_n)$,

(ii) $(\neg\varphi)^\varepsilon = \neg(\varphi)^\varepsilon$,

(iii) $(\varphi \wedge \psi)^\varepsilon = (\varphi)^\varepsilon \wedge (\psi)^\varepsilon$,

(iv) $(\Box\varphi)^\varepsilon = \Box(\varphi)^\varepsilon$,

(v) $(\exists x\varphi)^\varepsilon = \exists x(\varepsilon(x) \wedge \varphi^\varepsilon)$.

2. The construction

In [1] (Proposition 4.8.2.) one can find the claim that

$$\mathbb{VD} \Vdash \varphi \iff \mathbb{CD} \Vdash \varphi^\varepsilon \qquad (\star)$$

for any sentence φ which does not contain ε. Implication to the left is proven by authors, while the other direction is left to the reader. However, we observed that this implication fails. Indeed, let us consider the sentence:

$$\exists x(r(x) \vee \neg r(x)),$$

where r is an arbitrary unary predicate (distinct from ε). Then we obtain:

$$\begin{aligned}(\exists x(r(x) \vee \neg r(x)))^\varepsilon &= \exists x(\varepsilon(x) \wedge (r(x) \vee \neg r(x))^\varepsilon) \\ &= \exists x(\varepsilon(x) \wedge (r(x) \vee \neg r(x))).\end{aligned}$$

Clearly, $\exists x(r(x) \vee \neg r(x))$ is valid in all varying-domain models, however $\exists x(\varepsilon(x) \wedge (r(x) \vee \neg r(x)))$ is not valid in those constant-domain models in which ε is interpreted as empty and this falsifies $(\star)$[1].

Although the implication $\mathbb{VD} \Vdash \varphi \Longrightarrow \mathbb{CD} \Vdash \varphi^\varepsilon$ does not hold, we can still prove a weaker version. Before we do it, let us introduce a couple of definitions and facts. If $K \subseteq \mathbb{VD}$, by K_ε we denote the class of those models $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I})$ from K such that $\mathcal{I}(\varepsilon, t) \neq \emptyset$, for any $t \in \mathcal{U}$.

DEFINITION 2.1. Let $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}) \in \mathbb{CD}_\varepsilon$ and let w be any object such that $w \notin \mathcal{U}$. We define a model $\mathcal{M}^w \in \mathbb{VD}$ as $\mathcal{M}^w = (\mathcal{U}^w, \mathcal{R}^w, \mathcal{D}^w, \mathcal{I}^w)$, where $\mathcal{U}^w = \mathcal{U} \cup \{w\}$, $\mathcal{R}^w = \mathcal{R}$, and

$$\mathcal{D}^w(t) = \begin{cases} \mathcal{I}(\varepsilon, t) & \text{if } t \neq w, \\ \mathcal{D}(\mathcal{M}) & \text{if } t = w, \end{cases} \quad \text{and} \quad \mathcal{I}^w(r, t) = \begin{cases} \mathcal{I}(r, t) & \text{if } t \neq w, \\ \mathcal{D}(\mathcal{M})^{\tau(r)} & \text{if } t = w. \end{cases}$$

FACT 2.2. $\mathcal{M}^w \in \mathbb{VD}_\varepsilon$, for any $\mathcal{M} \in \mathbb{CD}_\varepsilon$.

FACT 2.3. Let $\mathcal{S}$ be any proposition of our meta-language (the very language of this paper). For any $\mathcal{U}, t, w$ and $\mathcal{R}$ as in Definition 2.1, the following assertions are equivalent:

(i) *For any* $t \in \mathcal{U}$, such that $u\mathcal{R}t$, $\mathcal{S}$

(ii) *For any* $t \in \mathcal{U} \cup \{w\}$, such that $u\mathcal{R}t$, $\mathcal{S}$

PROOF: ($\Longleftarrow$) Trivial.
($\Longrightarrow$) Let $t \in \mathcal{U} \cup \{w\}$. If $t \in \mathcal{U}$, by the assumption, thesis holds. If $t = w$, then, by definition of $\mathcal{R}$, $u\mathcal{R}t$ fails and therefore the thesis holds.

□

Now we can prove the following lemma.

LEMMA 2.4. *For any formula* φ *not containing* ε*, model* $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}) \in \mathbb{CD}_\varepsilon$*,* $w \notin \mathcal{U}$*,* $t \in \mathcal{U}$*, and valuation* v*,*

$$(\mathcal{M}, t) \Vdash \varphi^\varepsilon[v] \Longleftrightarrow (\mathcal{M}^w, t) \Vdash \varphi[v].$$

PROOF: We will prove it inductively.

[1] An error of which Prof. Fitting had been aware before we observed it as he said in personal correspondence, and gratefully offered a suggestion that non-emptyness of the existence predicate is a requirement—an idea which we develop in this article.

For an atomic formula $r(x_1, \ldots, x_n)$ we have:

$$
\begin{aligned}
&(\mathcal{M}, t) \Vdash r(x_1, \ldots, x_n)^{\varepsilon}[v] \iff (\mathcal{M}, t) \Vdash r(x_1, \ldots, x_n)[v] && \text{(by 1.2)}\\
&\iff \langle v(x_1), \ldots, v(x_n)\rangle \in \mathcal{I}(r, t) && \text{(by 1.1)}\\
&\iff \langle v(x_1), \ldots, v(x_n)\rangle \in \mathcal{I}^w(r, t) && \text{(by 2.1)}\\
&\iff (\mathcal{M}^w, t) \Vdash r(x_1, \ldots, x_n)[v] && \text{(by 1.1)}
\end{aligned}
$$

Crucial in this step is the fact that interpretations of predicates are the same in the new model for the 'old worlds' *and* that valuations are the same, i.e. every valuation into $\mathcal{M}$ is a valuation into $\mathcal{M}^w$ and vice versa.

For negation we get:

$$
\begin{aligned}
&(\mathcal{M}, t) \Vdash (\neg\psi)^{\varepsilon}[v] \iff (\mathcal{M}, t) \Vdash \neg\psi^{\varepsilon}[v] && \text{(by 1.2)}\\
&\iff (\mathcal{M}, t) \nVdash \psi^{\varepsilon}[v] && \text{(by 1.1)}\\
&\iff (\mathcal{M}^w, t) \nVdash \psi[v] && \text{(induction)}\\
&\iff (\mathcal{M}^w, t) \Vdash \neg\psi[v] && \text{(by 1.1)}
\end{aligned}
$$

For conjunction we get:

$$
\begin{aligned}
&(\mathcal{M}, t) \Vdash (\psi \wedge \chi)^{\varepsilon}[v] \iff (\mathcal{M}, t) \Vdash (\psi^{\varepsilon} \wedge \chi^{\varepsilon})[v] && \text{(by 1.2)}\\
&\iff (\mathcal{M}, t) \Vdash \psi^{\varepsilon}[v] \text{ and } (\mathcal{M}, t) \Vdash \chi^{\varepsilon}[v] && \text{(by 1.1)}\\
&\iff (\mathcal{M}^w, t) \Vdash \psi[v] \text{ and } (\mathcal{M}^w, t) \Vdash \chi[v] && \text{(induction)}\\
&\iff (\mathcal{M}^w, t) \Vdash (\psi \wedge \chi)[v] && \text{(by 1.1)}
\end{aligned}
$$

For box we have:

$$
\begin{aligned}
&(\mathcal{M}, t) \Vdash (\Box\psi)^{\varepsilon}[v] \iff (\mathcal{M}, t) \Vdash \Box\psi^{\varepsilon}[v] && \text{(by 1.2)}\\
&\iff \text{ for any } s \in \mathcal{U}, \text{ if } t\mathcal{R}s, \text{ then } (\mathcal{M}, s) \Vdash \psi^{\varepsilon}[v] && \text{(by 1.1)}\\
&\iff \text{ for any } s \in \mathcal{U}, \text{ if } t\mathcal{R}^w s, \text{ then } (\mathcal{M}^w, s) \Vdash \psi[v] && \text{(induction)}\\
&\iff \text{ for any } s \in \mathcal{U} \cup \{w\}, \text{ if } t\mathcal{R}^w s, \text{ then } (\mathcal{M}^w, s) \Vdash \psi[v] && \text{(by 2.3)}\\
&\iff (\mathcal{M}^w, s) \Vdash \Box\psi[v] && \text{(by 1.1)}
\end{aligned}
$$

For the quantifier we have:

$$\begin{aligned}
(\mathcal{M},t) \Vdash (\exists x\psi)^{\varepsilon}[v] &\iff (\mathcal{M},t) \Vdash \exists x(\varepsilon(x) \wedge \psi^{\varepsilon})[v] && \text{(by 1.2)}\\
&\iff \exists_{a\in\mathcal{D}(t)}\ (\mathcal{M},t) \Vdash (\varepsilon(x) \wedge \psi^{\varepsilon})[v(a/x)] && \text{(by 1.1)}\\
&\iff \exists_{a\in\mathcal{D}(t)}\ (\mathcal{M},t) \Vdash \varepsilon(x)[v(a/x)] \text{ and } (\mathcal{M},t) \Vdash \psi^{\varepsilon}[v(a/x)] && \text{(by 1.1)}\\
&\iff \exists_{a\in\mathcal{D}(t)}\ (\mathcal{M},t) \Vdash \varepsilon(x)[v(a/x)] \text{ and } (\mathcal{M}^{w},t) \Vdash \psi[v(a/x)] && \text{(induction)}\\
&\iff \exists_{a\in\mathcal{D}(t)}\ a \in \mathcal{I}(\varepsilon,t) \text{ and } (\mathcal{M}^{w},t) \Vdash \psi[v(a/x)] && \text{(by 1.1)}\\
&\iff \exists_{a\in\mathcal{D}(t)}\ a \in \mathcal{D}^{w}(t) \text{ and } (\mathcal{M}^{w},t) \Vdash \psi[v(a/x)] && \text{(by 2.1)}\\
&\iff \exists_{a\in\mathcal{D}^{w}(t)}\ (\mathcal{M}^{w},t) \Vdash \psi[v(a/x)] && (\mathcal{M} \in \mathbb{CD})\\
&\iff (\mathcal{M}^{w},t) \Vdash \exists x\psi[v] && \text{(by 1.1)}
\end{aligned}$$

□

Now we can state and prove the said weaker version of (⋆).

Theorem 2.5. *For any formula φ not containing ε, $\mathbb{VD}_{\varepsilon} \Vdash \varphi \Longrightarrow \mathbb{CD}_{\varepsilon} \Vdash \varphi^{\varepsilon}$.*

Proof: Let $\mathcal{M} = (\mathcal{U},\mathcal{R},\mathcal{D},\mathcal{I}) \in \mathbb{CD}_{\varepsilon}, t \in \mathcal{U}$ and v such that $(\mathcal{M},t) \nVdash \varphi^{\varepsilon}[v]$. Let w be any object such that $w \notin \mathcal{U}$. By Fact 2.2, $\mathcal{M}^{w} \in \mathbb{VD}_{\varepsilon}$, and therefore by Lemma 2.4 we achieve $(\mathcal{M}^{w},t) \nVdash \varphi[v]$. □

3. Conclusion

Let us recall the construction Fitting and Mendelsohn introduced in [1, p. 107].

Definition 3.1. Let $\mathcal{M} = (\mathcal{U},\mathcal{R},\mathcal{D},\mathcal{I}) \in \mathbb{VD}$. Then we define $\mathcal{M}^{\star} = (\mathcal{U}^{\star},\mathcal{R}^{\star},\mathcal{D}^{\star},\mathcal{I}^{\star})$, where $\mathcal{U}^{\star} = \mathcal{U}, \mathcal{R}^{\star} = \mathcal{R}, \mathcal{D}^{\star}(t) = \mathcal{D}(\mathcal{M})$, for any $t \in \mathcal{U}$, and $\mathcal{I}^{\star}(r,t) = \mathcal{I}(r,t)$, for any predicate r distinct from ε, and $\mathcal{I}^{\star}(\varepsilon,t) = \mathcal{D}(t)$, for any $t \in \mathcal{U}$.

Fact 3.2. $\mathcal{M}^{\star} \in \mathbb{CD}_{\varepsilon}$, for any $\mathcal{M} \in \mathbb{VD}$.

Lemma 3.3 ([1, p. 107]). *For any formula φ not containing ε,*

$$(\mathcal{M},t) \Vdash \varphi[v] \Longleftrightarrow (\mathcal{M}^{\star},t) \Vdash \varphi^{\varepsilon}[v].$$

Finally, this allows them to prove the following theorem.

Theorem 3.4 ([1, Proposition 4.8.2]). *For any formula φ not containing ε, $\mathbb{CD} \Vdash \varphi^{\varepsilon} \Longrightarrow \mathbb{VD} \Vdash \varphi$.*

The very same construction and the same proof suffice to justify that

Fact 3.5. For any formula φ not containing ε, $\mathbb{CD}_{\varepsilon} \Vdash \varphi^{\varepsilon} \Longrightarrow \mathbb{VD} \Vdash \varphi$.

Obviously we have

Fact 3.6. For any formula φ not containing ε, $\mathbb{VD} \Vdash \varphi \Longrightarrow \mathbb{VD}_{\varepsilon} \Vdash \varphi$.

As a corollary of the above facts and Theorem 2.5 we obtain:

Corollary 3.7. For any formula φ not containing ε, the following conditions are equivalent:

(i) $\mathbb{VD} \Vdash \varphi$

(ii) $\mathbb{VD}_{\varepsilon} \Vdash \varphi$

(iii) $\mathbb{CD}_{\varepsilon} \Vdash \varphi^{\varepsilon}$.

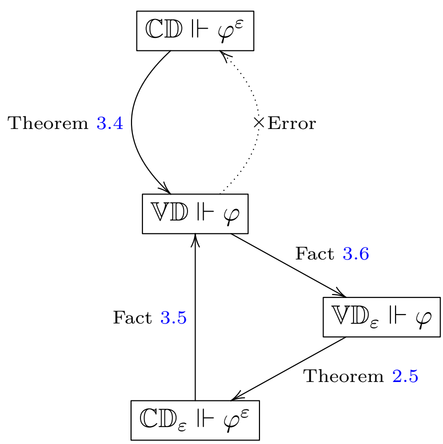

Figure 1. Summary

4. Further results

Corollary 3.7 invites us to asking a natural question: how, if at all, can we 'cut' classes $\mathbb{VD}$, $\mathbb{VD}_\varepsilon$ and $\mathbb{CD}_\varepsilon$ to hold the equivalence? In other words: when $K \Vdash \varphi \iff K \cap \mathbb{VD}_\varepsilon \Vdash \varphi \iff K \cap \mathbb{CD}_\varepsilon \Vdash \varphi^\varepsilon$ holds?

Obviously if $K = \emptyset$, then the equivalence in question is true. But we can do a little better.

We will say that a class of models K is *closed under $\star$-operation* (see Definition 3.1), or simply *$\star$-closed*, when for any model $\mathcal{M}$, $\mathcal{M} \in K$ implies $\mathcal{M}^\star \in K$. We will say that K is *closed under adding-new-points-operation*, or *add-closed* for short, when for any $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}) \in \mathbb{CD}_\varepsilon$, if $\mathcal{M} \in K$, then for some $w \notin \mathcal{U}$, $\mathcal{M}^w \in K$. Finally, we will say that K is *add*-closed* if it is both $\star$- and *add-closed*.

It turns out that operations introduced in Definitions 2.1 and 3.1 provide sufficient conditions for the examined equivalence to hold. Let us decompose the equivalence into conditionals so we can prove the following lemmas.

Lemma 4.1. *For any formula φ not containing ε and any $K \subseteq \mathbb{VD}$, if K is $\star$-closed, then $K \cap \mathbb{CD}_\varepsilon \Vdash \varphi^\varepsilon \Longrightarrow K \cap \mathbb{VD}_\varepsilon \Vdash \varphi$.*

Proof: Suppose $(\mathcal{M}, t) \nVdash \varphi[v]$, for some $\mathcal{M} \in K \cap \mathbb{VD}_\varepsilon$. By Lemma 3.3, $(\mathcal{M}^\star, t) \nVdash \varphi^\varepsilon[v]$. By Fact 3.2, $\mathcal{M}^\star \in \mathbb{CD}_\varepsilon$ and by the assumption that K is *$\star$-closed*, $\mathcal{M}^\star \in K \cap \mathbb{CD}_\varepsilon$. □

It is worth noting that the $\star$-operation does not affect the domain nor the accessibility relation of a model. Therefore if K is a class of models defined by the property of frames[2] on which those models are based, then the implication of Lemma 4.1 holds. Such classes of models, defined by properties of the accessibility relation like reflexivity, transitivity, symmetry etc, are in special interest of logicians, for they give rise to well-behaved and largely explored logical systems.

Lemma 4.2. *For any formula φ not containing ε and any $K \subseteq \mathbb{VD}$, if K is add-closed, then $K \cap \mathbb{VD}_\varepsilon \Vdash \varphi \Longrightarrow K \cap \mathbb{CD}_\varepsilon \Vdash \varphi^\varepsilon$.*

Proof: Suppose $(\mathcal{M}, t) \nVdash \varphi^\varepsilon[v]$, for some $\mathcal{M} = (\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I}) \in K \cap \mathbb{CD}_\varepsilon$. By Fact 2.2, $\mathcal{M}^w \in \mathbb{VD}_\varepsilon$ and by Lemma 2.4, $(\mathcal{M}^w, t) \nVdash \varphi$. Moreover, $\mathcal{M}^w \in K$ for some $w \notin \mathcal{U}$, since K is *add–closed*. □

[2] By a *frame* of a model $(\mathcal{U}, \mathcal{R}, \mathcal{D}, \mathcal{I})$ we mean a structure $(\mathcal{U}, \mathcal{R})$.

Lemma 4.3. *For any formula φ not containing ε and any $K \subseteq \mathbb{VD}$, if K is add*-closed, then $K \cap \mathbb{CD}_\varepsilon \Vdash \varphi^\varepsilon \Longrightarrow K \Vdash \varphi$.*

Proof: Suppose $(\mathcal{M}, t) \nVdash \varphi[v]$, for some $\mathcal{M} \in K$. By Lemma 3.3, $(\mathcal{M}^\star, t) \nVdash \varphi^\varepsilon[v]$. By Fact 3.2, $\mathcal{M}^\star \in \mathbb{CD}_\varepsilon$ and by the assumption that K is *add**-closed, $\mathcal{M}^\star \in K \cap \mathbb{CD}_\varepsilon$. □

Let us notice the following trivial facts.

Fact 4.4. For any $K \subseteq \mathbb{VD}$, $K \Vdash \varphi \Longrightarrow K \cap \mathbb{VD}_\varepsilon \Vdash \varphi$.

Fact 4.5. $K \cap \mathbb{VD}_\varepsilon = K_\varepsilon$

The above facts and lemmas entail:

Corollary 4.6. For any formula φ not containing ε and any $K \subseteq \mathbb{VD}$, if K is *add**–closed, then the following conditions are equivalent:

(i) $K \Vdash \varphi$

(ii) $K_\varepsilon \Vdash \varphi$

(iii) $K \cap \mathbb{VD}_\varepsilon \Vdash \varphi$

(iv) $K \cap \mathbb{CD}_\varepsilon \Vdash \varphi^\varepsilon$.

This corollary is a generalization of Corollary 3.7, for if we take $K = \mathbb{VD}$, the assumption of Corollary 4.6 becomes true and we get Corollary 3.7.

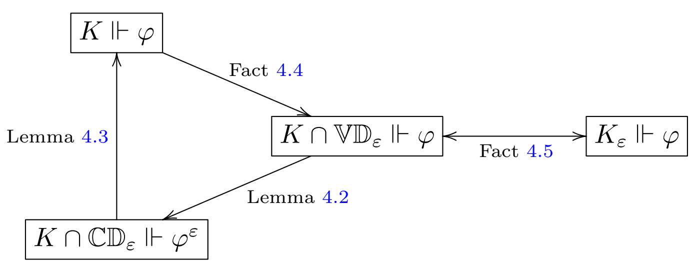

Figure 2. Summary

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