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**Modelling and Forecasting the Volatility of Thin Emerging Stock
Markets: the Case of Bulgaria**

Abstract

Modern Portfolio Theory associates the stock market risk with the volatility of return. Volatility is measured by the variance of the returns' distribution. However, the investment community does not accept this measure, since it weights equally deviations of the average returns, whereas most investors determine the risk on the basis of small or negative returns. In the last few years the measure Value at Risk (VaR) has been established and adopted widely by practitioners.

The issue of modelling and forecasting thin emerging stock markets' risk is still open. The subject of this present paper is the risk of the Bulgarian stock market. The aim of this research is to give the investment community a model for assessing and forecasting the Bulgarian stock market risk.

*The result of this research shows that the SOFIX index has basic characteristics that are observed in most of the emerging stock markets, namely: high risk, significant autocorrelation, non-normality and volatility clustering. Three models have been applied to assess and estimate the Bulgarian stock market risk: RiskMetrics, EWMA with *t*-distributed innovations and EWMA with GED distributed innovations. The results revealed that the EWMA with *t*-distributed innovations and the EWMA with GED distributed innovations evaluate the risk of the Bulgarian stock market adequately.*

1. Introduction

Modern Portfolio Theory associates the stock market risk with the volatility of return. The latter is measured by the variance of returns' distribution. However, the investment community does not accept this measure, since it weights equally the deviations of the average return, whereas most investors determine the risk on the basis of small or negative returns.

In the last decade the measure Value at Risk (VaR) has been established and adopted widely by practitioners. It is part of the New Basel Accord. Value-at-Risk is defined as the maximum expected loss for a given time horizon at a specified level of confidence.

The issue of modelling and forecasting thin emerging stock markets' risk is still open. The aim of this paper is to give the investment community an appropriate model for assessing and forecasting the Bulgarian stock market risk.

This paper is structured as follows: Literature review is presented in the next section. Data and methodology are outlined in section 3. Empirical results are presented in section 4. Summary and concluding remarks are given in the last section.

2. Literature Review

There are numerous papers on emerging equity markets that outline their different characteristics. Harvey (1995a) explained that high volatility of returns in emerging markets was caused by three factors: (1) lack of diversification in the country index, (2) high risk exposure to volatile economic factors and (3) time-variation in risk exposures and/or incomplete integration with the global capital market.

Harvey (1995b) found that the serial correlation in emerging markets' returns was much higher than the one observed in developed markets. He explained that this phenomenon was caused by lack of diversification and by the fact that trading depth induced spurious serial correlation. There were some emerging markets partially integrated into the world capital market. He concluded that factors contributing to market integration were connected with free access that foreigners had to the domestic capital market and free access that domestic investors had to international capital markets. Among potential barriers for integration there are:

- different access to capital markets,
- different tax systems,
- different diffusion of information from market to market.

Bekaert et al. (1998) argued that emerging markets are highly non-normal. Moreover, they found that seventeen out of twenty emerging stock markets exhibited positive skewness in the returns, while nineteen out of twenty were leptokurtic over the investigated period, April 1987 to March 1997. Furthermore, there was no strong evidence that the non-normality found in many emerging market returns became less prominent in the 1990s.

Bekaert et al. (1998) pointed out that correlation varied depending on both the state of the economy and the state of the equity market in each country examined. Correlation was observed to be higher during recessions and lower during recovery periods when comparing to the average correlation during both states of the economy. Moreover, the same asymmetry of correlation was observed during bear and bull markets: in bear markets correlation coefficients were higher, while in bull markets correlation coefficients were lower.

In recent years modelling the time-varying nature of the emerging stock markets' volatility has attracted the interest of researchers. Aggarwal, Inclan, and Leal (1999) examined the stock market volatility of 10 largest emerging markets in Asia and Latin America. They found that shifts in volatility of the examined emerging markets were related to significant country-specific political, social and economic events. Moreover, the time-varying stock market volatility was modelled by various GARCH models.

Balaban, Bayar, and Faff (2003) forecasted stock market volatility of fourteen stock markets. They employed eleven models and used symmetric and asymmetric loss functions to evaluate the performance of these models. According to symmetric loss functions the exponential smoothing model provided the best forecast. However, when asymmetric loss functions were applied, ARCH-type models provided the best forecast.

Several authors have investigated the volatility of Central and Eastern European stock markets: Kasch-Haroutounian and Price (2001), Glimore and McManus (2001), Poshakwale and Murinde (2001) and Murinde and Poshakwale (2002). They all found that significant autocorrelation, high volatility persistence, significant asymmetry, lack of relationship between stock market volatility and expected returns and non-normality of the returns' distribution were the basic characteristics of stock market volatility in transition countries.

We detected a gap in the literature that explains basic characteristics of stock market volatility in transition countries. Therefore, we contribute to the

literature by considering the Bulgarian stock market risk and evaluating/forecasting it by applying the Value-at-Risk approach with three different models. These models are presented in the following section.

3. Methodology

Supervision institutions like BIS and IOSCO introduced the Value-at-Risk approach (hereinafter, VaR) as a measure of the market risk in financial institutions. Jorion (1996, p.86) defined VaR as “the expected maximum loss (or the worst loss) over a target time horizon within a given confidence interval”. In practice, three approaches are established for the estimation of VaR:

- parametric (or variance-covariance),
- historical simulation,
- Monte Carlo simulation.

According to the survey prepared by Deloitte and Touche (2002) the most widely used approach is the parametric approach, popularized by JP Morgan (1996). The parametric approach has the benefit of easy implementation and quick computations. The main problem is related to VaR estimation of non-linear instruments (e.g. options).

The expected VaR estimation over the period $t+1$ could be calculated as follows:

$$VaR = |\delta(\alpha)| \times \sigma_{t+1} \quad (1)$$

where $\delta(\alpha)$ is α -quintile of the standardized distribution, σ_{t+1} is the standard deviation of r_{t+1} conditional on the time t information set.

In recent years, researchers have been widely using GARCH models for forecasting stock market volatility, (σ_{t+1}), in spite of the fact that the exponentially weighted moving average (hereinafter, EWMA) is the most popular model for stock market volatility forecasting among practitioners [Deloitte and Touche (2002)]. Dimson and Marsh (1990) gave another explanation for such popularity of the EWMA model. They stated that sometimes sophisticated models could provide worse forecasts than naïve models. The main advantage of the EWMA is the simplicity of the calculating procedure with a small number of available observations. The determination of the smoothing constant λ is the most critical issue. Once the smoothing constant is determined, there are needed two values to forecast the volatility: volatility at

time t , in other words (σ_t^2) and squared return at time t , (r_t^2). JP Morgan's *RiskMetrics* model is a variation of EWMA. They set $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data. Mathematical specification of it is following:

$$\sigma_{t+1}^2 = (1 - \lambda)r_t^2 + \lambda\sigma_t^2 \quad (2)$$

The exponentially weighted moving averages model is a special case of the GARCH (1,1) model. Bollerslev (1986) proposed GARCH models for modelling volatility clustering and hence the leptokurtosis that is induced in the unconditional distribution of stock returns. In fact, when there is small number of observations estimations of parameters in the GARCH model are inefficient.

On the basis of numerous empirical studies on emerging stock markets we expect that the Bulgarian stock market will possess the typical characteristics of an emerging stock market: volatility, clustering and leptokurtosis. The insufficient data set for GARCH modelling forces us to apply the EWMA model for modelling and forecasting the Bulgarian stock market volatility. Following Guermat and Harris (2002) we estimate the vector of parameters of the EWMA model, θ , using the Maximum Likelihood Method. The Log-likelihood function (Equation 3) is maximized by Marquardt's optimisation algorithm.

$$L(\theta = \sigma_{t+1}^2; \lambda; \nu_{t+1}; r_{-\infty}, \dots, r_t) = \sum_{s=0}^{\infty} (1 - \lambda)\lambda^s \ln f(r_{t-s}; \sigma_{t+1}; \nu_{t+1}) \quad (3)$$

where λ is the smoothing constant, and $f(r_{t-s}; \sigma_{t+1}; \nu_{t+1})$ is the density function.

The leptokurtosis induced in the unconditional distribution of stock market returns is captured by two models - EWMA-t and EWMA-ged. The first model assumes that stock market returns are Student-t distributed where the log-likelihood function of the t-distribution has the following specification:

$$L(\theta = \sigma_{t+1}^2; \lambda; \nu_{t+1}; r_{-\infty}, \dots, r_t) = \sum_{s=0}^{\infty} (1 - \lambda)\lambda^s \ln f(r_{t-s}; \sigma_{t+1}; \nu_{t+1}) \quad (4)$$

$$f_{\nu}(\theta) = \Gamma\left(\frac{\nu_{t+1} + 1}{2}\right) \Gamma\left(\frac{\nu_{t+1}}{2}\right)^{-1} \left((\nu_{t+1} - 2)\sigma_{t+1}^2\right)^{-1/2} \times \left(1 + r_{t+1}^2 \left(\sigma_{t+1}^2 (\nu_{t+1} - 2)\right)^{-1}\right)^{-(\nu_{t+1} + 1)/2}$$

where Γ is the Gamma function and ν represents the degrees of freedom.

Nelson (1995) proposed the Generalised Error Distribution (GED) for modelling fat-tailed distributions of stock returns. The log-likelihood function in this case is as follows:

$$\begin{aligned} L(\theta = \sigma_{t+1}^2; \lambda; \nu_{t+1}; r_{-\infty}, \dots, r_t) &= \sum_{s=0}^{\infty} (1-\lambda)\lambda^s \ln f(r_{t-s}; \sigma_{t+1}; \nu_{t+1}) \\ f[r_t(\theta); \nu_{t+1}] &= \nu_{t+1} \lambda^{-1} 2^{-(1+1/\nu_{t+1})} \Gamma(\eta^{-1})^{-1} \exp\left[-0.5|r_{t+1}(\theta)\lambda^{-1}|^{\nu_{t+1}}\right] \\ \lambda &= \left[2^{(-2/\nu_{t+1})} \Gamma(\nu_{t+1}^{-1}) \Gamma(3\nu_{t+1}^{-1})^{-1}\right]^{1/2} \end{aligned} \quad (5)$$

where Γ is the Gamma function and ν denotes the degrees of freedom.

The one-day ahead volatility (σ_{t+1}), is forecasted by using three models: *RiskMetrics*, EWMA-t and EWMA-GED. Forecasted volatility is replaced in Equation 1 to estimate Value-at-Risk. VaR estimations are calculated at three different confidence intervals: 95%, 97.50%, and 99%. We employ Kupiec's (1995) methodology for evaluation the adequacy of a particular model for VaR estimation. The null-hypothesis is that the number of violations follows a binomial distribution. The Kupiec's Likelihood Ratio is χ^2 distributed with one degree of freedom and has the following specification:

$$LR = 2\ln(\alpha^* (1 - \alpha^*)^{T-x}) - 2\ln(\alpha^x \times \beta^{T-x}) \quad (6)$$

where $\alpha^* = x/T$, x is the number of violations from VaR estimates, T is the total number of VaR estimates, α is the required level of risk (5%, 2.50% and 1%) and $\beta = 1 - \alpha$. The null-hypothesis cannot be rejected at a confidence interval p if the percentage of VaR violations is equal to α , or if:

$$\alpha - \sqrt{\alpha(1-\alpha) \frac{\chi_p^2(1)}{T_t}} < \alpha^* < \alpha + \sqrt{\alpha(1-\alpha) \frac{\chi_p^2(1)}{T_t}} \quad (7)$$

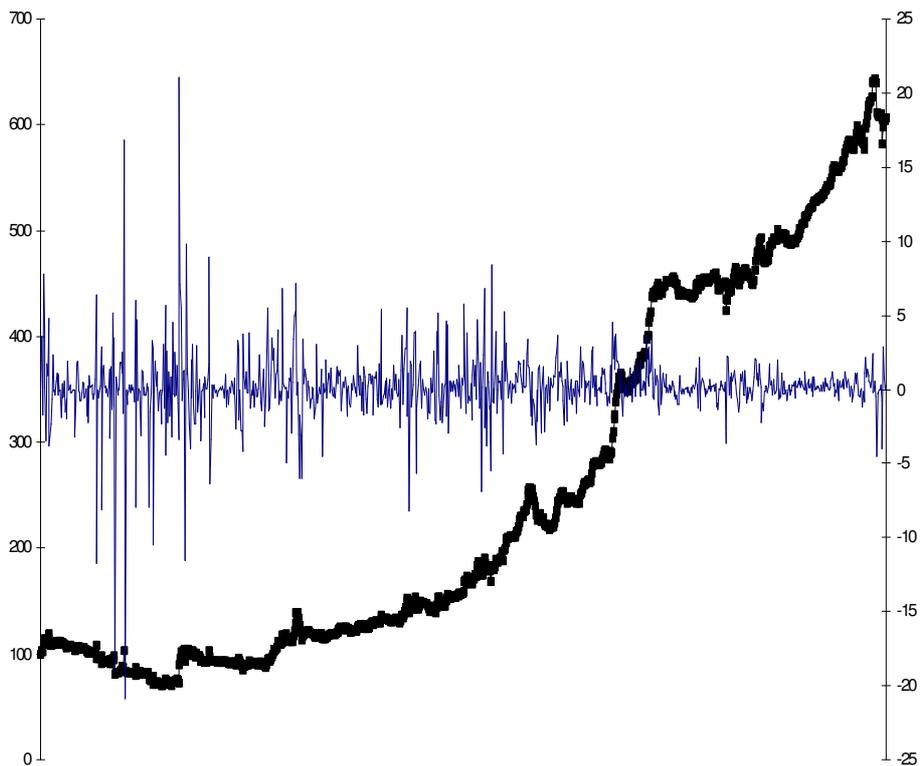
where T_t is the total number of VaR estimates, $\chi_p^2(1)$ is the critical value of the chi-squared distribution with one degree of freedom at a probability level p . When the percentage of violations is below the lower bound, then the VaR approach overestimates the risk, while when the percentage of violations is above the upper bound, then the VaR approach underestimates the risk.

4. Empirical Results

We consider the official stock market index of the Bulgarian Stock Exchange – Sofia. We examine the daily returns of the SOFIX index over the period 24 October 2000 – 19 November 2004. The data is provided by the Bulgarian Stock Exchange.

Figure 1 presents index values (right scale) and index returns (left scale) over the considered period. We observe a downward trend over the period November 2000 – November 2001 and high volatility. The trend is upward after November 2001 and the volatility is lower.

Figure 1. Daily values of SOFIX index



Source: data provided by the Bulgarian Stock Exchange.

Tables 1 and 2 help us to derive the basic distributional characteristics of the Bulgarian stock market, represented by the SOFIX index. We documented a high and positive average daily return equal to 0.2034% (44.42% on annual basis) over the considered period. The stock market has high risk, measured by the standard deviation of returns' distribution which is equal to 35.25% on annual basis. The Jarque-Berra statistics is significant, which leads to the rejection of the null hypothesis of normality. The kurtosis is high and significant while the skewness is negative. Thus, negative extreme returns are more likely to occur than the normal distribution forecasts. The leptokurtosis in the returns' distribution could be caused by volatility clustering.

Table 1. Descriptive Statistics of the Bulgarian Stock Market Index SOFIX 2000-04

Mean	0.1777%
Median	0.1104%
Maximum	21.0733%
Minimum	-20.8995%
Standard deviation	2.2295%
Skewness	-0.4566
Kurtosis	29.4941*
Jarque-Berra	29692*

Note: * denotes significance at the 5% risk level

Source: as to Fig.1, own calculations.

The autocorrelation coefficients of stock market returns and squared stock market returns are depicted in Table 2. We can observe significant autocorrelation coefficients. The significant autocorrelation in squared returns series proves the presence of volatility clustering that could be caused by high kurtosis values. Our findings of significant autocorrelation in the returns series are consistent with Harvey (1995a). This could be caused by nonsynchronous trading.

Table 2. Autocorrelation Coefficients of Stock Market Returns and Squared Stock Market Returns

	Lag	1	2	6	12	24
r	Autocorrelation coefficient	-0.1190*	0.0580*	0.0360*	0.0550*	-0.0560*
	<i>Ljung-Box</i> Q-statistics	14.5190	17.9390	25.2090	43.1130	64.5540
r ²	Autocorrelation coefficient	0.2490*	0.0120*	0.0250*	0.2370*	0.0090
	<i>Ljung-Box</i> Q-statistics	63.2870	63.4400	64.5510	150.7700	163.3400

Note: * denotes significance at the 5% risk level

Source: as to Fig.1, own calculations.

The above drawn characteristics of the SOFIX index coincide with those ones typical for emerging stock markets documented by Harvey (1995a) and Bekaert et al. (1998). The Bulgarian stock market has high risk, non-normal distributed returns and significant autocorrelation that are caused by nonsynchronous trading.

The results from applying the GARCH models meet our expectations that the parameter estimates will be inefficient and insignificant due to the small number of observations. These results are not published because of space limitations but are available upon request. Thus, we continue calculating the smoothing constant of EWMA-t and EWMA-GED models.

Table 3 shows the parameter estimates of the two latter models. The smoothing constants are significant at the 5% risk level. The EWMA-GED model has higher smoothing constant than the EWMA-t model. This implies that the past values of the volatility have greater impact on current volatility. The degrees of freedom are significant. This proves the “fat-tailed” distribution of the stock market return.

Table 3. Parameter Estimates

	EWMA-t	EWMA-ged
λ	0.8947*	0.9134*
t-statistic	89.9658	96.8492
ν	3.2582*	0.7922*
t-statistic	20.2908	23.0236
Log likelihood	-1816.9670	-1795.4440

Note: * denotes significance at the 5% risk level

Source: as to Fig.1, own calculations.

We employ the approaches of Bams and Wielhouwer (2001) as well as Guermat and Harris (2002). First we forecast the stock market volatility. Then we calculate the VaR for one-day horizon at three different confidence intervals. The evaluation results regarding the VaR calculation are presented in Table 4. It reports the number of violations, Kupiec's Likelihood Ratio, and upper and lower bounds.

Table 4. Evaluation Results of the VaR Calculations

<i>95% Confidence interval</i>			
	<i>EWMA-t</i>	<i>EWMA-ged</i>	<i>RiskMetrics</i>
x	16	43	33
α^*	1.58%	4.25%	3.26%
α	5.00%	5.00%	5.00%
Kupiec's LR	33.59	1.26	7.31
Upper bound	6.34%	6.34%	6.34%
Lower bound	3.66%	3.66%	3.66%
<i>97.50% Confidence interval</i>			
x	9	23	27
α^*	0.89%	2.27%	2.67%
α	2.50%	2.50%	2.50%
Kupiec's LR	14.26	0.22	0.11
Upper bound	3.59%	3.59%	3.59%
Lower bound	1.41%	1.41%	1.41%
<i>99% Confidence interval</i>			
x	4	9	12
α^*	0.40%	0.89%	1.19%
α	1.00%	1.00%	1.00%
Kupiec's LR	4.85	0.13	0.33
Upper bound	1.79%	1.79%	1.79%
Lower bound	0.21%	0.21%	0.21%

Note: Figures in bold face denote that VaR estimates of that particular model exceed from the upper and lower bounds.

Source: as to Fig.1, own calculations.

Both the EWMA-t and the *RiskMetrics* models overestimate the risk at the 95% confidence interval. The percentage of violations is below the lower bound. The EWMA-GED model evaluates the risk adequately. Any violations are within bounds. The EWMA-t model also overestimates the risk at the 97.50% confidence interval. The *RiskMetrics* and the EWMA-GED models evaluate the Bulgarian stock market risk adequately. The EWMA-GED has lower violations than the *RiskMetrics* model. All models examined in this study evaluate the risk

of the Bulgarian stock market at the 99% confidence interval adequately however; the EWMA-t model has the lowest number of violations.

5. Summary and Conclusions

The study examines the risk of the Bulgarian stock market, measured by the Value-at-Risk approach. The SOFIX index possesses the basic characteristics of a typical emerging stock market. Based on the results obtained in the present study we reach a number of important conclusions.

Firstly, exponentially weighting moving average models allow both modelling and forecasting the time-varying volatility and kurtosis in returns. These models could be applied in thin stock markets with insufficient number of observations.

Secondly, the assumed probability distribution should be able to reflect the fat tailed behaviour of stock market returns and we propose the Student-t distribution and the Generalised Error distribution as the most appropriate ones.

Finally, the EWMA-GED model evaluates the risk of the Bulgarian stock market adequately at both 95% and 97.50% confidence intervals. We propose the EWMA-GED model since it allows to capture the fat-tailed distribution of stock market returns.

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Streszczenie

MODELOWANIE I PROGNOZOWANIE ZMIENNOŚCI NA WSCHODZĄCYCH RYNKACH GIEŁDOWYCH: PRZYPADEK BUŁGARII.

Przedmiotem artykułu jest modelowanie i prognozowanie wskaźników giełdowych. Szczególna uwaga poświęcona jest analizie ryzyka na bułgarskiej giełdzie papierów wartościowych. Celem tej analizy jest opracowanie wiarygodnego modelu oceny i prognozowania ryzyka na bułgarskiej giełdzie papierów wartościowych.

Przeprowadzone analizy wskazują, że indeks giełdowy SOFIX charakteryzuje się typowymi cechami: wysokim ryzykiem autokorelacji i nienormalnym rozczłonkowaniem. W celu oszacowania i oceny ryzyka giełdy w Bułgarii wykorzystano trzy modele: RiskMetrics, EWMA oraz zmodyfikowany model EWMA. Z analiz wynika, że dwa ostatnie modele dosyć dokładnie szacują ryzyko na bułgarskiej giełdzie papierów wartościowych.