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UFUK YOLCU*, EREN BAS**

The Forecasting Of Labour Force Participation And The Unemployment Rate In Poland And Turkey Using Fuzzy Time Series Methods

Abstract

Fuzzy time series methods based on the fuzzy set theory proposed by Zadeh (1965) was first introduced by Song and Chissom (1993). Since fuzzy time series methods do not have the assumptions that traditional time series do and have effective forecasting performance, the interest on fuzzy time series approaches is increasing rapidly. Fuzzy time series methods have been used in almost all areas, such as environmental science, economy and finance. The concepts of labour force participation and unemployment have great importance in terms of both the economy and sociology of countries. For this reason there are many studies on their forecasting. In this study, we aim to forecast the labour force participation and unemployment rate in Poland and Turkey using different fuzzy time series methods.

Keywords: *fuzzy time series, forecasting, labour force participation, unemployment*

* Ph.D. Associate Professor at Ankara University, Faculty of Sciences, Department of Statistics, e-mail: uyolcu@ankara.edu.tr

** Ph.D. Assistant Professor at Giresun University, Faculty of Arts and Sciences, Department of Statistics, e-mail: eren.bas@giresun.edu.tr

1. Introduction

Fuzzy time series procedures, which have attracted the attention of many researchers in recent years, have a quite wide area of use, such as information technology, economy, environmental sciences and hydrology.

Fuzzy time series are divided into two parts; time-invariant and time-variant. It is assumed that time-invariant fuzzy time series' behavior does not change in time, while time-variant fuzzy time series act the opposite. Since the vast majority of the studies in the literature are about the time-invariant fuzzy time series, we limited the scope of our study to the time invariant-fuzzy time series.

Fuzzy time series was first proposed in 1993 (Song and Chissom 1993a) and depended on fuzzy set theory (Zadeh 1965). Fuzzy time series forecasting procedures consist of three fundamental stages: fuzzification, determination of fuzzy relations, and defuzzification.

The length of the interval which is required for the partition of the universe of discourse has a significant impact on forecasts. For this reason, many studies in literature on the determination of interval length are available. In the majority of these, it is determined as a fixed length. In some of them, fixed lengths were determined arbitrarily (Song and Chissom 1993a,b, 1994; Chen 1996, 2002). In the others, a fuzzy C-means (FCM) procedure was used for fuzzification (Cheng et al. 2008; Li et al. 2008; Cagcag Yolcu 2013).

In fuzzy time series analyses, the determination of fuzzy relations is also an important stage since it affects the forecasting performance. In the literature, there are some studies which contribute to that stage. While matrix algebra was used in the first studies (Song and Chissom 1993a, b, 1994), fuzzy logic group tables were utilized in the next study (Chen 1996). This technique has been used in subsequent studies. Moreover, the artificial neural network was also used in the determination of fuzzy relations (Huarng and Yu 2006b; Aladag et al. 2009; Egrioglu et al. 2009a,b,c; Yu and Huarng 2008, 2010; Alpaslan et al. 2012). In addition in another study, while fuzzy logic relationship tables were used in the identification of fuzzy relations, estimating based on the next state for training set and master voting scheme for test set were used (Yolcu et al. 2014).

In the defuzzification stage, the centroid method is frequently used. For many data sets encountered in real life a high-order fuzzy time series forecasting model would be more appropriate to be analyzed, while a first-order fuzzy time series forecasting model can be enough to fit to some fuzzy time series data. In the some papers, the first-order fuzzy time series forecasting model was used (Song and Chissom 1993a,b, 1994; Chen 1996; Yu and Huarng 2008, 2010).

The high-order fuzzy time series forecasting model was employed to analyze the data sets in a number of studies (Chen 2002; Aladag et al. 2009; Egrioglu et al. 2009a,b,c, 2010).

Labour force participation and the unemployment rate, which has a direct impact on economic and social development, has become an issue in most countries. For this reason the concepts of labour force participation and unemployment are of great importance in terms of both the economy and sociology for countries. According to the International Labour Organization (ILO) standards, the official working age is 15. Therefore, the population aged 15 and over is taken into account in the analyses related to employment and unemployment. The labour force participation rate and the unemployment rate is one of the most important criterions of a country's development. However the concept of the non-institutional civilian population should be addressed before the concepts of labour force participation rate and the unemployment rate. The non-institutional civilian population is the population excluding those residing in places such as university dormitories, orphanages, nursing homes, hospitals, prisons, barracks and so on. The non-institutional civilian population is divided into three subgroups:

a. Employees

b. Unemployed

c. Not included in the labour force

The sum of the unemployed and those who are employed in a country is called the labour supply.

$$Labour\ Supply = Employees + Unemployed$$

The labour force is the actual number of people available for work and labour force participation rate refers to those who want to work in the civilian non-institutional population and is calculated in Equation 1.

$$Labor\ force\ participation\ rate = \frac{Labour\ Supply}{Non\text{-}Institutional\ Civilian\ Population} \times 100 \qquad (1)$$

The labour participation rate decreases during periods of economic recession and increases in periods of expansion. Unemployment includes those who desire to and have the ability to work within the current wage level, but cannot find work in the labour force.

The unemployment rate refers to the proportion of those in the labour supply who are not working, and is calculated Equation 2.

$$\text{Unemployment rate} = \frac{\text{Unemployed}}{\text{Labour Supply}} \times 100 \qquad (2)$$

The concepts of labour force participation and unemployment have great importance in terms of both the economy and sociology of countries. The rapid growth experienced in labour force participation in recent years created an upward pressure on the unemployment rate. In this context, calculation of the labour force participation rate for the upcoming years is thought to be important. There are also some studies showing the effectiveness of the labour force participation rate of the generation gap and the aging population in developed countries. For this reason, the methods used in this paper were applied for each generation separately.

Starting from this point of view, we also aim to forecast the labour force participation and unemployment rate in Poland and Turkey with different fuzzy time series methods.

2. Fuzzy Time Series

In contrast to conventional time series methods, various theoretical assumptions do not need to be checked in fuzzy time series approaches. The most important advantage of the fuzzy time series approaches is the ability to work with a very small set of data. The definition of fuzzy time series is given as follows:

Let U be the universe of discourse, where $U = \{u_1, u_2, \cdots, u_n\}$. A fuzzy set A_i of U can be defined in Equation 3.

$$A_i = \frac{\mu_{A_i}(u_1)}{u_1} + \frac{\mu_{A_i}(u_2)}{u_2} + \cdots + \frac{\mu_{A_i}(u_n)}{u_n} \tag{3}$$

Where μ_{A_i} is the membership function of the fuzzy set A_i and $\mu_{A_i}; U \to [0, 1]$. In addition to, $\mu_{A_i}(u_j)$, $j = 1, 2, \cdots, n$ denotes is a generic element of fuzzy set A_i; $\mu_{A_i}(u_j)$ is the degree of belongingness of u_1 to A_i; $\mu_{A_i}(u_j) \in [0,1]$.

Definition 1. Fuzzy time series Let $Y(t)$ $(t = \cdots, 0, 1, 2, \cdots)$ a subset of real numbers, be the universe of discourse by which fuzzy sets $f_i(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \cdots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2. Fuzzy time series relationships assume that $F(t)$ is caused only by $F(t-1)$, then the relationship can be expressed as: $F(t) = F(t-1) * R(t, t-1)$, which is the fuzzy relationship between $F(t)$ and $F(t-1)$, where $*$ represents as an operator. To sum up, let $F(t-1)=A_i$ and $F(t)=A_j$. The fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be denoted as $A_i \rightarrow A_j$ where A_i (current state) refers to the left-hand side and A_j (next state) refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationships.

Definition 3. Let $F(t)$ be a fuzzy time series. If $F(t)$ is a caused by $F(t-1),\ F(t-2),\cdots,F(t-m),\quad F(t-m)$, then this fuzzy logical relationship is represented in Equation 4.

$$F(t-m),\ F(t-m+1),\ \cdots,\ F(t-2),\ F(t-1)\rightarrow F(t) \tag{4}$$

and it is called the m^{th} order fuzzy time series forecasting model.
Where $F(t-m),\ F(t-m+1),\ \cdots,\ F(t-2),\ F(t-1)$ refers to the current state and $F(t)$ refers to the next state.

3. Methods

Some of methods which were used in the application process are given below with an algorithm.

3.1. Chen's First Order Fuzzy Time Series Method

Chen improved the approach proposed by Song and Chissom. Chen's method uses a simple operation, instead of complex matrix operations, in the establishment step of fuzzy relationships. The algorithm of Chen's method can be given as follows:

Algorithm 1.

Step 1. Define the universe of discourse and intervals for rules abstraction.

Based on the issue domain, the universe of discourse can be defined as: $U=[starting,\ ending]$. As the length of interval is determined U can be partitioned into several equally length intervals.

Step 2. Define fuzzy sets based on the universe of discourse and fuzzify the historical data.

Step 3. Fuzzify observed rules.

Step 4. Establish fuzzy logical relationships and group them based on the current states of the data of the fuzzy logical relationships.

For example, $A_1 \to A_2$, $A_1 \to A_1$, $A_1 \to A_3$, can be groupedas: $A_1 \to A_2, A_3, A_1$.

Step 5. Forecast.

Let $F(t-1) = A_i$

Case 1. There is only one fuzzy logical relationship in the fuzzy logical relationship sequence. If $A_i \to A_j$, then $F(t)$ forecast value is equal to A_j.

Case 2. If $A_i \to A_i, A_j, \ldots, A_k$, then $F(t)$, forecast value, is equal to $A_i, A_j, \ldots, A_k$.

Step 6. Defuzzify.

Apply the "Centroid" method to get the results. This procedure (also called center of area, center of gravity) is the most often adopted method of defuzzification.

3.2. Chen's High Order Fuzzy Time Series Method

Chen proposed a method based on high order fuzzy time series which enable obtaining forecasts. The method proposed by Chen produces more accurate forecasts than the first order fuzzy time series methods (Chen 2002). The model given in definition 3 can be analyzed by the high order fuzzy time series approach. The steps of the algorithm are given below.

Algorithm 2.

Step 1. Define the universe of discourse and subintervals. $D_{\min}$ and $D_{\max}$ variables are defined based on min and max values in the data set. Then choose two arbitrary positive numbers which are D_1 and D_2 in order to divide the interval evenly. $U = [D_{\min} - D_1, D_{\max} + D_2]$.

Step 2. Define the fuzzy sets based on the universe of discourse and fuzzify the historical data.

Step 3. Fuzzify the observed rules.

Step 4. Establish fuzzy logical relationships and group them based on the current states of the data of the fuzzy logical relationships. Based on the linguistically defined variables, k^{th} order fuzzy logical relationship $A_{ik}, A_{i(k-1)}, \ldots, A_{il} \to A_j$ can be established. For example, the values of the year $i-1$ and i corresponds to fuzzy values A_a and A_b. Also, the values of the year $i+1$ corresponds to fuzzy value A_j. Therefore, 2th order fuzzy logical relationship can be written as $A_a, A_b \to A_j$. In a similar manner, the more high order fuzzy logical relationships and fuzzy logical groups for 3th, 4th and other high orders are constructed.

Step 5. Forecast and Defuzzify.

In this step, fuzzy values are defuzzified and real forecasts are obtained.

If the k^{th} order fuzzified history time series for year i are $A_{ik}, A_{i(k-1)}, \ldots,$ and A_{il}, where $k \geq 2$, and there is the following fuzzy logical relationship in the k^{th} order fuzzy logical relationship groups shown as follows:

$$A_{ik}, A_{i(k-1)}, \ldots, A_{il} \to A_j,$$

where $A_{ik}, A_{i(k-1)}, \ldots, A_{il}$ and A_j, are fuzzy sets, and the maximum membership value of A_j occurs at interval u_j, and the midpoint of u_j is m_j, then the forecasted time series of year i is m_j.

If the k th order fuzzified history time series for year i are $A_{ik}, A_{i(k-1)}, \ldots,$ and A_{il}, where $k \geq 2$, and there is the following fuzzy logical relationship in the k^{th} order fuzzy logical relationship groups shown as follows:

$$A_{ik}, A_{i(k-1)}, \ldots, A_{il} \to A_{j1}$$

$$A_{ik}, A_{i(k-1)}, \ldots, A_{il} \to A_{j2}$$

$$\vdots$$

$$A_{ik}, A_{i(k-1)}, \ldots, A_{il} \to A_{jp}$$

where $A_{ik}, A_{i(k-1)}, \ldots, A_{il}, A_{j1}, A_{j2}, \ldots,$ and A_{jp}, are fuzzy sets, then we can see that there is an ambiguity to forecast the time series of the year i (i.e. the fuzzy data of year i may be A_{j1} or A_{j2} or… A_{jp}). In this case, we must find higher order fuzzified history time series for year i, such that there is no ambiguity to forecast the time series of the year i. Assume that there exists an integer m that can resolve this ambiguity, where $m \geq k$, such that m^{th} order fuzzified time series of year i are $A_{im}, A_{i(m-1)}, \ldots,$ and A_{i1}, and there is the following fuzzy logical relationship in the m^{th} order fuzzy logical relationship groups, shown as follows:

$$A_{im}, A_{i(m-1)}, \ldots, A_{il} \to A_{j1},$$

where $A_{im}, A_{i(m-1)}, \ldots, A_{il}$ and A_j are fuzzy sets, and the maximum membership value of A_j occurs at interval u_j, and the midpoint of u_j is m_j, then the forecasted time series of year i is m_j.

If the k^{th} order fuzzified history time series for year i are $A_{ik}, A_{i(k-1)},...,$ and A_{il}, where $k \geq 2$, and there is the following fuzzy logical relationship in the k^{th} order fuzzy logical relationship groups in which the right hand side of the fuzzy logical relationship is empty shown as follows:

$$A_{ik}, A_{i(k-1)},..., A_{il} \rightarrow \#,$$

where $A_{ik}, A_{i(k-1)},...,$ and A_{il}, are fuzzy sets, and the maximum membership values of $A_{ik}, A_{i(k-1)},...,$ and A_{il} occur at intervals $u_{ik}, u_{i(k-1)},...,$ and u_{il}, respectively, and the midpoint of $u_{ik}, u_{i(k-1)},...,$ and u_{il} are $m_{ik}, m_{i(k-1)},...,$and m_{il}, respectively then the forecasted time series of year i is calculated as follows:

$$\frac{1\times m_{ik} + 2\times m_{i(k-1)} + \cdots + k\times m_{i1}}{1+2+\cdots+k}$$

3.3. Aladag et al.'s Fuzzy Time Series Method

Algorithm 3.

In order to construct a high order fuzzy time series model, various feed forward artificial neural networks (FF-ANN) architectures are employed to define the fuzzy relation. The stages of the proposed method based on neural networks are given below.

Step 1. Define and partition the universe of discourse

The universe of discourse for observations, $U = [starting, ending]$, is defined. After the length of intervals, l, is determined, the U can be partitioned into equal-length intervals $u_1, u_2,...,u_b$, $b = 1,...$ and their corresponding midpoints $m_1, m_2,...,m_b$, respectively.

$$u_b = [starting + (b-1)\times l, starting + b\times l],$$

$$m_b = \frac{[starting + (b-1)\times l, starting + b\times l]}{2}$$

Step 2. Define fuzzy sets.
Each linguistic observation, A_i, can be defined by the intervals $u_1, u_2,...,u_b$.

$$A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$$

Step 3. Fuzzify the observations.

For example, a datum is fuzzified to A_i, if the maximal degree of membership of that datum is in A_i.

Step 4. Establish the fuzzy relationship with feed forward neural network.

An example will be given to explain Step 4 more clearly for the second order fuzzy time series. Because of dealing with second order fuzzy time series, two inputs are employed in neural network model, so that lagged variables $F(t-2)$ and $F(t-1)$ are obtained from fuzzy time series $F(t)$. These series are given in Table 1. The index numbers (i) of A_i of $F(t-2)$ and $F(t-1)$ series are taken as input values whose titles are Input-1 and Input-2 in Table 1 for the neural network model. Also, the index numbers of A_i of $F(t)$ series are taken as target values whose title is Target in Table 1 for the neural network model. When the third observation is taken as an example, inputs values for the learning sample $A_i\ [A_6, A_2]$ are 6 and 2. Then, the target value for this learning sample is 3.

Table 1. Notations for second order fuzzy time series

Observation No	F_{t-2}	F_{t-1}	F_t	Input– 1	Input– 2	Target
1	---	---	A_6	---	---	---
2	---	A_6	A_2	---	---	---
3	A_6	A_2	A_3	6	2	3
4	A_2	A_3	A_7	2	3	7
5	A_3	A_7	A_4	3	7	4
6	A_7	A_4	A_2	7	4	2

Source: Own preparation

Step 5. Defuzzify results

The defuzzified forecasts are middle points of intervals which correspond to fuzzy forecasts obtained by neural networks in the previous stage.

3.4. Yolcu et al.'s Fuzzy Time Series Method

Algorithm 4.

Step 1. Time series are fuzzified by FCM clustering.

Let c be the number of the fuzzy set, such that $2 \le c \le n$ where n is the number of observations. The FCM clustering algorithm in which the number of fuzzy sets is c is applied to the time series consists of crisp values. After this application, the center of each fuzzy set is determined. Then, the membership degrees for each observation, which denote a degree of belonging to a fuzzy set for that observation, are calculated with respect to the obtained values of center of fuzzy sets. Finally, ordered fuzzy sets, $L_r, r = 1,2,...,c$ are obtained according to the ascending ordered centers, which are denoted by $v_r, r = 1,2,..., c$

For better understanding, we consider a time series data with 8 observations such as 20, 30, 40, 30, 20, 50, 60, 80. Let *c*, the number of fuzzy sets, be 3. When we applied the FCM method to this data, the centroid of the fuzzy sets and the membership degrees of each observation, which denotes the belonging degree of that observation to the related fuzzy set, are given in Table 2. According to Table 2, the membership degree of belonging to the second fuzzy set (L_2) of the first observation $t = 1$ is $\mu_{L_2}(x_{(1)}) = 0.0293$.

Step 2. Define the fuzzy relationship with FF-ANN.

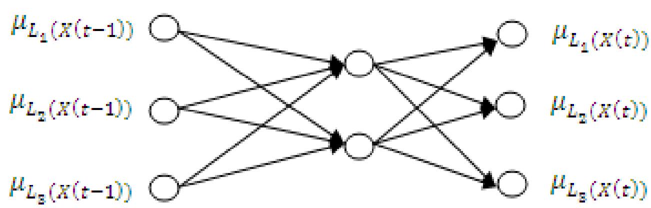
The number of neurons in the input and output layers of FF-ANN, used for determining fuzzy relationships, is equal to the number of fuzzy sets, c. What the number of neurons in the hidden layer can be are decided via the trial and error method to avoid a possible loss in the ability of generalization of FF-ANN. The architecture of the network was shown in Figure 1 with 3 fuzzy sets and 2 neurons in the hidden layer. In Figure 1, $\mu_{L_i}(X(t))$ denotes the membership degree of belonging to ith fuzzy set of related observation of time series $X(t)$. Then, the target values of FF-ANN are every membership degrees of belonging to c fuzzy sets of the observation of time series at t while the inputs of the networks are every membership degrees of belonging to c fuzzy sets of the observation of time series at t–1.

Table 2. An example of fuzzification

		Center of Set 1 (v_1)	Center of Set 2 (v_2)	Center of Set 3 (v_3)
		25.4718	51.3607	79.4108
		Memberships of Observations to Fuzzy Sets		
t	$X(t)$	**Set 1** (L_1)	**Set 2** (L_2)	**Set 3** (L_3)
1	20	0.9625	0.0293	0.0082
2	30	0.9494	0.0427	0.0080
3	40	0.3608	0.5901	0.0490
4	30	0.9494	0.0427	0.0080
5	20	0.9625	0.0293	0.0082
6	50	0.0031	0.9948	0.0021
7	60	0.0497	0.7932	0.1571
8	80	0.0001	0.0004	0.9995

Source: The memberships and centers were obtained via FCM algorithm based on X(t) by using MATLAB.

Figure 1. The architecture of the feed forward network with 3 fuzzy sets



Source: Own preparation.

The following logistic activation function is used in all layers of FF-ANN, whose architecture was explained in Equation 5.

$$f(x) = (1 + \exp(-x))^{-1} \tag{5}$$

In the FF-ANN with this architectural structure optimal weights are obtained by training with the learning algorithm Levenberg-Marquardt. Thus, the trained FF-ANN has learned the relationship among the membership degrees of the subsequent observations of time series.

For example, suppose that we consider the time series given in Table 2. When we defined the architectural structure as given in Figure 1, the input and the targets of ANN would be as Table 3.

Table 3. An example of determine fuzzy relation

Training Sample	t	Input 1 $\mu_{L_1(X(t-1))}$	Input 2 $\mu_{L_2(X(t-1))}$	Input3 $\mu_{L_3(X(t-1))}$	Target 1 $\mu_{L_1(X(t))}$	Target 2 $\mu_{L_2(X(t))}$	Target 3 $\mu_{L_3(X(t))}$
1	2	0.9625	0.0293	0.0082	0.9494	0.0427	0.0080
2	3	0.9494	0.0427	0.0080	0.3608	0.5901	0.0490
3	4	0.3608	0.5901	0.0490	0.9494	0.0427	0.0080
4	5	0.9494	0.0427	0.0080	0.9625	0.0293	0.0082
5	6	0.9625	0.0293	0.0082	0.0031	0.9948	0.0021
6	7	0.0031	0.9948	0.0021	0.0497	0.7932	0.1571
7	8	0.0497	0.7932	0.1571	0.0001	0.0004	0.9995

Source: Own preparation.

Step 3. Defuzzify fuzzy forecasts.

When it is desired to get the fuzzy forecast for the observation at *t*, firstly the membership degrees of the observation at *t*–1 are obtained from the equation (5) associated with the centers of the fuzzy sets v_r $(r = 1,2,...,c)$ which are defined by the method of FCM clustering. Then, these membership degrees are given as input of FF-ANN and the output is generated. These outputs are actually the membership degrees for the fuzzy forecasting value of the observation at *t*. It should be here noted that the sum of the degrees of membership is not equal to 1, contrary to the FCM method. In the defuzzification stage, the degrees of memberships of the fuzzy forecast are put in Equation 6 given below and are transformed into weights. And finally, the defuzzified forecast of the observation at *t* is calculated as in Equation 7, given below.

$$w_{it} = \frac{\hat{u}_{it}}{\hat{u}_{1t} + \hat{u}_{2t} + \cdots + \hat{u}_{ct}} \tag{6}$$

$$\hat{X}_t = \sum_{i=1}^{c} w_{it} v_i \tag{7}$$

$\hat{u}_{it}$, $i = 1, 2, \cdots, c$ denotes the membership degrees for the fuzzy forecasting value of the observation at t obtained from the output of FF-ANN, w_{it} , $t = 1, 2, \cdots, c$ are weights used for defuzzifying forecasts.

Let’s go back to the sample data set in Table 1. Consider that we have the output of ANN for third observation *(t=3)* like 0.35, 0.61, 0.12. In this case, we obtained

$$w_{13} = \frac{\hat{u}_{13}}{\hat{u}_{13} + \hat{u}_{23} + \hat{u}_{33}} = \frac{0.35}{0.35 + 0.61 + 0.12} = 0.3241$$

$$w_{23} = \frac{\hat{u}_{23}}{\hat{u}_{13} + \hat{u}_{23} + \hat{u}_{33}} = \frac{0.61}{0.35 + 0.61 + 0.12} = 0.5648$$

$$w_{33} = \frac{\hat{u}_{33}}{\hat{u}_{13} + \hat{u}_{23} + \hat{u}_{33}} = \frac{0.12}{0.35 + 0.61 + 0.12} = 0.1111$$

$$\hat{X}_3 = \sum_{i=}^{3} w_{i3} v_i = 0.3251 \times 25.4718 + 0.5648 \times 51.3607 + 0.1111 \times 79.4108$$

4. Application

Fuzzy time series methods have been used in almost all areas such as environmental science, economy and finance. The concepts of labour force participation and unemployment are of great importance in terms of both the economy and sociology of countries. For this reason, there are many studies on the forecasting of these data. In this study, we aim to forecast the labour force participation and unemployment rate in Poland and Turkey using different fuzzy time series methods. The fuzzy time series methods which were used in the application are given below;

Song and Chissom (1993b) : SC93

Chen (1996) : C96

Chen (2002) : C02

Cheng et al. (2008) : C08

Aladag et al. (2009) : A09

Yolcu et al (2013) : Y13

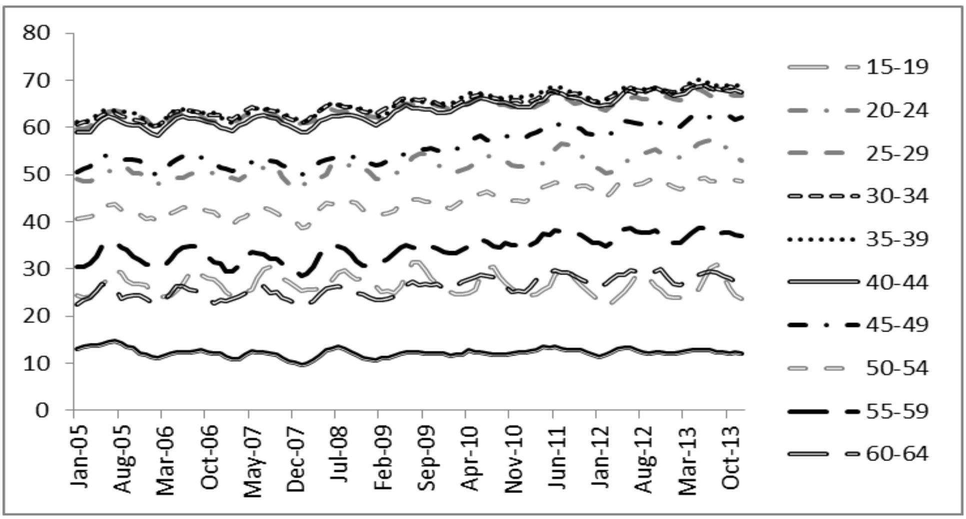
The forecasts obtained from the application were evaluated according to the two different error criteria – the root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE), given in Equations 8 and 9.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}\left(X_t - \hat{X}_t\right)^2}{n}} \tag{8}$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \tag{9}$$

In the application, first of all the Turkey labour force participation rate time series data with 108 observations between January 2005 and December 2013 in 11 different age groups, whose graph in Figure 2 is forecasted. In the analysis, 12 observations of the previous year were given as a test set.

Figure 2. The graph of the labour force participation rate in Turkey



Source: The World Bank (http://data.worldbank.org).

The error criteria with the best results obtained from the analysis are summarized in Tables 4 and 5.

Table 4. The best RMSE values for the labour force participation rate in Turkey

Age	SC93	C96	C02	C08	A09	Y13
15-19	1.4245	1.4245	1.2165	2.8243	0.9394	0.8933
20-24	1.0468	0.7311	1.0219	1.6446	0.9784	0.8339
25-29	0.8366	0.4701	0.5243	0.8399	0.5585	0.5388
30-34	0.5725	0.4409	0.4049	1.0036	0.4082	0.3851
35-39	1.1127	0.6853	0.6724	1.9305	0.6108	0.5746
40-44	0.5765	0.5159	0.5925	0.9062	0.5573	0.5391
45-49	1.1577	0.6807	0.6810	1.5436	0.6649	0.6539
50-54	0.4316	0.4316	0.6625	0.7779	0.4088	0.3736
55-59	0.5557	0.5557	0.7544	1.9594	0.5656	0.5581
60-64	0.6246	0.6495	0.7069	0.9785	0.5392	0.5287
65+	0.2343	0.2343	0.3069	0.4465	0.1994	0.1830

Source: The results were obtained by using MATLAB.

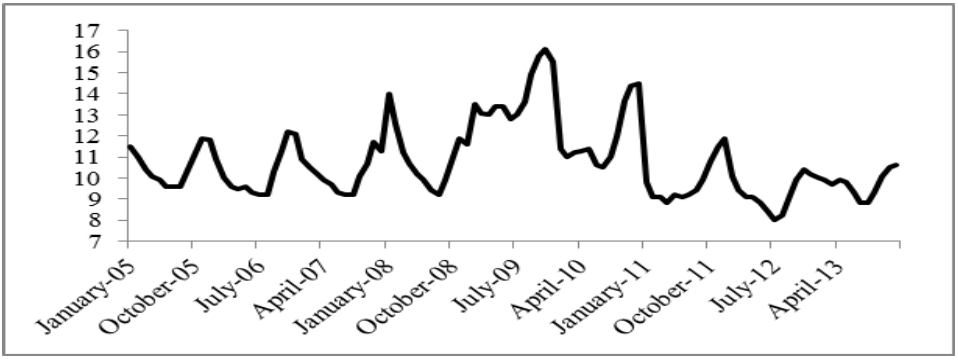
Table 5. The best MAPE values for the labour force participation rate in Turkey

Age	SC93	C96	C02	C08	A09	Y13
15-19	4.26%	4.26%	3.66%	2.82%	2.78%	1.12%
20-24	1.68%	1.14%	1.48%	1.65%	1.50%	1.12%
25-29	0.88%	0.57%	0.64%	1.07%	0.57%	0.64%
30-34	0.77%	0.59%	0.51%	1.31%	0.44%	0.48%
35-39	1.41%	0.72%	0.73%	2.53%	0.67%	0.55%
40-44	0.58%	0.54%	0.70%	1.14%	0.65%	0.64%
45-49	1.63%	0.89%	0.86%	2.29%	0.88%	0.74%
50-54	0.68%	0.68%	1.12%	2.06%	0.75%	0.61%
55-59	1.29%	1.29%	1.63%	4.14%	1.17%	1.25%
60-64	1.90%	1.98%	1.85%	3.19%	1.64%	1.39%
65+	1.29%	1.29%	2.18%	2.68%	1.12%	1.17%

Source: The results were obtained by using MATLAB.

Secondly, the Turkey unemployment rate time series data with 108 observations between January 2005 and December 2013 in 11 different age groups, whose graph is shown in Figure 3, is forecasted. In the analysis, 12 observations of the previous year were given as a test set.

Figure 3. Graph of the unemployment rate in Turkey



Source: The World Bank (http://data.worldbank.org).

The error criteria with the best results obtained from the analysis are summarized in Table 6.

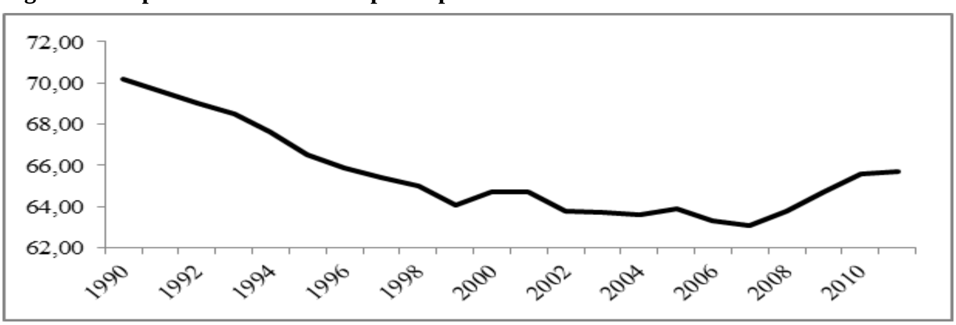
Table 6. The best error criterion values for labour force participation rate in Turkey

Methods	RMSE	MAPE	Methods	RMSE	MAPE
SC93	0.3787	3.37%	C08	0.6483	5.34%
C96	0.3787	3.37%	A09	0.3299	2.63%
C02	0.4318	3.92%	Y13	0.3389	2.74%

Source: The results were obtained by using MATLAB.

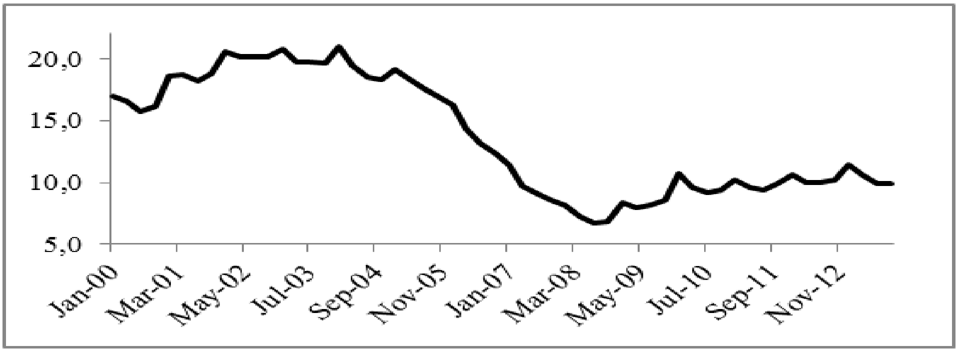
Also in the application, the Poland labour force participation rate time series data, with 22 observations between the years 1990 and 2011, and the Poland unemployment rate time series data observed quarterly between January 2000 and October 2013, the graphs of which are given in Figures 4 and 5 respectively, were analyzed. In the analysis, the last 4 and 12 observations were given as test set.

Figure 4. Graph of the labour force participation rate in Poland



Source: The World Bank (http://data.worldbank.org).

Figure 5. Graph of the unemployment rate in Poland



Source: The World Bank (http://data.worldbank.org).

The error criteria obtained from the best results for these two time series are summarized in Table 7.

Table 7. The best error criterion values for the time series in Poland

	Labour force participation		Unemployment rate	
Methods	**RMSE**	**MAPE**	**RMSE**	**MAPE**
SC93	0.5232	0.52%	0.7937	5.64%
C96	0.5788	0.69%	0.7937	5.64%
C02	0.7025	1.01%	0.5583	3.76%
C08	1.1306	1.51%	0.9781	8.02%
A09	0.468	0.58%	0.4539	3.49%
Y13	0.4058	0.49%	0.4365	3.63%

Source: The results were obtained by using MATLAB.

When all tables of the application stage were examined, it can be said that Y13 method is the best method among all methods used in this paper. This means that the Y13 method gives the best forecasting results, with a smaller error for the estimation of the labour force participation and unemployment rate of Turkey and Poland for these data sets, when compared with some methods in the literature for use in forecasting.

5. Conclusions and Discussion

Fuzzy time series procedures, which have attracted the attention of many researchers in recent years, have quite a wide area of use, such as information technology, economy, environmental sciences and hydrology. Most of the real time series data can be evaluated by fuzzy time series approaches since the uncertainty in these time series can be considered as uncertainty defined in fuzziness.

The unemployment rate and labour force participation rate are the one of the most important criterion of a country's development. Estimating these rates accurately and properly is very important in terms of taking anticipatory measures. The estimation of these rates also give direction to the economic development of the countries and shed light on the precautions to be taken for the country's economy.

In this study, we aimed to forecast the labour force participation and unemployment rate in Poland and Turkey with different fuzzy time series methods.

As a result of all the analyses, it can be said that fuzzy time series forecasting models have superior forecasting performance in the forecasting of time series such as the unemployment rate and labour force participation rate and can be used as an effective predictive tool for this type of time series.

It can be also said that an important conclusion of this paper is that when fuzzy time series methods are improved, they will give even better forecasting results. Then problems such as the estimation of unemployment rate and labour force participation rate, which have an important place in the national economy, can be predicted more reliably. Thus, some precautions can be taken to overcome these kinds of problems in the upcoming years, which will contribute to the economic development of countries.

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Streszczenie

PROGNOZOWANIE AKTYWNOŚCI ZAWODOWEJ I STOPY BEZROBOCIA W POLSCE I TURCJI PRZY UŻYCIU METODY ROZMYTYCH SZEREGÓW CZASOWYCH

Metody rozmytych szeregów czasowych oparte na teorii zbiorów rozmytych zaproponowanej przez Zadeh (1965) zostały użyte po raz pierwszy w badaniach Song i Chissom (1993). Od tego czasu przy wykorzystaniu metod rozmytych szeregów nie obowiązują założenia wymagane dla tradycyjnych szeregów czasowych. Szeregi rozmyte stanowią jednak skuteczne narzędzie prognozowania, a zainteresowanie nimi jest coraz większe. Stosowane są w niemal wszystkich dziedzinach naukowych, takich jak ochrona środowiska, finanse i ekonomia. Szczególne znaczenie w obszarze ekonomii i socjologii mają zjawiska aktywności zawodowej i bezrobocia. Z tego powodu istnieje wiele badań z zakresu ich prognozowania. W niniejszym artykule wykorzystano właśnie różne metody rozmytych szeregów czasowych dla sporządzenia prognozy aktywności zawodowej i stopy bezrobocia w Polsce i Turcji.

***Słowa kluczowe**: rozmyte szeregi czasowe, prognozowanie, aktywność zawodowa, bezrobocie*