A NOTE ON GÖDEL-DUMMET LOGIC LC

Abstract

Let $A_0, A_1, ..., A_n$ be (possibly) distinct wffs, $n$ being an odd number equal to or greater than 1. Intuitionistic Propositional Logic IPC plus the axiom $(A_0 \rightarrow A_1) \lor ... \lor (A_{n-1} \rightarrow A_n) \lor (A_n \rightarrow A_0)$ is equivalent to Gödel-Dummett logic LC. However, if $n$ is an even number equal to or greater than 2, IPC plus the said axiom is a sublogic of LC.

Keywords: Intermediate logics, Gödel-Dummet logic LC.

1. Introduction

Propositional Intuitionistic Logic IPC can be axiomatized as follows (cf. [5] and references therein):

Axioms:

A1. $A \rightarrow (B \rightarrow A)$

A2. $[(A \rightarrow (B \rightarrow C)) \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]]$

A3. $(A \land B) \rightarrow A; (A \land B) \rightarrow B$

A4. $A \rightarrow [B \rightarrow (A \land B)]$

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*http://grobv.unileon.es.
†https://sites.google.com/site/sefusmendez.
A5. \( A \rightarrow (A \lor B); B \rightarrow (A \lor B) \)

A6. \( (A \lor B) \rightarrow [(A \rightarrow C) \rightarrow [(B \rightarrow C) \rightarrow C]] \)

A7. \( (A \rightarrow \neg B) \rightarrow (B \rightarrow \neg A) \)

A8. \( \neg A \rightarrow (A \rightarrow B) \)

**Rule of inference:**

**Modus Ponens (MP):** If \( A \) and \( A \rightarrow B \), then \( B \)

The following wffs and rule (derivable in IPC) are used in the sequel:

- **t1.** \( A \rightarrow A \)
- **t2.** \( (B \rightarrow C) \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)] \)
- **t3.** \( (A \rightarrow B) \rightarrow [(B \rightarrow C) \rightarrow (A \rightarrow C)] \)

**Transitivity (Trans):** If \( A \rightarrow B \) and \( B \rightarrow C \), then \( A \rightarrow C \)

In what follows, regardless of a particular order or association of the \( n \) implicative wffs \( A_1, ..., A_n \) connected by \( \lor \) as the sole connective, in general, we simply write \( A_1 \lor A_2 \ldots \lor A_n \).

By IPC+, we refer to the negationless fragment of IPC, axiomatized by A1 through A6 and MP. Well then, in [4] it is noted that Gödel-Dummett logic LC (cf. [2], [3]) can be axiomatized by adding any of the following axiom schemes to IPC:

- **a1.** \( (A \rightarrow B) \lor (B \rightarrow A) \)
- **a2.** \( (A \rightarrow B) \lor [(A \rightarrow B) \rightarrow A] \)
- **a3.** \( (A \rightarrow B) \lor [(A \rightarrow B) \rightarrow B] \)
- **a4.** \( [A \rightarrow (B \lor C)] \rightarrow [(A \rightarrow B) \lor (A \rightarrow C)] \)
- **a5.** \( [(A \land B) \rightarrow C] \rightarrow [(A \rightarrow C) \lor (B \rightarrow C)] \)
- **a6.** \( [(A \rightarrow B) \rightarrow B] \land [(B \rightarrow A) \rightarrow A] \rightarrow (A \lor B) \)

We remark that Dummett’s original axiomatization of LC is the result of adding a1 to IPC (cf. [2]). We will occasionally refer to a1 as “Dummett’s axiom”.
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The authors of [4] add: “An even larger number of equivalents [axioms] arises by the fact that in IPC ⊢ A ∨ B iff ⊢ (A → B) ∧ (B → C) → C (DR), and, more generally, ⊢ D → A ∨ B iff ⊢ D ∧ (A → C) ∧ (B → C) → C (EDR)” ([2], p. 1).

The aim of this note is to increase the number of equivalent axioms recorded above by showing that, for any odd number \( n \) equal to or greater than 1 and (possibly) distinct wffs \( A_1, A_2, \ldots, A_n \), addition of

\[ A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \]

to IPC is an axiomatization of LC.

As a by-product of the fact just stated, it also will be shown that if in the preceding wff \( n \) is an even number equal to or greater than 2, addition of it to IPC results in an intermediate logic included in (but not including) LC.

To the best of our knowledge, neither of these facts is recorded in the literature.

2. IPC plus \((A \rightarrow B) \lor [(B \rightarrow C) \lor (C \rightarrow A)]\)

Let \( A_0, A_1, \ldots, A_n, A_{n+1}, A_{n+2} \) be (possibly) distinct wffs, \( n \) being an even number equal to or greater than 2. Consider now the following wffs:

\( \alpha. \) \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0 \)

\( \beta. \) \( A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \)

\( \gamma. \) \( A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_{n+1} \lor A_{n+1} \rightarrow A_{n+2} \lor A_{n+2} \rightarrow A_0 \)

We prove:

PROPOSITION 2.1 (IPC \(_+\) & \( \beta \) proves \( \alpha \)). The wff \( \alpha \) is provable in IPC\(_+\) plus \( \beta \).

PROOF:

1. \( A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \) \( \beta \)

By changing in (1), for each \( i \geq 3 \), \( A_i \) by \( A_1 \) (resp., \( A_2 \)) if \( i \) is an odd number (resp., even number), we get

2. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_1 \lor A_2 \rightarrow A_0 \)
or equivalently

3. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0 \lor A_2 \rightarrow A_1 \)

Moreover, by changing in (1), for each \( i \geq 3 \), \( A_i \) by \( A_0 \) (resp. \( A_1 \)) if \( i \) is an odd number (resp., even number), we get

4. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0 \lor A_1 \rightarrow A_0 \)

Next, we proceed as follows. Obviously, we have

5. \((A_2 \rightarrow A_0) \rightarrow (\alpha)\)

In addition,

6. \((A_1 \rightarrow A_0) \rightarrow [(A_2 \rightarrow A_1) \rightarrow (A_2 \rightarrow A_0)]\)
7. \((A_1 \rightarrow A_0) \rightarrow [(A_2 \rightarrow A_1) \rightarrow (\alpha)]\)
8. \((\alpha) \rightarrow [(A_2 \rightarrow A_1) \rightarrow (\alpha)]\)

Then,

9. \((A_2 \rightarrow A_1) \rightarrow (\alpha)\)

Now, by using

10. \((\alpha) \rightarrow (\alpha)\)

3, 9 and A6, we derive

11. \(A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0 \)

as it was to be proved.

\[\square\]

**Proposition 2.2 (IPC + \& \( \alpha \) proves \( \beta \)).** The wff \( \beta \) is provable in IPC+ plus \( \alpha \).

**Proof:** Firstly, we show,

(I) The wff \( \delta \), \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_3 \lor A_3 \rightarrow A_4 \lor A_4 \rightarrow A_0 \), is provable in IPC+ plus \( \alpha \):

1. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0 \) \( \alpha \)
2. \( A_2 \rightarrow A_3 \lor A_3 \rightarrow A_4 \lor A_4 \rightarrow A_2 \) \( \alpha \)

We trivially have:
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3. \((A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2) \rightarrow (\delta)\)

4. \((A_2 \rightarrow A_3 \lor A_3 \rightarrow A_4) \rightarrow (\delta)\)

5. \((A_4 \rightarrow A_0) \rightarrow (\delta)\)

Then, we get

6. \([A_4 \rightarrow A_2) \rightarrow (\delta)] \rightarrow (\delta)\) A6, 2, 4

In addition,

7. \((A_2 \rightarrow A_0) \rightarrow [(A_4 \rightarrow A_2) \rightarrow (A_4 \rightarrow A_0)]\) t2

8. \((A_2 \rightarrow A_0) \rightarrow [(A_4 \rightarrow A_2) \rightarrow (\delta)]\) t2, Trans, 5, 7

9. \((A_2 \rightarrow A_0) \rightarrow (\delta)\) Trans, 6, 8

Finally,

10. \(A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_3 \lor A_3 \rightarrow A_4 \lor A_4 \rightarrow A_0\) A6, 1, 3, 9

(II) Given (I), the wff \(\varepsilon, A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_3 \lor A_3 \rightarrow A_4 \lor A_4 \rightarrow A_5 \lor A_5 \rightarrow A_6 \lor A_6 \rightarrow A_0\), is provable in IPC+ plus \(\alpha\) similarly as \(\delta\) has been proved above. We can use \(\delta, \alpha\) and \(t_2\) in the forms \(A_4 \rightarrow A_5 \lor A_5 \rightarrow A_6 \lor A_6 \rightarrow A_4\) and \((A_4 \rightarrow A_0) \rightarrow [(A_6 \rightarrow A_4) \rightarrow (A_6 \rightarrow A_0)\], respectively.

(III) In this way, the wff \(\gamma\), displayed at the beginning of the section, can be obtained given \(\beta\) (i.e., \(A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n\)), and \(\alpha\) and \(t_2\) in the forms \(A_n \rightarrow A_{n+1} \lor A_{n+1} \rightarrow A_{n+2} \lor A_{n+2} \rightarrow A_n\) and \((A_n \rightarrow A_0) \rightarrow [(A_{n+2} \rightarrow A_n) \rightarrow (A_{n+2} \rightarrow A_0)\], respectively.

Once (I), (II) and (III) are proved, it is clear that \(\beta\) is derivable from IPC+ plus \(\alpha\).

Given Propositions 2.1 and 2.2, we have the following corollary.

**Corollary 2.3** (IPC & \(\alpha\) is equivalent to IPC & \(\beta\)). Let \(A_0, A_1, \ldots, A_n\) be (possibly) distinct wffs, \(n\) being an even number equivalent to or greater than 2. The systems IPC plus \(\alpha\) (i.e., \(A_0 \rightarrow A_1 \lor A_1 \rightarrow A_2 \lor A_2 \rightarrow A_0\)) and IPC plus \(\beta\) (i.e., \(A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0\)) are deductively equivalent.

The section is ended by proving that Dummett’s axiom \((A \rightarrow B) \lor (B \rightarrow A)\) (a1) is not provable from IPC plus \((A \rightarrow B) \lor [(B \rightarrow C) \lor (C \rightarrow A)]\).
Let us provisionally name LC\(_2\) the result of adding \((A \rightarrow B) \lor [(B \rightarrow C) \lor (C \rightarrow A)]\) to IPC. We have:

**Proposition 2.4 (Dummett's axiom is not provable in LC\(_2\)).** Dummett’s axiom \((A \rightarrow B) \lor (B \rightarrow A)\) is not provable in LC\(_2\), that is, the result of adding \((A \rightarrow B) \lor [(B \rightarrow C) \lor (C \rightarrow A)]\) to IPC.

**Proof:** Consider the following set of truth-tables (4 is the only designated value):

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This set verifies all axioms of IPC (A1-A8) plus \((A \rightarrow B) \lor [(B \rightarrow C) \lor (C \rightarrow A)]\) and the rule MP, but falsifies Dummett’s axiom: let \(v\) be any assignment to the propositional variables such that \(v(p) = 2\) and \(v(q) = 1\), for distinct propositional variables \(p\) and \(q\). Then, \(v[(p \rightarrow q) \lor (q \rightarrow p)] = 3\).

It follows from this proposition that LC is not included in LC\(_2\). Instead, in the following section, it is proved that LC\(_2\) is included in LC.

### 3. A sequence of axioms equivalent to Dummett’s axiom

Let \(A_0, A_1, \ldots, A_n\) be distinct wffs, \(n\) being an odd number equal to or greater than 1. Now, consider the following wffs:
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\[ \varepsilon. \, A_0 \rightarrow A_1 \lor A_1 \rightarrow A_0 \]

\[ \theta. \, A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \]

We prove:

**Proposition 3.1** (IPC\_ & \( \theta \) proves \( \varepsilon \)). The wff \( \varepsilon \) is provable from IPC\_ plus \( \theta \).

**Proof:**

1. \( A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \)

By changing in (1), for each \( i \geq 2 \), \( A_i \) by \( A_0 \) (resp., \( A_1 \)) if \( i \) is an even number (resp., odd number), we get

2. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_0 \lor \ldots \lor A_0 \rightarrow A_1 \lor A_1 \rightarrow A_0 \)

that is,

3. \( A_0 \rightarrow A_1 \lor A_1 \rightarrow A_0 \)

i.e., the characteristic axiom of LC.

**Proposition 3.2.** Consider the following wff \( \eta \), \( A_0 \rightarrow A_1 \lor \ldots \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0 \), where \( A_0, A_1, \ldots, A_{n-1}, A_n \) are (possibly) distinct wffs. This wff \( \eta \) is provable in LC (notice that \( n \) is any natural number equal to or greater than 1).

**Proof:**

1. \( (A_n \rightarrow A_{n-1}) \rightarrow [(A_{n-1} \rightarrow A_{n-2}) \rightarrow (A_n \rightarrow A_{n-2})] \)
2. \( (A_n \rightarrow A_{n-2}) \rightarrow [(A_{n-2} \rightarrow A_{n-3}) \rightarrow (A_n \rightarrow A_{n-3})] \)
3. \( (A_n \rightarrow A_{n-1}) \rightarrow [(A_{n-1} \rightarrow A_{n-2}) \rightarrow [(A_{n-2} \rightarrow A_{n-3}) \rightarrow (A_n \rightarrow A_{n-3})]] \)

In this way, we have

4. \( (A_n \rightarrow A_{n-1}) \rightarrow [(A_{n-1} \rightarrow A_{n-2}) \rightarrow [(A_{n-2} \rightarrow A_{n-3}) \rightarrow (A_n \rightarrow A_{n-3})]] \)

Now, we obviously have

5. \( A_n \rightarrow A_0 \rightarrow (\eta) \)

and
6. \((A_{n-1} \rightarrow A_n) \rightarrow (\eta)\)

So, by t2, t3, (4) and (5), we derive

7. \((A_n \rightarrow A_{n-1}) \rightarrow [(A_{n-1} \rightarrow A_{n-2}) \rightarrow \ldots \rightarrow [(A_1 \rightarrow A_0) \rightarrow (\eta)] \ldots] \)

And by A1, (6) and Trans, we obtain

8. \((A_{n-1} \rightarrow A_n) \rightarrow [(A_{n-1} \rightarrow A_{n-2}) \rightarrow \ldots \rightarrow [(A_1 \rightarrow A_0) \rightarrow (\eta)] \ldots] \)

Now, by Dummett’s axiom, we have

9. \((A_{n-1} \rightarrow A_n) \lor (A_n \rightarrow A_{n-1})\)

whence

10. \((A_{n-1} \rightarrow A_{n-2}) \rightarrow [(A_{n-2} \rightarrow A_{n-3}) \rightarrow \ldots \rightarrow [(A_1 \rightarrow A_0) \rightarrow (\eta)] \ldots] \)

follows by A6, (7), (8) and (9).

Next, notice that, for any \(k\) \((0 \leq k \leq n - 1)\),

11. \((A_k \rightarrow A_{k+1}) \rightarrow (\eta)\)

is clearly provable.

Finally, proceeding from (10) and (11), similarly as we have proceeded from (4), (7), (8) and (9) to (10), we eventually derive

12. \(A_0 \rightarrow A_1 \lor \ldots A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0\)

that is, the wff \(\eta\), as it was to be proved.

\[ \square \]

Given Propositions 3.1 and 3.2, we immediately have the following corollary.

**Corollary 3.3 (IPC & \(\theta\) is equivalent to LC).** Let \(A_0, A_1, \ldots, A_n\) be (possibly) distinct wffs, \(n\) being an odd number equivalent to or greater than 1. The result of adding the wff \(\theta\) (i.e., \(A_0 \rightarrow A_1 \lor \ldots A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0\)) to IPC is a system deductively equivalent to LC.

On the other hand, given Propositions 2.4 and 3.2, the following corollary is immediate.
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Corollary 3.4 (LC\(_2\) is included in LC). The system LC\(_2\), that is, IPC plus the axiom \((A \rightarrow B) \lor ((B \rightarrow C) \lor (C \rightarrow A))\) is included in (but does not include) LC.

4. A couple of remarks

This note is ended with a couple of remarks.

1. The proofs of Propositions 2.1, 2.2, 3.1 and 3.2 are given within the context of IPC\(_+\), but it is possible that weaker systems are sufficient. For example, MaGIC (cf. [7]) does not find a set of truth-tables verifying Ticket Entailment (cf. [1]) plus Dummett’s axiom but falsifying \((A \rightarrow B) \lor (B \rightarrow C) \lor (C \rightarrow D) \lor (D \rightarrow A)\).

2. An IPC-model is the following structure \((K, R, \models)\), where \(K\) is a non-empty set, \(R\) is a reflexive and transitive binary relation defined on \(K\) and \(\models\) is a (valuation) relation such that for each \(a \in K\), propositional variable \(p\) and wffs \(A, B\), the following conditions (clauses) are fulfilled:

   (i). \((Rab \land a \models p) \Rightarrow b \models p\)
   (ii). \(a \models A \land B\) iff \(a \models A\) and \(a \models B\)
   (iii). \(a \models A \lor B\) iff \(a \models A\) or \(a \models B\)
   (iv). \(a \models A \rightarrow B\) iff for all \(b \in K\), \((Rab \land b \models A) \Rightarrow b \models B\)
   (v). \(a \models \neg A\) iff for all \(b \in K\), \(Rab \Rightarrow b \not\models A\)

   We have: for any set of wffs \(\Gamma\) and wff \(A\), \(\Gamma \vdash_{IPC} A\) iff \(\Gamma \models A\) (\(\Gamma \models A\) iff for any IPC-model \(M\) and \(a \in K\), \(a \models A\) if \(a \models \Gamma\), where \(a \models \Gamma\) iff \(a \models B\) for all \(B \in \Gamma\)) (cf. [5] or [6] and references therein).

   Well then, let us name LC\(_n\) the result of adding the axiom \(A_0 \rightarrow A_1 \lor ... \lor A_{n-1} \rightarrow A_n \lor A_n \rightarrow A_0\) to IPC; and let LC\(_n\)-models be the result of adding the following condition to IPC-models: for any \(a_0, a_1, ..., a_n \in K\), if \(Ra_0a_1\) and \(Ra_0a_2\) and ..., and \(Ra_0a_n\), then, \(Ra_1a_n\) or \(Ra_2a_1\) or ..., or \(Ra_n a_{n-1}\). For instance, LC\(_2\)-models (i.e., models for IPC plus the axiom \((A \rightarrow B)\lor(B \rightarrow C)\lor(C \rightarrow A)\)) are defined when adding to IPC-models the condition, for any \(a, b, c, d \in K\).
\[(\text{Rab} \& \text{ Rac} \& \text{ Rad}) \Rightarrow (\text{Rbd} \text{ or } \text{Rcb} \text{ or } \text{Rdc})\]. It is not difficult to prove that \(\text{LC}_n\) is (strongly) sound and complete w.r.t. \(\text{LC}_n\)-models.

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**References**


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Gemma Robles
Universidad de León
Departamento de Psicología, Sociología y Filosofía
Campus de Vegazana, s/n, 24071
León, Spain
e-mail: gemma.robles@unileon.es

José M. Méndez
Universidad de Salamanca
Campus Unamuno
Edificio FES, 37007
Salamanca, Spain
e-mail: sefus@usal.es