10.2478/cer-2018-0027

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Exogenous and Endogenous Growth Models: a Critical Review

Abstract

The main divisions of the theoretical economic growth literature that we study today include exogenous and endogenous growth models that have transitioned through a number of notions and criticisms. Proponents of exogenous growth models argue that technological progress is the key determinant of long-run economic growth as well as international productivity differences. Within the endogenous growth models, there are two notions that are propagated. The first postulates that capital used for innovative purposes can exhibit increasing returns to scale and thus account for the international productivity differences we observe today. The key determinants include knowledge, human capital, and research and development. The second argues that factors that affect the efficiency of capital, and hence cause capital flight, can also explain international productivity differences. These factors that affect the efficiency of capital include government spending, inflation, real exchange rates, and real interest rates. Our study results reveal that there is still no agreement on the dominant theoretical economic growth model amongst economists that can fully account for international productivity differences. We conclude that the future of theoretical economic growth is far from over and more work needs to be done to develop more practical structural economic growth models.

Keywords: *economic growth; exogenous growth models; endogenous growth models*

JEL: *B22; E13; N10; O40*

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1. Introduction

What drives an economy's growth path has always been an enigma. Klenow and Rodriguez-Clare (1997) reviewed the developments in existing research on theoretical and empirical economic growth. In their study, they identified four main challenges in relation to the future research of theoretical growth economics. These include the need to link theory and evidence; the need to develop methodologies that would empirically distinguish competing theories of endogenous growth; the need to develop new growth theories that would explain international productivity differences; and the need to improve on data availability. Many of the theoretical growth models provide a basis for how economic growth is determined in each country. But empirical studies that have shown this fail to account for the international productivity differences that we observe today. There are two main theoretical divisions in the economic growth literature – exogenous and endogenous growth models. The fundamental differences between the neoclassical exogenous and endogenous growth models are more on the basis of the behaviour of the aggregate production function as it relates to capital accumulation. A more hybrid version of the two competing theories has emerged through what are called new growth theories. These studies lean towards more empirical analysis than theoretical and, more importantly, on the factors that affect the efficiency of resource utilisation.

The challenge of all theoretical studies on economic growth is that they have emphasised understanding why some countries experience persistent per capita income growth while others have not (Lucas 1993). This has resulted in a number of variations to theoretical growth models, all of which explain different components of macroeconomic factors affecting the growth in per capita income. The exogenous growth models largely pioneered by Solow (1956) postulate that productivity growth can only be explained through direct investment, population growth and technological progress. In Solow's argument, direct investment and population growth only have level effects on output and do not affect the long-run growth rate. Technological progress, therefore, becomes the only factor that affects the long-run growth rate of any economy, and thus accounts for productivity differences between nations in the world. What is also common in theoretical economic growth models after Solow (1956) is that increasing returns to capital is associated with innovation. Endogenous growth theorists extend this thought by arguing that capital investment, if modelled correctly, can also exhibit increasing returns to scale if capital is used for innovative purposes, such as investment in innovative and intellectual capital (Frankel 1962; Romer 1986; Grossman and Helpman 1991; Aghion and Howitt 1992; Stokey 1995), and knowledge and skills (Lucas 1988). Another class of endogenous growth models have extended the thought by including factors that affect the efficiency of capital, such as real interest rates (Gelb

1989), fiscal policy (Barro 1990; Barro and Sala-i-Martin 1992), inflation (Bruno and Easterly 1998), and real exchange rates (Rodrik 2008).

The aim of this paper, therefore, is to focus on the third issue raised by Klenow and Rodriguez-Clare (1997), and to examine whether developments in the theoretical growth literature are addressing the issue of developing growth models that can account for international productivity differences. The rest of the paper is discussed as follows; Section 2 reviews exogenous growth theories and their criticisms. Section 3 examines the endogenous growth theories and their criticisms. Lastly, section 4 provides a summary and concluding remarks.

2. Exogenous theories of economic growth

2.1. The Domar (1946) Multiplier-Effect Economic Growth Model

The first neoclassical mathematical proposition on the process of economic growth was propounded by Evsey Domar in his 1946 paper on '*Capital Expansion, Rate of Growth and Employment*'. Domar looked at the relationship between capital accumulation and full employment. The axiom put forward by Domar was that an economy will be in equilibrium when its productive capacity is equal to its national income. Domar adopted a classical doctrine where the labour force and its productivity were key to the economic growth paradigm. His postulate was based on the assumption that the growth rate of national income was a combined effect of the growth of labour and its productivity (Domar 1946). The approach adopted by Domar was based on the general equilibrium theory, where demand meets supply. He developed his model in a closed economic setting, disregarding the possibility of having external economies or diseconomies. From the supply side, the rate of growth of the production function was a function of the productive capacity-capital ratio of the following order with respect to time (Domar 1946, p. 138):

$$\frac{dP}{dt} = I\sigma \tag{1a}$$

where:
I = Investment per year
σ = Productive capacity

From the demand side, Domar defines the rate of growth of national income as a function of the rate of growth of investment over time through the multiplier driven by the marginal propensity to save, *s*. The demand function, therefore, is represented as (Domar 1946, p. 138):

$$\frac{dY}{dt}=\frac{dI}{dt}\cdot\frac{1}{s} \tag{1b}$$

where:
Y = National Income
s = Marginal propensity to save

Domar's general equilibrium position was, therefore, where supply meets demand (Domar 1946, p. 138):

$$\frac{dY}{dt}=\frac{dP}{dt} \tag{2a}$$

$$\frac{dI}{dt}\cdot\frac{1}{s}=I\sigma \tag{2b}$$

By directly integrating both sides of equation (2b) with respect to time, Domar obtained the following equilibrium growth path for a closed economy (Domar 1946, p. 138):

$$I\left(t\right)=I_0e^{s\sigma t} \tag{3}$$

In Domar's world, for the economy to remain in equilibrium, this required the actual rate of investment, denoted r, to grow at the same rate as the required equilibrium rate of $s\sigma$. From equation (3), differentiating with respect to time, we get (Domar 1946, p. 138):

$$\frac{dI}{dt}=s\sigma I_0e^{s\sigma t} \tag{4a}$$

From equation (1b), it follows that (Domar 1946, p. 138):

$$\frac{dY}{dt}=\sigma I_0e^{s\sigma t} \tag{4b}$$

Domar then equates the actual investment rate to the productive capacity, $r=\sigma$. Then equation (6b) becomes (Domar 1946, p. 138):

$$\frac{dY}{dt}=rI_0e^{s\sigma t} \tag{4c}$$

For Domar's equilibrium to hold (Domar 1946, p. 138)

$$\frac{dY/dt}{dP/dt} = \frac{rI_0 e^{s\sigma t}}{s\sigma I_0 e^{s\sigma t}} \tag{5a}$$

The fundamental Domar growth model equation is, therefore (Domar 1946, p. 138):

$$\frac{dY/dt}{dP/dt} = \frac{r}{s\sigma} \tag{5b}$$

In restating Domar's proposition, for an economy to remain in a state of full employment, the actual rate of growth of investment should equal the productive multiplier, $r = s\sigma$. However, Domar's approach provided solutions that were paradoxical. Situations where $r > s\sigma$ implied a demand-creating effect that implied a shortage of capacity. For the equilibrium to hold, this meant that the rate of investment should fall towards the productive multiplier. However, from a firm's perspective, demand is greater than the existing productive capacity. The behaviour of firms would make them increase rather than reduce the level of investment. Similarly, for cases where $r < s\sigma$, it implied a capacity-generating effect or some productive capacity lying idle. In Domar's equilibrium, this meant an increase in investment. But from a firm's perspective, this would entail a reduction in capacity, which would decrease further the actual rate of investment. Overall, Domar's economic growth path lead to a failure to attain full employment if the solution deviates from the equilibrium path given in (5b).

2.2. Solow's (1956) Exogenous Economic Growth Model

In 1956, Robert Solow developed an alternative economic growth model to address the weaknesses of the Domar growth proposition. The Domar economic growth framework had three major problems. The first is related to the instability of the equilibrium growth path, which meant that once the system diverted from its equilibrium path, it would continue being in a disequilibrium position. The balancing of Domar's long-run equilibrium path on a '*knife-edge*' equilibrium growth rate developed scenarios that, for a small slip in the fixed proportions given in equation (3b), would lead to a continued failure to satisfy the rule of full capacity utilisation either by creating persistent unemployment or inflation (Solow 1956, p. 65). The second problem related to the reliance of the multiplier effect in Domar's model, which in Solow's view was a short-term tool used to sort long-run problems (Solow 1956, p. 66). Lastly, in Domar's model, the only factor of production that affected the growth in output was the productive capacity, which was a function of the stock of capital alone. Under Solow's proposition, he reintroduced labour as an important factor used in the production function. In the Domar model, the proposition he adopted was to allow the combination of capital and labour to be in fixed proportions. In Solow's growth model, capital and labour were combined in varying proportions. The mathematical representation of Solow's growth model is based on three equations. He first assumes that output is a function of both capital and labour (Solow 1956, p. 66).

$$Y = F\left(K,\ L\right) \qquad for\ K, L > 0 \tag{6a}$$

In intensive form per effective labour, equation (6a) is represented as

$$Y = Lf\left(k\right) \qquad for\ k = K / L \tag{6b}$$

To solve the two unknowns, Solow assumes the rates of change for $K\ and\ L$ over time to be the following. For capital,

$$\frac{dK}{dt} = \dot{K} = sY \tag{7a}$$

Alternatively, from equation (6b) using the product rule

$$\dot{K} = \dot{k}L + k\dot{L} \tag{7b}$$

The labour force grows exponentially, given by the following labour function

$$\frac{dL}{dt} \cdot \frac{1}{L} = \frac{\dot{L}}{L} = \lambda \quad for\ \lambda > 0 \tag{7c}$$

The Solow growth model is, therefore, solved as follows. Substituting equation (6b), (7b) and (7c) into (7a) we get

$$\dot{k}L + k\lambda L = sLf\left(k\right) \tag{8a}$$

Dividing equation (8a) throughout by L and solving for $\dot{k}$ we get the fundamental equation of the Solow growth model

$$\dot{k} = sf\left(k\right) - \left(\lambda + \delta\right)k \tag{8b}$$

In the Solow growth model, the optimal growth path, where output, consumption and capital are maximized, will be at the intersection where $sf\left(k\right) = \left(\lambda + \delta\right)k$. The long-run growth rates are determined by exogenous elements in a closed economy. The transitional dynamics of the Solow growth model allows the growth path to converge to some optimum level even when exogenous shocks (s, λ) are experienced within the economy (see Barro and Sala-i-Martin 2004, pp. 37–40). The strength of the Solow growth model, unlike the Domar model, depends on its convergence whenever there is an external shock. Solow argues that shifts in the production function caused by increases (or decreases) in the rates of savings, population growth and technological progress have temporal level effects. Once the shifts to the balanced growth path are made, the economy returns to its steady state growth path. For savings in a closed economic setting, there is a limit to which households can save their income for future consumption. Similarly, there is a limit to how much a population can grow or decrease. Solow's model proposition argues that it is only technological progress that would continue to generate level-effect economic growth in the long-run per capita income and consumption. As such, technological progress in the Solow neoclassical growth model is the only source of sustained long-run growth of per capita income (Solow 1956), and a source of international productivity differences. Because Solow's growth model is designed in a closed economic setting, this implies that the rate of growth driven by technology is determined outside his model and is independent of any government policy intervention that may be instituted.

2.3. Criticisms of the Exogenous Growth Models

Some of the criticisms under the exogenous growth theories are discussed in this section. The first problem relates to a measure of the dissatisfaction with the neoclassical model. Mankiw et al. (1995) showed that, based on its present structure, the Solow neoclassical growth model leaves a high magnitude of income per capita unexplained when comparing international income differences. Mankiw et al. (1995, p. 282) showed that the magnitude of income differences in the neoclassical Solow growth model can be represented by the following equation on variations in income per capita:

$$dy^* / y^* = \left[\sigma / 1 - \sigma\right]\left[ds / s - d\left(\eta + g + \delta\right) / \left(\eta + g + \delta\right)\right] \tag{9}$$

where:
$\sigma = f'\left(k^*\right)k^* / f\left(k^*\right)$ is the steady-state capital share.

Based on their calculations and using data from national income accounts, they showed that the Solow neoclassical growth model can only explain income disparities of not more than two when in fact the world income disparities can even be five or ten times more than what the Solow growth model predicts. This confirms that the exogenous growth models suffer from omitted variable bias. Second, the Solow residual, which represents the total factor productivity or the portion of output that cannot be explained by amount of inputs used in the production process, has come under heavy criticism, especially by the endogenous and new growth theorists, that it harbours efficiency factors that have a significant impact on the utilization of inputs in the production process (Mosley et al. 1987). According to a World Bank report, the three components of sustained growth comprise stable macroeconomic conditions, an appropriate price structure and regulatory environment, and efficient institutions that can convert savings into productive investments. These efficiency factors include a low and stable inflation rate, a competitive exchange rate, a well-developed and sound financial system, and clear and sustainable tax rules. These efficiency factors are capable of inducing savings and investment, thereby preventing capital flight (World Bank 1990, p. 100). In the exogenous growth framework, however, this burden has been assumed away through the Solow residual. To address this problem, the Solow growth model has been augmented by a number of researchers with additional factors that may have an influence on long-run economic growth. For example, Mankiw et al. (1992, p. 420) found that the inclusion of a human capital accumulation variable improved model results with implied physical and human capital shares averaging one-third, respectively, as predicted by the Solow growth model. Knight et al. (1993) included outward orientation as an important input in the growth process. The inclusion of trade policies is based on the notion that export and import sectors promote a country's openness and facilitate the transfer of technology of advanced capital goods, as well as the diffusion of knowledge and skills (Easterly and Wetzel 1989, p. 10; Knight et al. 1993, p. 515).

3. Endogenous Theories of Economic Growth

In this section, we review four main sources of increasing returns to scale capital modelled by endogenous growth theorists namely: innovative capital (Frankel 1962; Cass 1965); intellectual capital (Romer 1986; Grossman and Helpman 1991; Aghion and Howitt 1992); and human capital (Lucas 1988).

3.1. Physical Capital-Based Endogenous Growth Theories

3.1.1. Frankel's (1962) AK Model

Endogenous growth models that focus on the accumulation of physical capital assume that the savings (or investment) rate has a permanent positive contribution to the long-run growth rate. Frankel (1962) argued that aggregate production functions can also exhibit increasing returns to scale if a portion of the capital employed is used for innovative capital that contributes to technological progress. Such innovative capital can be in the form of improvements in the organization, quality of labour, technical change, or external economies of scale, etc. (Frankel 1962, p. 1001). The aggregate production function is assumed to take the following form:

$$Y = aK \tag{10}$$

The aggregate production function given in equation (10) is a special case of the Cobb-Douglas function where the elasticities for capital and labour are $\beta = 1$ and $\alpha = 0$, respectively. In equation (10), the constant, a, is referred to as an output-capital ratio which is positive. This implies that an increase in the savings (or investment) rate will lead to a permanent increase in the long-run growth rate (Frankel 1962, p. 1012–1013). AK models, therefore, assume a unitary elasticity of physical capital on long-run economic growth.

3.1.2. Cass's (1965) Endogenous Economic Growth Model

In 1965, David Cass developed a growth theory where the central principle was to determine an optimum feasible growth path that focused on maximising social welfare. His argument was based on the assumption that for any economy, the social objective is to ensure that consumer goods were adequately provided over time. For this to be the case, the optimal feasible growth path could not be determined without maximising the utility of current consumption per capita (Cass 1965, p. 234). In the Cass (1965) framework, the determination of the savings rate is, therefore, endogenous, contrary to Solow's prediction. In his model setting, national income is used to satisfy consumption and investment over time. The general equilibrium in intensive form is, therefore, where the output is equated to the sum of consumption and investment (Cass 1965, p. 234).

$$y(t) = c(t) + z(t) \tag{11a}$$

The fundamental growth equation (or production function) is the same as the Solow function in equation (8b). Cass argues that for any particular growth path, total welfare is represented by the following social welfare function (Cass 1965, p. 234):

$$\frac{1}{\gamma}\int_{0}^{\infty} U\left(c(t)\right)e^{-(\rho-n)t}dt \tag{11b}$$

where:
$U(\cdot)$ = Utility index of current consumption per capita
ρ = Discount factor for future welfare
$e^{-(\cdot)}$ = Family size multiplier
n = Population growth
$\frac{1}{\gamma}$ = Weight of current population

In the Cass model, the objective is to specify a feasible growth path that maximizes the social welfare function presented in equation (11b). The optimum savings rate is endogenously determined in a virtual setting determined through a saddle path represented by a monotone function that can either steadily increase, decrease or remain constant as the per capita capital stock rises (Cass 1965, p. 238). In a closed economy, given competitive markets and rational economic agents, preferences of per capita consumption streams are given by the following consumer utility function:

$$\frac{1}{\gamma}\int_0^\infty \frac{\left[c(t)^{1-\sigma}-1\right]}{1-\sigma} e^{-(\rho-n)t} dt \tag{11c}$$

To determine optimal allocations, therefore, is to maximize the utility function (11c) subject to the production function given in equation (11a). This involves solving a Hamiltonian function of the order (see Barro and Sala-i-Martin 2004, p. 89):

$$H(z,\ \theta,\ c,t) = \frac{1}{\gamma}\frac{\left[c(t)^{1-\sigma}-1\right]}{1-\sigma} + \theta\left[y(t)-c(t)\right] \tag{11d}$$

The key message from Cass's (1965) model is that policy choices can influence the rate of savings and investment. In its simplest terminology, the first term on the right-hand side of the Hamiltonian function (11d) is the sum of the current period utility valued at a constant relative risk aversion σ, and the second term is the rate of increase in capital valued at a shadow price $\theta(t)$. The objective, therefore, is to maximize the Hamiltonian with respect to per capita consumption, provided that the social planner correctly chooses the shadow price $\theta(t)$ and the constant relative risk aversion σ. By using microfoundations to determine the equilibrium growth path based on utility, the inherent weakness of the Cass endogenous growth model is based on the derivation of an optimal growth path that is subject to maximising a consumer utility function where the key parameters of its function are determined outside the model $(\theta(t),\ \rho,\ \sigma)$. The abstractive nature of the general equilibrium analysis of the Cass (1965) endogenous growth model is, therefore, one of its biggest limitations. The key assumption made is that the social planner, consumers, and firms are rational agents that have perfect foresight of future prices and correctly choose the parameters $(\theta(t),\ \rho,\ \sigma)$ rationally. In addition, the stability or instability in determining the saddle point in the Cass (1965) model assumes perfect foresight of the parameters $(\theta(t),\ \rho,\ \sigma)$ by the social planner, which is rarely the case. Since the determination of these parameter values are based on subjective judgement, the social planner cannot know whether the assumptions made for the equilibrium path are correct or even how to correct a miscalculation concerning the optimum equilibrium path. As Rothbard (1995, p. 379–381) also notes, utility cannot be measured, it is indivisible, a subjective abstract evaluation and ranking by an individual. Given its subjective and intangible nature, the concept of comparing and aggregating utility across individuals would be equally impossible.

3.2. Intellectual Capital Based Endogenous Growth Theories

3.2.1. Romer's (1986) Knowledge-based Endogenous Growth Model

One caveat of the Frankel endogenous AK model is the assumption of a savings rate that increases in fixed proportions. The need to endogenize the drivers of economic growth continued with Romer in 1986, when he developed a fully specified model of long-run economic growth with the savings rate assumed to be endogenously generated by an intertemporal utility maximization supported by technology. In his assertion, the economic growth path of the technology-driven endogenous model was led by the accumulation of intangible knowledge that was measurable by what he termed '*forward-looking, profit-maximising agents*' (Romer 1986, p. 1003). The possibility of having increasing returns driven by an intangible capital good in Romer's model allowed for the possibility of having a fully specified model (which he claimed to be competitive) where per capita income grows without bound through a monotone function steadily increasing over time (Romer 1986, p. 1003). He further claimed that since developed countries are usually custodians of technological advancement more than poor countries, rich countries will tend to grow faster. The key deviation from the Ramsey-Koopmans-Cass neoclassical endogenous growth model was adding the concept of increasing marginal returns to the economic growth path based on an intangible capital good, knowledge (Romer 1986, p. 1004). By adopting all assumptions in the Ramsey-Koopmans-Cass model and adding another factor – intangible knowledge – Romer considers a discrete-time model of economic growth with two periods. The consumer utility function is given by $U\left(c_1, c_2\right)$ for a single good consumed in two periods. Consumers have an initial endowment of the consumption good in period 1 but in period 2 the consumption good is a function of the intangible capital good, knowledge, denoted κ, and a vector of other tangible factors like capital and labour denoted $\mathbf{x}$ (Romer 1986, p. 1014).

The second crucial assumption in Romer's model relates to the assumption of having a competitive equilibrium that is not Pareto optimal. He argues that it is essentially the existence of externalities that hold the model in a general equilibrium which can be sustained by government taxes and subsidies (Romer 1986, p. 1004). For this competitive equilibrium to hold, Romer assumes that the overall production function is a concave function for all factors of production. Even when some factors of production have a concave function, the intangible capital good, knowledge, exhibits a convex function with increasing returns to scale (Romer 1986, p. 1015). He assumed that knowledge was a principle and measurable intangible input of production and a factor that violated the Inada (1963) conditions of diminishing marginal rate of substitution. The possibility of holding all these controversial assumptions is presumed to be left in the hands of a benevolent dictator who sets '*economy-wide*' values of the intangible knowledge that is a convex function in the production function (Romer 1986, p. 1015). The setup for maximising consumption is the same as in the Cass (1965) model, where Romer comes up with a similar Hamiltonian function with parameters to be determined outside the model that include the shadow price, rate of time preference, degree of constant relative risk aversion and an additional factor, intangible knowledge. These are all at the mercy of the benevolent dictator or social planner. For a detailed derivation of the discrete-time model, see Romer (1986, pp. 1014–1034).

3.2.2. Schumpeterian Endogenous Growth Models of Industrial Innovation

Another set of endogenous growth models based on intellectual capital was developed by Grossman and Helpman (1991) and Aghion and Howitt (1992). These endogenous growth models focused on quality-improving innovations based on Schumpeter's (1942) creative-destruction theory. The Schumpeterian theory postulates that aggregate output is accumulated through a continuum of improvements in intermediate products. Grossman and Helpman (1991, p. 43) argue that product quality improvements raise total factor productivity in the manufacturing of consumer and capital goods over time. Aghion and Howitt (1992, p. 323), on the other hand, model the role of industrial innovations that improve the quality of products. Both theories are based on Schumpeter's (1942) theory of creative destruction, where there are quality ladders in which each new innovation is built based on the previous one (Grossman and Helpman 1991, p. 44) or "*better products render previous ones obsolete*" (Aghion and Howitt 1992, p. 323). In these Schumpeterian models, the long-run growth rate, therefore, depends on the share of GDP spent on research and development and not necessarily on the share of output that is saved (Stokey 1995).

3.3. Human Capital-Based Endogenous Growth Theories – Lucas (1988)

Another variant of the endogenous growth model that adds an important factor of production, human capital, was developed by Lucas (1988). Lucas defines human capital as developments in skill level where the productivity of a single worker can be increased by increasing his/her skill level. Lucas postulates that human capital has two effects: human capital effects on existing factors of production and the production function; and time allocation that affects human capital accumulation (Lucas 1988, p. 17). His model framework conforms to the classical triad first introduced by JB Say's praxeological deduction on the importance of labour in the accumulation of output or income. The Lucas (1988) human capital growth model has two solutions to solve: an optimal path and an equilibrium path. The optimal path aims to maximize consumer utility subject to the production function and the endogenous human capital accumulation function. The equilibrium path involves maximising the endogenous human capital accumulation function (Lucas 1988, p. 20). The consumer utility function is as presented in equation (13c) with a closed economic system and an exogenous population growth rate λ. The production function is modified and includes human capital based on skill level defined as follows (Lucas 1988, p. 18):

$$L(t)c(t)+\dot{K}(t)=AK(t)^{\beta}\left[u(t)\ h(t)L(t)\right]^{1-\beta}h_a(t)^{\gamma} \tag{12a}$$

Where:
$h_a(t)^{\gamma}$ = External effects of human capital
A = Level of technology assumed to be constant
$h(t)$ = Human capital accumulation

If $\gamma=0,$ then there are no external effects and the balanced growth path for the Lucas human capital growth model is a concave function with a straight line that passes through the origin. However, if $\gamma>0,$ then the Lucas model become a convex function. The accumulation of human capital function is based on the level of effort devoted to the accumulation of human capital linked to the rate of change in human capital. Lucas adapts a similar formulation by Uzawa (1965) and Rosen (1976), where the growth rate of human capital accumulation is (Lucas 1988, p. 19):

$$\dot{h}(t)=h(t)\delta\left[1-u(t)\right]. \tag{12b}$$

Based on the three equations (11a), (12a) and (12b), Lucas solves the following Hamiltonian function that maximises consumer utility, subject to the production function (12a) and human capital accumulation function (10b) (Lucas 1988, p. 20):

$$\begin{gathered}H(c,\ u,\ K,\ h,\ \theta_1,\theta_2,\ t)=\frac{L(t)\left[c(t)^{1-\sigma}-1\right]}{1-\sigma}+\\ \theta_1\left[AK(t)^{\beta}\left(u(t)\ h(t)L(t)\right)^{1-\beta}h_a(t)^{\gamma}-L(t)c(t)\right]+\\ \theta_2\left[\delta h(t)\left(1-u(t)\right)\right]\end{gathered} \tag{12c}$$

r finding solutions to the relevant parameters, see Lucas (1988, pp. 21–26). The solutions to Lucas' human capital growth model, however, are based on one key restriction that, for the model to hold, the value of the constant relative risk aversion factor should be equal to or greater than one. The Lucas model based on these restrictions provides an economic growth model where human capital leads to growth effects rather than level effects and is dependent on the value of the rate of preference, ρ, the growth rate for human capital accumulation, δ, and the constant relative risk aversion parameter, σ (Lucas 1988, p. 23). In summary, the key assumption of the Lucas human capital endogenous growth model assumes increasing returns based on human capital development function that is convex. The fundamental problems with the endogenous growth models discussed in this section are, to some extent, the same problems we articulated based on the Cass (1965) model – the problem of determining the optimal values of the intangible parameters that become subjective to the valuation of the benevolent dictator or social planner. In addition, and perhaps what Lucas (1988, p. 12) argued, even the parameters in the endogenous growth models do not lead to growth effects – that is, altering growth rates along the balanced growth paths – but to level effects – changes that shift the balanced growth path without affecting the slope of the long-run growth path.

3.4. Other Endogenous Based Growth Theories

Another set of growth theories emerged in the economic growth literature, especially in the 1990s, and these models allow for the efficiency of investment and human capital in the economy to be influenced by policies and institutional settings. These models reinforce the notion that economic policies have a long-term effect on the long-run equilibrium growth path (Bassanini et al. 2001, p. 6; Acemoglu et al. 2005). Therefore, policies that contribute to the efficiency of savings and investment by lowering distortions in resource allocation will generally encourage economic growth (Easterly and Wetzel 1989, p. 20). The theoretical arguments are grounded in growth hypotheses, such as the Wagner Law or Peacock-Wiseman hypothesis, on government spending and growth (Peacock and Wiseman 1961); the Mundell-Tobin hypothesis on inflation and growth (Mundell 1963; Tobin 1965); the Balassa-Samuelson hypothesis on purchasing power parity, the growth of real exchange rates, trade and growth (Balassa 1964; Samuelson 1964); the McKinnon-Shaw hypothesis on financial repression and growth (McKinnon 1973; Shaw 1973); and the Acemoglu et al. (2005) political institutions hypothesis. The Wagner law or the Peacock-Wiseman hypothesis postulate that public expenditure rises at a faster rate than national output. The law predicts that any economy that transitions to higher income levels will experience an increase in public expenditure as a share of GDP. This increase is due to the increased social activities of the state, increased government administration and protection, and the welfare functions of the state (Peacock and Wiseman 1961; Kolluri et al. 2000). Keynes (1936) also advocated for a deliberate use of government budget and expenditures to stimulate demand and employment during times when the aggregate demand for goods and services was lower than supply. According to the Keynesian theory, government spending can be used to stimulate the economy by inducing investment through government investment in infrastructure (fiscal policy) and a reduction in real interest rates (monetary policy). Keynes argued that there were some situations where a depressed economy would not quickly respond to the price mechanism to return to equilibrium. Thus, the use of countercyclical fiscal policies that aim at reducing the amplitude of the business cycle during a depression is recommendable to influence short-run growth while market forces do so in the long-run (Keynes 1936).

The impact of inflation on economic growth has been discussed in the literature through the Mundell-Tobin hypothesis (Mundell 1963; Tobin 1965). The impact of inflation on economic growth is through the efficiency of savings and investment, where inflation acts as a tax on savings and investment (Bassanini et al. 2001). While the monetary growth theory literature on inflation and growth posits a positive impact of inflation on the accumulation of physical capital and growth, i.e., the Mundell-Tobin hypothesis, there are others who posit a negative relationship between inflation and growth (Sidrauski 1967; Stockman 1981; Fischer 1983). Other studies have postulated no impact between inflation and growth (Friedman and Schwartz 1963). Recent models on the relationship between inflation and economic growth postulate that a negative relationship exists only at certain thresholds – lower inflation can be associated with a positive impact on growth whereas higher rates of inflation may be associated with a negative impact on growth (Bullard and Keating 1995; Bruno and Easterly 1998; Khan and Senhadji 2001). The relationship between economic growth and changes in the exchange rate is through the Purchasing Power Parity (PPP) theory, which postulates that the equilibrium exchange rate between currencies should be equivalent to their relative prices (Balassa 1964; Samuelson 1964). Any deviation is inflationary and expected to be corrected through exchange rate adjustments. The relationship between real exchange rates and economic growth has always been an important subject in the economic growth literature. If an economy faces rapid economic growth, the local currency is expected to appreciate, while depreciation is an indication of economic stagnation or weak performance of the economy (Ito et al. 1999, p. 110). Financial restrictions have a negative impact on growth rates, as artificial ceilings on interest rates lead to a reduction in savings, capital accumulation and hence the efficient allocation of resources (McKinnon 1973; Shaw 1973). Furthermore, credit rationing impacts growth negatively by channelling credit to areas that may not be productive, hence leading to slower growth (Gelb 1989). The empirical evidence has shown that higher real interest rates are positively associated with growth (Balassa 1989; Gelb 1989), while financial repression or low or negative real interest rates are negatively associated with growth (World Bank 1990).

Finally, Acemoglu et al. (2005) argue that differences in economic institutions adopted by economies are one of the fundamental causes of differences in economic development across nations. These institutions are critical in determining the types of incentives which have a positive impact on economic growth, or the disincentives which negatively affect growth rates that economic agents are likely to face, and thus determine the structure of future economic activities. They also argue that the ultimate resolve regarding which economic direction to follow depends on which economic groups or factions have greater political power to influence political institutions that govern as well as to influence the distribution of resources. Such political institutions, and hence the distribution of resources, are the key state variables of institutions' theoretical framework and are assumed to be dynamic over time as new factions emerge to change the political landscape.

3.5. Criticisms of the Endogenous Growth Models

Recent studies on endogenous growth theories have extended the neoclassical framework by relaxing the important assumptions of exogenous growth theories, such as diminishing returns to scale, the exogeneity of technological change, and investment in human capital. A fundamental departure from the exogenous growth models is founded on the assumption that the capital employed exhibits constant or increasing and not diminishing returns. Frankel (1962) assumed this would be achieved through the accumulation of innovative capital; Romer (1986) assumed that this is possible through knowledge externalities; Grossman and Helpman (1991), Aghion and Howitt (1992), and Stokey (1995) assumed the existence of constant returns to scale would emanate from economic innovation; and Lucas (1988) assumed this would be possible through increasing the efficiency of human capital in the form of skills or learning by doing. Other endogenous growth studies have further included factors that affect the efficiency of capital as possible growth determinants that can lead to capital flight, and hence contribute towards international differences in per capita income. These efficiency factors include fiscal policy (government spending, taxation, etc.), monetary policy (inflation), financial development (real interest rates), outward orientation (real exchange rates), and political institutions. There are five challenges that we observe with endogenous growth models that are yet to be addressed. The first challenge is that most of the empirical endogenous growth models faced the problem of how to estimate the optimum values of the rate of time preference, constant relative risk aversion (CRRA), and shadow prices. These parameters are usually assumed to be determined by a social planner or a benevolent dictator (Romer 1986; Lucas 1988). Lucas (1988) assumed this value to be greater than one for his model to be valid. King and Rebelo (1990) assumed a value of CRRA to be either one-half or two for them to be able to assess the impact of a permanent change in the tax rate, while Stokey (1995), in estimating the impact of research and development on economic growth, assumed parameter values of the rate of time preference and the CRRA to be equal to one. Overall, the present structural endogenous growth models still harbour important parameters that are difficult to estimate empirically.

The second challenge relates to the fact that the empirical estimation of endogenous growth models is also usually done through simulations. As noted by Schumpeter (1942, p. 83), any ex-post appraisal of an economic innovation cannot be achieved since the process of realising any transformation from an economic innovation takes considerable time to fully reveal its ultimate effects, and can take decades if not centuries. Hence, the only meaningful evaluation is an ex-ante simulation or forecasts. Furthermore, he argued that a meaningful analysis of an economic innovation is only within a particular industrial domain and becomes inconclusive beyond that (Schumpeter 1942, p. 83). The third challenge associated with endogenous growth modelling and empirical verification relates to finding a suitable proxy for human capital accumulation that would represent the accumulation of knowledge, skills and learning by doing as an additional source of innovative capital. However, models that rely on intangible variables, such as knowledge and skills, are hard to measure empirically. There is still no agreement on the best proxy for human capital as evidenced by different and unreliable approaches such as cost-based (Jorgenson and Fraumeni 1989), output-based (Barro and Lee 1994), and income-based approaches (Mulligan and Sala-i-Martin 1997). In addition, Levine and Renelt (1992, p. 945) argued that though investment in human capital is important, the proxies used, such as school enrolment or average years of schooling, do not control for quality, and investment in human capital is more than just formal schooling. Others have argued that an appropriate measure would be to estimate the value of all forgone earnings when an individual is engaged in training (Kwon 2009). This again has its own drawbacks, especially in determining the potential income that an individual is expected to earn.

A fourth challenge noted by Diamond (1990, p. 221) shows that the stability or instability in determining the saddle point in endogenous growth models assumes perfect foresight of the parameters $\left(\theta(t),\ \rho,\ \sigma\right)$ by the social planner, which is rarely the case. The key assumption made is that planners, consumers and firms are rational agents that have perfect foresight of future prices and correctly choose the parameters $\left(\theta(t),\ \rho,\ \sigma\right)$ rationally. In practice, the social planner cannot know whether the assumptions made for the equilibrium path are correct or even how to correct a miscalculation concerning the optimum equilibrium path based on consumer utility. As Solow (1956) noted, building a credible theory of investment or even consumption over time based on the axiom of perfect foresight is difficult. Eventually, the endogenous growth models present level effects just like the Solow neoclassical growth model and not growth effects. Level effects have temporal effects on the economic growth path of an economy and the endogenous models, while taking a different approach to developing alternative theories of economic growth based on maximising consumer utility, did not achieve the intended objective of endogenizing growth effects. The final challenge, as noted by Rothbard (1995, p. 379–381), states that utility or efficiency cannot be measured, it is indivisible, and a subjective abstract-valuation and ranking by an individual. Given its subjective and intangible nature, the concept of comparing and aggregating consumption utility across individuals, or efficiency across political, social or economic institutions, would be equally impossible and difficult to empirically estimate.

4. Summary and Conclusion

The paper examined theoretical developments in the economic growth literature between exogenous and endogenous growth models that we study today. Within these growth models, we identified six (6) key determinants that can account for international productivity differences. These include technology (Solow, 1956), innovative and intellectual capital (Frankel 1962; Romer 1986; Grossman and Helpman 1991; Aghion and Howitt 1992), human capital (Lucas 1988), fiscal policy (Peacock and Wiseman 1961), monetary policy (Mundell 1963; Tobin 1965; Sidrauski 1967; Stockman 1981; Fischer 1983), trade (Balassa 1964; Samuelson 1964), financial factors (McKinnon 1973; Shaw 1973), and political, social and economic institutions (Acemoglu et al., 2005). There are two main issues for future theoretical economic growth research that will influence the future debate on growth models that have the capability of explaining international productivity differences. In response to the four critical issues raised by Klenow and Rodriguez-Clare (1997), much as there has been an improvement in ensuring data availability especially for most countries in the world such as the data on human capital development (Barro and Lee 2013), Penn World Tables (Feenstra et al. 2015), or World Development Indicators (World Bank 2015), the empirical measurement of factors such as innovative capital, intellectual capital, human capital, and political, social and economic institutions continues to be a challenge. Second, there is still a need to link theory and evidence as well as to continue the process of developing new growth theories that seek to account for international productivity differences. In our view, an important empirical framework is one that measures capital shares as proposed by Mankiw et al. (1992). However, how to incorporate input shares based on endogenous growth modelling is still a challenge as the theoretical literature has to estimate clearly the share that each factor will contribute if analysed in one single growth equation. We conclude that the theoretical growth debate on factors accounting for international productivity differences is far from over.

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Streszczenie

EGZOGENICZNE I ENDOGENICZNE MODELE WZROSTU: PRZEGLĄD KRYTYCZNY

Badane obecnie główne działy literatury teoretycznej dotyczącej wzrostu gospodarczego obejmują egzogeniczne i endogeniczne modele wzrostu, które doczekały się wielu różnych definicji i spotkały z krytycznymi uwagami. Zwolennicy egzogenicznych modeli wzrostu twierdzą, że postęp technologiczny jest kluczowym wyznacznikiem długoterminowego wzrostu gospodarczego, jak również różnic w produktywności w skali międzynarodowej. W modelach endogenicznego wzrostu propagowane są dwa poglądy. Pierwszy z nich mówi, że kapitał wykorzystywany do celów innowacyjnych może wywoływać korzyści skali, a tym samym przyczyniać się do powstawania obserwowanych obecnie różnic w pro-

duktywności w skali międzynarodowej. Najważniejszymi czynnikami są tutaj: wiedza, kapitał ludzki oraz badania i rozwój. Drugi pogląd mówi, że międzynarodowe różnice w produktywności można również wytłumaczyć czynnikami, które wpływają na efektywność kapitału, a tym samym powodują ucieczkę kapitału,. Czynniki wpływające na efektywność kapitału obejmują: wydatki rządowe, inflację, realne kursy walut i realne stopy procentowe. Wyniki przeprowadzonych badań pokazują, że wśród ekonomistów nadal nie ma konsensusu odnośnie dominującego teoretycznego modelu wzrostu gospodarczego, który może w pełni uwzględniać różnice w produktywności w skali międzynarodowej. W konkluzji stwierdzono, że prace dotyczące teorii wzrostu gospodarczego są dalekie od zakończenia i należy kontynuować prace zmierzające do opracowania bardziej praktycznych modeli strukturalnych wzrostu gospodarczego.

Słowa kluczowe*: wzrost gospodarczy; egzogeniczne modele wzrostu; endogeniczne modele wzrostu*