

The Bayesian Method in Estimating Polish and German Industry Betas. A Comparative Analysis of the Risk between the Main Economic Sectors from 2001–2020

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Abstract

This paper examines the long-term dependence between the Polish and German stock markets in terms of industry beta risk estimates according to the Capital Asset Pricing Model (CAPM). The main objective of this research is to compare the Polish and German beta parameters of five Polish and three German sector indices using the Bayesian methodology in the period 2001–2020. The study has two detailed aims. First, to develop a modified, Bayesian approach (SBETA model) that generates significantly more precise beta than the traditional model. Second, to compare the results of different time-varying industry betas in the Polish and German economies, giving a simple investment recommendation, i.e., which sector could be classified as aggressive or defensive.

The betas were time-varying in both markets but less persistent in the German industries, which seems characteristic of an advanced economy. The Banking sector betas were the highest in both markets, implying the aggressive nature of that industry in the last twenty years. For the Polish market industry, the betas of Construction, IT, Food and Drinks, and Telecom were classified as defensive. For the German economy, the Technologies (IT) sector was also classified as aggressive, but Telecom

was defensive. The results give a valuable insight into the systematic risk levels in Poland and Germany, reflecting the investors' learning process and indicating that Polish Banking and German technologies outperformed the market in the last twenty years.

Keywords: industry beta, CAPM, Bayesian method, Markov Chain Monte Carlo, Polish Stock Market, German Stock Market

JEL: C11, G10, G11, G15

Introduction

In finance, risk estimation is crucial for all market practitioners when investing in the stock market and researchers dealing with portfolio theory. Estimating risk is taken into consideration because a stock's profitability goes together with the appropriate risk level. The estimates of the beta parameter have various applications in finance, but they are extremely important both for portfolio managers, to build investment strategies, and individual investors. From a portfolio management perspective, risk estimation based on the beta parameter is crucial for building effective hedging strategies, asset pricing, and performance assessment. Various methods of risk estimation have been carried out in highly developed economies, mostly the US, as well as emerging markets in Europe and Asia.

The main aim of this study is to estimate the industry beta coefficient of five Polish and three German sectors over a long period in different economic conditions. The research compares these countries' stock markets in terms of industry beta risk estimates. The German market has an influential role in emerging European stock market movements due to their significant investment flows in Central and Eastern Europe. Banking, Construction, IT, Food and Drinks, and Telecom represent the main divisions of the Polish economy, and for Germany, it is Banking, Technologies (IT), and Telecom. The detailed aims of the study are: 1) to develop a modified Bayesian approach to estimate a beta parameter that is variable in time and that generates significantly more appropriate estimates and 2) to compare the results of different time-varying industry betas in the Polish and German economies and give a simple investment recommendation of which sector could be classified as aggressive or defensive over a long period, regardless of the economic cycle. We believe that our study demonstrates the usefulness of the beta coefficient as a proper measure of risk when building investment strategies on international markets. The Bayesian method of beta estimation reflects investors' learning process, and, therefore, our industry betas are highly precise relative to the competing traditional method.

This paper is organized as follows. Section 2 presents the literature review and the current state of research. Section 3 describes the research sample. Section 4 develops the model. Section 5 describes the results, and Section 6 concludes the paper. The Appendix provides details.

Literature review

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) describes the relationship between the expected return and risk of investing in a security. The Security Market Line (SML) represents the relationship between risk (beta parameter) and expected return; therefore, the beta can be viewed as a measure of systematic risk. According to CAPM, the beta should be the only source of excess returns in market equilibrium. Investors using beta assume that it is invariant over time. However, it might be unstable for many reasons, for example, a change in the company's strategy or capital structure, or microeconomic factors such as a change in its dividend policy or financial leverage. Such instances may influence beta stability over time. As a consequence, beta variability over time has been empirically examined worldwide, e.g., the American market (Fabozzi and Francis 1978), the European market (Wells 1994; Chaveau and Maillet 1998), Central and Eastern Europe (Dębski, Feder-Sempach, Szczepocki 2020), and the Polish market (Dębski, Feder-Sempach, and Świdorski 2015, 2016; Wdowiński 2004).

An influential study was carried out on US industry portfolios by Jostova and Philipov (2005), who analyzed a mean-reverting stochastic process for beta estimation. They demonstrated that stochastic beta (SBETA) estimates outperform competing beta estimates and produce superior beta forecasts. Stochastic beta estimates reduce hedging errors by up to 30% compared to OLS, and they provide much stronger support for the conditional CAPM. They can also be useful when dealing with asset pricing anomalies such as the size, book-to-market, and idiosyncratic volatility effects in a cross-section of stock returns.

Some research has also been conducted on the German stock market. Ebner and Neumann (2005) estimated the time-varying betas of 48 stock returns, rejecting the traditional market model with strong evidence of beta instability. Lewellen and Nagel (2006) studied time-variation in risk and expected returns by using Conditional CAPM. They showed that betas vary considerably over time, with relatively high-frequency changes from year to year, but not enough to generate significant unconditional pricing errors. It suggests that conditional CAPM performs nearly as poorly as the unconditional CAPM.

Another interesting study was done on European stock markets by Mergner and Bulla (2008), who investigated the time-varying betas of 18 pan-European sectors in the period 1987–2005 using weekly data. The outcome of the study showed that the random walk process in connection with the Kalman filter was the best at describing and forecasting the time-varying industry systematic risk in a European market.

Central and Eastern Europe industry stock exchange data were commonly used to describe the behavior of CAPM beta to estimate the systematic risk of selected industries. Kurach and Stelmach (2014) studied five industry indices from the Warsaw Stock

Exchange and demonstrated that using the Kalman filter for beta estimation better shows an image of changing expectations about systematic risk.

Another study of the time-varying systematic risk of beta was made by Będowska-Sójka (2017), who investigated various approaches to assessing systematic risk on the Polish capital market from 2001–2015 using data from the Banking and IT sectors. The beta was estimated in a few different approaches: two Multivariate GARCH model specifications, Dynamic Conditional Correlation (DCC) (Engle 2002) and BEKK (Engle and Kroner 1995) models, the Kalman filter technique, and the estimates from linear regression models. The results showed that unobserved component beta together with beta from the DCC model have higher predictive accuracy than beta from the BEKK model or static beta. What was really interesting was the positive correlation of beta within the industry and the negative correlation of beta for stocks from different sectors. Będowska-Sójka stated that beta coefficients were more accurate for stocks from the banking sector than the IT sector.

Lastly, Cepeda-Cuervo et al. (2016) showed how to fit beta regression models applying the Bayesian method, while French (2016) compared time-varying betas and a traditional constant beta model for five ASEAN countries and US sectors. Similar research was conducted by Tsuji (2017) on international CAPM time-varying betas from the Asia Pacific and Japanese stock markets, and by Elshqirat and Sharifzadeh (2018) on the Jordanian Stock Exchange. Recent research with the Bayesian method was presented by Phuoc and Pham (2020), who indicated that CAPM using the non-parametric Bayes estimator is superior to the parametric Bayes estimator; consequently, the non-parametric estimator should be used in the asset pricing model.

Research sample

We apply our framework to Polish and German industry portfolios in a long run of 20 years. The research sample consists of five industry indices listed on the Warsaw Stock Exchange: WIG-Banking, WIG-Construction, WIG-IT, WIG-Food and Drinks, and WIG-Telecom, and three industry indices listed on German Exchanges: PRM-Banking, PRM-Technologies (IT), and PRM-Telecom, which represent the main divisions of the economy (if the industry index did not have enough 20-year time series data, we dropped it). As a risk-free rate, we took one-year Polish and German treasury bills yield, as it is consistent with the CAPM (Sharpe 1964; Lintner 1965; Mossin 1966; Fama and French 1993). We collected the closing prices of eight industry indices from the Polish and German markets. The WIG and DAX COMPOSIT indices were used as a proxy. The industry indices were collected beginning from the first trading day of January 2001 to the first trading day of January 2021, which gives 1044 weekly observations. We took all the industry indices with full time series available in the Refinitiv EIKON database. No data were interpolated, and the stock exchange time series were complete. The Polish and German T-bills yields were collected

beginning from December 2001 for the Polish market and May 2003 for the German, using the database. Missing time series data were interpolated.

Based on these prices, the logarithmic weekly rates of return were calculated as $R_{it} = \ln P_{it} - \ln P_{it-1}$, where R_{it} is the logarithmic rate of return on the i -th index at time t , and P_{it} is the price of the i -th index at time t . The rates of return on industry indices were calculated without dividends. For each industry, we received 1043 observations of return in the period 2001–2020 (20 years). Data were obtained from the Refinitiv EIKON database, and all tables are labeled with the RIC (Reuters Instrument Code). They formed the basis for the present research. Figure 1 presents the time series of the WIG and DAX index.



Figure 1. Time series of WIG and DAX index, 2001–2020

Source: own elaboration based on Refinitiv EIKON data.

The model and estimation method

We estimated CAPM with time-varying market risk premia, using the model based on the SBETA proposed by Jostova and Philipov (2005). The stochastic beta approach requires the model to be written in the form of two equations – the empirical market model and one that describes the dynamics of the time-varying beta. For this paper, we consider the following version of the market model

$$R_{it} - R_{ft} = \alpha + \beta_t (R_{Mt} - R_{ft}) + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2) \quad (1)$$

where R_{it} is the logarithmic return on portfolio i , R_{Mt} is the logarithmic return on the market portfolio, and R_{ft} is the logarithmic return on the risk-free asset. Here, α is the proportion of the excess return on considered portfolio¹ i that is left unexplained by the portfolio's systematic exposure, β_t is sensitivity to movements of market excess returns, and σ_ε^2 is the variance of the market model error term. Although α , β_t , and σ_ε^2 should have i subscripts to denote that they differ across portfolios, we omit them for ease of notation.

The form of the second equation depends on the form of the stochastic process that the beta parameter is assumed to follow. There are a few propositions: random walk (Faff et al. 2000; Ebner and Neumann 2005; Das and Ghoshal 2010; Będowska-Sójka 2017), autoregressive model of order 1 with the mean reversion (Faff et al. 2000; Jostova and Philipov 2005; Kurach and Stelmach 2014), the autoregressive-moving-average (ARMA) model (Yao and Gao 2004), and random coefficient (Faff et al. 2000; Ebner and Neumann 2005). The most popular specification is the random walk of the beta parameter. However, the random walk assumption means that the beta parameter is not stationary, which seems contrary to the fact that portfolio excess returns are a linear combination of parameters and market excess returns (equation (1)), and empirical research shows that both portfolio and market returns are typically stationary (see, for example, Dębski et al. (2018)). Jostova and Philipov (2005) also present an economic argument for the mean reversion of beta: the theoretical models of the business life cycle of Berk et al. (1999) and Gomes et al. (2003) imply that “beta reverts to its long-term mean over a period of time consistent with the business cycle.” They support their claim with the empirical findings of Zhang (2005) and Petkova and Zhang (2005). We decided to follow the arguments of Jostova and Philipov (2005) and use the autoregressive process with the mean reversion of the form

$$\beta_t = \mu + \phi(\beta_{t-1} - \mu) + \eta_{it}, \quad \eta_{it} \sim N(0, \sigma_\eta^2) \quad (2)$$

where: μ is a long-term level of beta, ϕ is a persistence parameter, and σ_η^2 is the variance of the conditional beta. We assume that $|\phi| < 1$ to ensure stationarity of the beta process. Again, μ , ϕ , σ_η^2 should all have i subscripts, but we omit them for simplicity. All three parameters of the mean-reverting autoregressive process can be used to obtain the unconditional mean and variance of the beta (Jostova and Philipov 2005)

$$E(\beta_t) = \frac{\mu(1-\phi)}{1-\phi} = \mu, \quad (3)$$

1 Our study assumes that $\alpha \neq 0$, in contrast to the model of Jostova and Philipov (2005), who examined the US market.

$$Var(\beta_t) = \frac{\sigma_\eta^2}{1 - \phi^2}. \quad (4)$$

This formulation of the data generating process for the beta parameter also has the theoretical advantage of being the generalization of many other processes. When the persistence parameter ϕ is equal to zero, one obtains a beta process with no persistence and perfect mean reversion. This is the random coefficient assumption. By contrast, when μ is equal to zero, the beta-generating process simply becomes an autoregressive process of order 1. Additionally, in the limiting case of $\phi \rightarrow 1$, the autoregressive process becomes a random walk.

Based on equation (2), the market model may be rewritten in the form of a conditional normal distribution

$$R_{it} - R_{ft} | R_{Mt} - R_{ft} \sim N\left(\alpha + \beta_t (R_{Mt} - R_{ft}), \sigma_\varepsilon^2\right) \quad (5)$$

The beta-generating process (equations (3)) may also be rewritten in the form of a conditional normal distribution

$$\beta_t | \beta_{t-1} \sim N\left(\mu + \phi(\beta_{t-1} - \mu), \sigma_\eta^2\right) \quad (6)$$

Therefore, the likelihood of the model may be written as

$$L\left(\alpha, \mu, \phi, \sigma_\varepsilon^2, \sigma_\eta^2 | R_{it}, R_{Mt}, R_{ft}\right) \sim \prod_{t=1}^T N\left(\alpha + \beta_t (R_{Mt} - R_{ft}), \sigma_\varepsilon^2\right) \\ \times \prod_{t=1}^T N\left(\mu + \phi(\beta_{t-1} - \mu), \sigma_\eta^2\right) \quad (7)$$

The joint posterior distribution of the parameter set $\theta = (\alpha, \mu, \phi, \sigma_\varepsilon^2, \sigma_\eta^2)$ is, by the Bayes theorem, proportional to the product of the joint prior distribution and the likelihood function

$$p(\theta | R_{it}, R_{Mt}, R_{ft}) \propto p(\theta) L(\theta | R_{it}, R_{Mt}, R_{ft}) \quad (8)$$

Regarding the prior distribution $p(\theta)$, we assume prior independence of all parameters. For the α parameter, we use a normal prior with mean 0 and standard deviation 10. Typically, the value of parameter α is close to 0. For the long-term mean parameter μ , we use a normal prior with mean 1 and standard deviation 10. The value of 1 is often referred to as the “grand mean of all betas” (Blume 1976). We set $\phi = 2\tilde{\phi} - 1$ and specify a Beta (20,1.5) prior for $\tilde{\phi}$, which has a mean of 0.93 and a standard deviation of 0.055. For the parameters σ_ε and σ_η , we use Inverse-Gamma (2.5,0.025), which has a mean of 0.167 and a standard deviation of 0.024. The priors follow typical specifications of basic stochastic volatility models with a mean re-

version of log-volatility (see Kim et al. 1998; Yu 2004) and are slightly different from the priors used by Jostova and Philipov (2005).

We use the open-source Bayesian software STAN, which interfaces with the R by RStan package (Stan Development Team 2020). STAN implements an efficient estimation of posterior probability using the No-U-Turn Sampler (NUTS). This sampler is the Markov chain Monte Carlo (MCMC) algorithm that avoids random walk behavior, and thus it is less prone to the correlation of parameters that impede many MCMC methods. NUTS is an extension of the Hamiltonian Monte Carlo (HMC) algorithm proposed by Hoffman and Gelman (2014). Two other popular open-source Bayesian programs, BUGS and JAGS, perform MCMC, updating one scalar parameter at a time. STAN moves in the entire space of all the parameters using NUTS, thus avoiding some difficulties that occur with one dimension at a time sampling in high dimensions. In all the presented results, we used a burn-in period of 5,000 iterations and a follow-up period of 20,000 iterations.

Results

The results show empirical evidence of the behavior of industry betas for the five Polish and three German industrial sectors. We evaluate the degree of time variation, persistence, and volatility exhibited by industry beta parameters over a twenty-year period (2001–2020, weekly data). The long time series of data suggests that betas respond to macroeconomic variables of the Polish and German economies and global risk factors arriving randomly on the international financial market. It means that the stochastic process of beta is implied, and our research captures a wide variety of economic events in the 20 years.

Polish industry betas

Table 1 summarizes the estimation results for the SBETA parameters for each Polish industry. i.e., Banking, Construction, IT, Food and Drinks, and Telecom.

The Banking sector has average systematic risk, a long-term beta μ level of 1.060, and a persistence parameter ϕ of 0.939. It means that the Banking sector in Poland can be classified as aggressive, and with a beta greater than 1, it tends to be more volatile than the whole Polish stock market. Compared to other Polish sectors, Banking should be chosen by investors who are willing to take on more risk or when an upward trend is forecast in the long run. The persistence parameter of 0.939 is close to unity, meaning that the Banking beta does not revert to a long-term mean for a long period. It can be affected by frequent and dynamic events, such as changing economic conditions or company structures. Additionally, the Banking industry in Poland has a positive alpha parameter, i.e., 0.152, indicating a higher risk-adjusted return. It means that this sector outperformed the market.

The Construction sector has average systematic risk, a long-term beta μ level of 0.910, and a persistence parameter ϕ of 0.986. It means that the Construction sector can be classified as defensive; with a beta less than 1 but more than 0, it tends to be less volatile than the market. This sector should be chosen by investors who are more risk-averse. The persistence parameter of 0.986 is also close to unity, so it means that the Construction beta is highly persistent (the strength of the long-term mean reversion is very weak).

The IT sector has average systematic risk, a long-term beta μ level of 0.916, and a persistence parameter ϕ of 0.386. The IT sector can also be classified as defensive, with a beta smaller than 1; however, the persistence is low, at 0.386, so the strength of long-term mean reversion is high.

The Food and Drinks sector has average systematic risk, a long-term beta μ level of 0.909, and a persistence parameter ϕ of 0.988. It means that this sector can be classified as defensive and less volatile than the market. The very high persistence parameter of 0.988 also means that the Food and Drinks beta is highly persistent.

The last sector, Telecom, has average systematic risk, a long-term beta μ level of 0.996, and a persistence parameter ϕ of 0.975; therefore, it is classified as defensive, less volatile than the market, and the beta is highly persistent.

Looking at Table 1 and Figure 2, one can compare the volatility of beta based on the σ_η parameter. The highest value is observed in the IT sector, which has a more jagged line. Four out of the five sectors have smooth graphs. There is also a clearly visible difference between WIG.IT, with low persistence, and the rest of sector which has higher persistence. After shocks, the beta of WIG.IT quickly returns to the long-term mean, whereas the betas of the other sectors persist far from the long-term mean for quite a long time.

Table 1. Summary of the posterior distribution for the model parameters – Polish industries

	Parameter	Mean	Standard error of the mean	Standard deviation	Quantiles				
					5%	25%	50%	75%	95%
WIG.BANKS	α	0.152	0.001	0.063	0.048	0.109	0.152	0.194	0.254
	ϕ	0.939	0.003	0.026	0.840	0.926	0.953	0.971	0.988
	μ	1.060	0.001	0.032	1.001	1.045	1.064	1.081	1.104
	σ_ϵ	1.641	0.002	0.039	1.577	1.614	1.640	1.667	1.705
	σ_η	0.035	0.002	0.014	0.020	0.026	0.032	0.040	0.062
WIG.CON- STRUCTION	α	-0.177	0.001	0.076	-0.303	-0.229	-0.178	-0.126	-0.053
	ϕ	0.986	0.001	0.005	0.965	0.982	0.989	0.993	0.997
	μ	0.910	0.000	0.043	0.841	0.881	0.908	0.937	0.983
	σ_ϵ	2.432	0.001	0.054	2.344	2.395	2.431	2.467	2.523
	σ_η	0.024	0.001	0.008	0.013	0.018	0.023	0.029	0.040

	Parameter	Mean	Standard error of the mean	Standard deviation	Quantiles				
					5%	25%	50%	75%	95%
WIG.IT	α	-0.179	0.001	0.070	-0.293	-0.227	-0.179	-0.132	-0.064
	ϕ	0.386	0.003	0.062	0.189	0.301	0.383	0.469	0.594
	μ	0.916	0.000	0.023	0.878	0.901	0.916	0.931	0.953
	σ_ε	2.030	0.002	0.067	1.923	1.984	2.029	2.074	2.142
	σ_η	0.217	0.002	0.026	0.172	0.200	0.218	0.234	0.257
WIG.FOOD & DRINKS	α	-0.310	0.001	0.077	-0.436	-0.363	-0.311	-0.258	-0.182
	ϕ	0.988	0.001	0.006	0.967	0.985	0.992	0.996	0.999
	μ	0.909	0.001	0.054	0.828	0.870	0.905	0.944	1.002
	σ_ε	2.542	0.000	0.057	2.450	2.503	2.541	2.579	2.639
	σ_η	0.024	0.001	0.008	0.014	0.018	0.022	0.027	0.038
WIG.TELECOM	α	-0.151	0.001	0.082	-0.285	-0.205	-0.151	-0.098	-0.015
	ϕ	0.975	0.001	0.010	0.938	0.966	0.979	0.989	0.996
	μ	0.996	0.003	0.053	0.919	0.959	0.991	1.029	1.091
	σ_ε	3.030	0.001	0.068	2.920	2.985	3.029	3.075	3.143
	σ_η	0.033	0.002	0.013	0.017	0.024	0.031	0.040	0.058

Source: own elaboration.

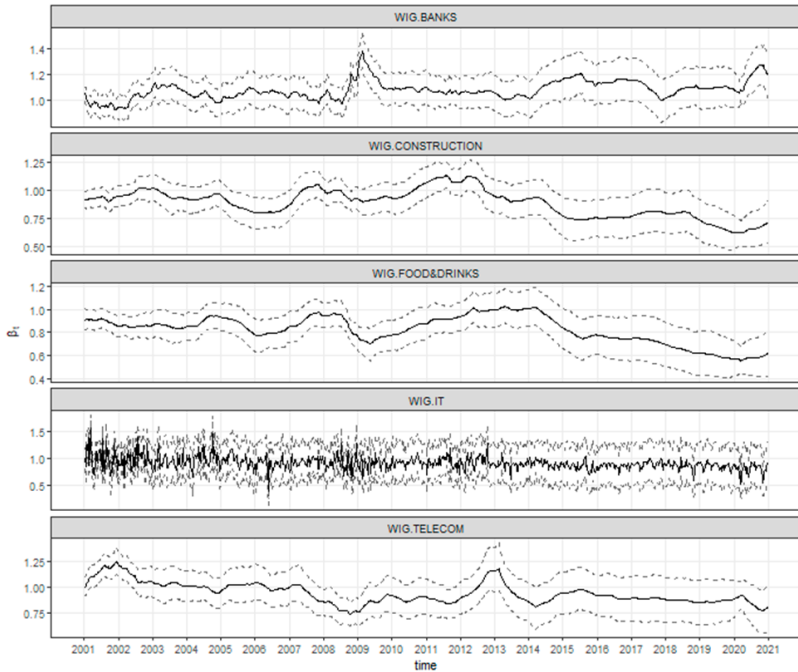


Figure 2. The posterior mean of β_{it} (thick line) with two thin dashed lines representing the 90% Bayesian posterior interval. Polish industries, weekly data from January 2001 to December 2020
 Source: own elaboration based on Refinitiv EIKON data.

German industry betas

Table 2 summarizes the estimation results for the SBETA parameters for each German industry – Banking, Technologies (IT), and Telecom.

Table 2. Summary of the posterior distribution for the model parameters – German industries

Index	Parameter	Mean	Standard error of the mean	Standard deviation	Quantiles				
					5%	25%	50%	75%	95%
PRM.BANKS	α	-0.080	0.000	0.071	-0.196	-0.128	-0.080	-0.032	0.038
	ϕ	0.452	0.002	0.044	0.306	0.392	0.453	0.513	0.596
	μ	1.225	0.000	0.040	1.158	1.198	1.226	1.252	1.290
	σ_ε	2.801	0.002	0.086	2.662	2.742	2.799	2.858	2.944
	σ_η	0.472	0.002	0.050	0.389	0.437	0.472	0.506	0.555
PRM.TECHN	α	0.112	0.000	0.071	-0.005	0.064	0.112	0.161	0.230
	ϕ	0.605	0.013	0.084	0.295	0.499	0.625	0.726	0.856
	μ	1.221	0.001	0.037	1.160	1.197	1.222	1.246	1.281
	σ_ε	2.975	0.009	0.090	2.829	2.914	2.972	3.035	3.123
	σ_η	0.243	0.013	0.074	0.110	0.197	0.247	0.292	0.362
PRM.TELE-COM	α	-0.091	0.000	0.066	-0.198	-0.136	-0.091	-0.046	0.017
	ϕ	0.561	0.002	0.042	0.418	0.507	0.564	0.618	0.693
	μ	0.868	0.000	0.038	0.806	0.843	0.868	0.894	0.931
	σ_ε	2.401	0.002	0.072	2.284	2.351	2.400	2.449	2.520
	σ_η	0.393	0.003	0.044	0.318	0.364	0.393	0.422	0.465

Source: own elaboration.

The Banking sector has average systematic risk, a long-term beta μ of 1.225, and a persistence parameter ϕ of 0.452. It means that the Banking sector in Germany can also be classified as aggressive. It tends to be more volatile than the whole German stock market; therefore, it should be interesting to investors who take on more risk. The persistence parameter of 0.452 means that the strength of long-term mean reversion is high.

The Technologies (IT) sector has average systematic risk, a long-term beta μ level of 1.221, and a persistence parameter ϕ of 0.605. It is also classified as aggressive for low-risk averse investors, with average persistence. Similar to the Banking industry in Poland, German Technologies has a positive alpha parameter of 0.112, indicating a higher risk-adjusted return. It means that this sector outperformed the market.

The last analyzed German industry is Telecom. With a long-term beta μ of 0.868 and a persistence parameter ϕ of 0.561, the Telecom sector can be classified as defensive and less volatile than the market, and with moderate persistence.

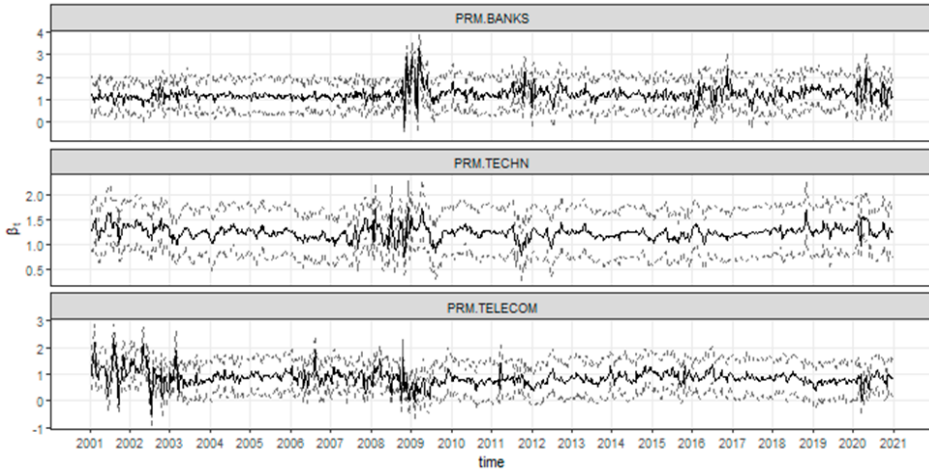


Figure 3. The posterior mean of β_{it} (thick line) with two thin dashed lines representing the 90% Bayesian posterior interval. The German industries, weekly data from January 2001 to December 2020
 Source: own elaboration based on Refinitiv EIKON data.

Looking at Table 2 and Figure 3, one can compare the volatility of beta based on the σ_{η} parameter. The highest value is observed for the Banking sector, which has a more jagged line, but the differences are small, and all lines look different than for Poland. According to the persistence parameter, the beta of the Banking sector has the highest strength of mean-reversion. However, all sectors are less persistent than the Polish sectors (with the exception of WIG.IT)

Conclusions

The overall research objective was to examine the long-term dependence between the Polish and German stock markets in terms of industry risk estimates according to CAPM. The main aim of this article was to compare industry beta parameters of five Polish and three German sector indices using Jostova and Philipov's (2005) Bayesian methodology SBETA model in a long run. The novelty, in terms of Bayesian modeling, lies in the use of the No-U-Turn Sampler (NUTS) via open-source Bayesian software STAN. This Markov chain Monte Carlo sampler is very efficient because it is able to produce high-dimensional proposals that are accepted with high probability. The empirical investigation of Polish and German industries over the past 20 years provides evidence that betas are time-varying in both markets but less persistent in the German industries. Only one of the five Polish sectors had a beta with low persistence that quickly reverted to the long-term mean. In general, the German industry betas

showed a lower level of persistence in comparison with the Polish ones, which is usually observable in more advanced markets.

Comparing the level of risk, the Banking sector betas were the highest in both the Polish and German markets, implying the aggressive nature of that industry in the last twenty years. For the Polish market, the industry betas of the Construction, IT, Food and Drinks, and Telecom sectors were classified as defensive, but with betas more than 0.9 but less than 1, close to the market index. Looking at the German economy, the Technologies (IT) sector was also classified as aggressive, but Telecom was defensive. Comparing the volatility of industry betas in both markets, it is visible that German industries were more volatile, and the graphs of the beta parameters were more jagged than the Polish ones.

When comparing the alpha levels, the banking sector in Poland and Technologies (IT) in Germany outperformed the market by having positive results. This finding can be explained by the idiosyncrasy of the Polish and German economies. In the last twenty years, banking has been seen as a leading sector in Poland, with a high influx of foreign capital, while Germany is the world leader in technology and innovation.

To conclude, we showed that this modeling approach has several advantages. The first advantage of the SBETA model is its ability to separate and measure the beta characteristics in the long run and ensure that investors get highly desired investment recommendations based on a solid methodology. The comparison of different time-varying industry betas in the Polish and German economies gives some insights into the level of systematic risk and how it is perceived by investors. These findings indicate that the application of the SBETA model is associated with research impact and added value to international investors and portfolio managers.

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Oszacowaniach polskich i niemieckich współczynników beta z użyciem metody bayesowskiej – porównanie dla głównych indeksów sektorowych w latach 2001–2020

Celem artykułu jest porównanie długookresowych zależności w poziomie branżowego ryzyka systematycznego, mierzonego współczynnikiem beta, na polskim i niemieckim rynku giełdowym. Poziom ryzyka został oszacowany dla pięciu sektorów polskich i trzech niemieckich na podstawie modelu CAPM z wykorzystaniem metody bayesowskiej w okresie 2001–2020. Cele szczegółowe artykułu to rozwinięcie i udoskonalenie nowego podejścia bayesowskiego (model SBETA) do szacowania poziomu ryzyka i porównanie wielkości współczynnika beta zmiennego w czasie na obu rynkach wraz z prostą rekomendacją inwestycyjną, tj. sektor agresywny lub defensywny.

Wyniki wskazują, że współczynniki beta niemieckich sektorów miały niższy poziom persystencji, co jest charakterystyczne dla rynków rozwiniętych. Sektor bankowy okazał się najbardziej agresywny, najwyższy poziom bety, zarówno na polskim i niemieckim rynku giełdowym. Polskie indeksy sektorowe budownictwo, IT, artykuły spożywcze i telekomunikacja zostały zakwalifikowane do defensywnych. Niemieckie indeksy, Technologiczny (IT) został zakwalifikowany do agresywnych ale telekomunikacja do defensywnych. Na podstawie obliczeń wskazano, że polski sektor bankowy i niemiecki technologiczny przyniosły wyższe dochody niż cały rynek w analizowanym okresie. Wyniki mają bardzo duże znaczenie dla oceny poziomu ryzyka systematycznego na polskiej i niemieckiej giełdzie papierów wartościowych i dają jasne rekomendacje inwestorom międzynarodowym.

Słowa kluczowe: beta sektorowa, CAPM, metoda bayesowska, symulacja Monte Carlo, Polska, Niemcy



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